9.02.25

Determinant

$$\begin{vmatrix} 2 & 3 \\ 4 & 5 \end{vmatrix}$$

element

C

R

2×2 (order) $= 2^{nd}$ order

Raw column

$$\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & -6 \\ 7 & 8 & 9 \end{vmatrix}$$

$3 \times 3 = 3^{rd}$ order

$$\begin{vmatrix} a_1 & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

$a_{11} = 0, \pm 1, \pm 2, \ldots$

R C

a_{22}

R C

Position of the element.

Q1. Find the value of

$$\begin{vmatrix} 5 & 6 \\ 9 & -8 \end{vmatrix}$$

$\Rightarrow \begin{vmatrix} 5 & 6 \\ 9 & -8 \end{vmatrix} = 5 \times (-8) - 9 \times 6$

$= -40 - 54$

$= -94$.

Q2. Find the value of

$$\begin{vmatrix} 1 & 3 & 5 \\ 2 & 3 & 4 \\ 1 & 3 & 5 \end{vmatrix}$$

$$= \cancel{1 \begin{vmatrix} 3 & 4 \\ 3 & 5 \end{vmatrix}}$$

$$= \begin{vmatrix} 1 & 3 & 5 \\ 2 & 3 & 4 \\ 1 & 3 & 5 \end{vmatrix}$$

$$= 1 \begin{vmatrix} 3 & 4 \\ 3 & 5 \end{vmatrix} - 3 \begin{vmatrix} 2 & 4 \\ 1 & 5 \end{vmatrix} + 5 \begin{vmatrix} 2 & 3 \\ 1 & 3 \end{vmatrix}$$

$$= 1(15-12) - 3(10-4) + 5(6-3)$$

$$= 1 \times 3 - 3 \times 6 + 5 \times 3$$

$$= 3 - 18 + 15$$

$$= 3 - 3$$

$$= 0$$ Ans.

Q3. Find the value of the determinant.

$$\begin{vmatrix} 2 & 3 & -1 \\ 1 & 0 & 2 \\ 1 & 2 & 3 \end{vmatrix}$$

$$= \begin{vmatrix} 2 & 3 & -1 \\ 1 & 0 & 2 \\ 1 & 2 & 3 \end{vmatrix}$$

$$= 2\begin{vmatrix} 0 & 2 \\ 2 & 3 \end{vmatrix} - 3\begin{vmatrix} 1 & 2 \\ 1 & 3 \end{vmatrix} - 1\begin{vmatrix} 1 & 0 \\ 1 & 2 \end{vmatrix}$$

$$= 2[0-4] - 3[3-2] - 1[2-0]$$

$$\cancel{= 2 \times -0 - 3 \times (+1) \times -1 \times 2}$$

$$\cancel{= -0 - 3 - 2}$$

$$\cancel{= -0 - 5}$$

$$\cancel{= -5}$$

$$= -8 - 3 - 2$$

$$= -11 - 2$$

$$= -13 \text{ Ans}$$

Q4. find the value of $\begin{vmatrix} 1 & 1 & 1 \\ 4 & 6 & 9 \\ 5 & 6 & 7 \end{vmatrix}$

$$= \begin{vmatrix} 1 & 1 & 1 \\ 4 & 6 & 9 \\ 5 & 6 & 7 \end{vmatrix}$$

$$= 1\begin{vmatrix} 6 & 9 \\ 6 & 7 \end{vmatrix} - 1\begin{vmatrix} 4 & 9 \\ 5 & 7 \end{vmatrix} + 1\begin{vmatrix} 4 & 6 \\ 5 & 6 \end{vmatrix}$$

$$\begin{array}{r} 54 \\ -42 \\ \hline 12 \end{array} \qquad \begin{array}{r} 45 \\ -28 \\ \hline 17 \end{array} \qquad 30$$

$$= 1(42-54) - 1(28-45) + 1(24-30)$$

$$= 1(-12) - 1(-17) + 1(-6)$$

$$= -12 + 17 - 6$$

$$= -12 + 11$$

$$= -1 \text{ Ans}$$

Q5. Find $\begin{vmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & \omega \\ \omega^2 & 1 & 1 \end{vmatrix}$ where ω is a complex root of unity

$$= \begin{vmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & \omega \\ \omega^2 & 1 & 1 \end{vmatrix}$$

$$= 1\begin{vmatrix}\omega^2 & \omega\\ 1 & 1\end{vmatrix} - \omega\begin{vmatrix}\omega & \omega\\ \omega^2 & 1\end{vmatrix} + \omega^2\begin{vmatrix}\omega & \omega^2\\ \omega^2 & 1\end{vmatrix}$$

$$= 1(\omega^2-\omega) - \omega(\omega-\omega^3) + \omega^2(\omega-\omega^4)$$

$$= 1(\omega^2-\omega) - \omega(\omega-1) + \omega^2(\cancel{\omega}-\cancel{\omega})$$

$$= 1(\omega^2-\omega) - \omega(\omega-1) + 0$$

$$= \cancel{\omega^2} + \cancel{\omega} - \cancel{\omega^2} + \cancel{\omega}$$

$= 0$ Ans.

Formula

$\omega^3 = 1$

$\omega^7 = \omega$

$\omega^9 = 1$

$\omega^5 = \omega^2$

$\omega^{17} = \omega^2$

$\omega^{18} = 1$

$1+\omega+\omega^2 = 0$

$1+\omega = -\omega^2$

$1+\omega^2 = -\omega$

$\omega+\omega^2 = -1$

Q6. If ω is an imaginary cube root of unity, then the value of $\begin{vmatrix}\omega^3 & \omega^4 & \omega^5\\ \omega^2 & 1 & \omega\\ \omega^7 & \omega^5 & \omega^6\end{vmatrix} = ?$

$$= \begin{vmatrix}\omega^3 & \omega^4 & \omega^5\\ \omega^2 & 1 & \omega\\ \omega^7 & \omega^5 & \omega^6\end{vmatrix}$$

$$= \begin{vmatrix}1 & \omega & \omega^2\\ \omega^2 & 1 & \omega\\ \omega & \omega^2 & 1\end{vmatrix}$$

$$= 1\begin{vmatrix}1 & \omega\\ \omega^2 & 1\end{vmatrix} - \omega\begin{vmatrix}\omega^2 & \omega\\ \omega & 1\end{vmatrix} + \omega^2\begin{vmatrix}\omega^2 & 1\\ \omega & \omega^2\end{vmatrix}$$

$= 1(1-\omega^3) - \omega(\omega^2-\omega^2) + \omega^2(\omega^4-\omega)$

$= 1(\cancel{1}-\cancel{1}) - \omega(0) + \omega^2(\cancel{\omega}-\cancel{\omega})$

$= 0 - 0 + 0$

$= 0$ Ans

Q7. Find the value of $\begin{vmatrix} 1 & \omega^3 & \omega^2 \\ \omega^3 & 1 & \omega \\ \omega^2 & \omega & 1 \end{vmatrix}$

$= \begin{vmatrix} 1 & \omega^3 & \omega^2 \\ \omega^3 & 1 & \omega \\ \omega^2 & \omega & 1 \end{vmatrix}$

$= 1\begin{vmatrix} 1 & \omega \\ \omega & 1 \end{vmatrix} - \omega^3\begin{vmatrix} \omega^3 & \omega \\ \omega^2 & 1 \end{vmatrix} + \omega^2\begin{vmatrix} \omega^3 & 1 \\ \omega^2 & \omega \end{vmatrix}$

$= 1(1-\omega^2) - \omega^3(\omega^3-\omega^3) + \omega^2(\omega^4-\omega^2)$

$= 1(1-\omega^2) - \omega^3 \times 0 + \omega^2(\omega'-\omega^2)$

$= 1-\omega^2+\omega^3-\omega^4$

$= 1-\omega^2+1-\omega$

$= 1+1-\omega^2-\omega$

$= 2-(\omega^2+\omega)$

$= 2-(-1)\left[\because \omega+\omega^2 = -1\right]$

$= 2+1$

$= 3$ Ans

Q8. Find the minor of the element 2 if $\Delta = \begin{vmatrix} 0 & 2 & 4 \\ -3 & 0 & -1 \\ 5 & -2 & 7 \end{vmatrix}$

$\Rightarrow \Delta = \begin{vmatrix} 0 & 2 & 4 \\ -3 & 0 & -1 \\ 5 & -2 & 7 \end{vmatrix}$

$\Rightarrow \therefore$ The minor of element 2 is

$= \begin{vmatrix} -3 & -1 \\ 5 & 7 \end{vmatrix}$

$= -21-(-5)$

$= -21+5$

$= -16$ Ans

Q9. Find the minor of element 4 if $\Delta = \begin{vmatrix} 0 & 2 & 4 \\ -3 & 0 & -1 \\ 5 & -2 & 7 \end{vmatrix}$

$\Rightarrow \therefore$ The minor of element 4 is

$\cancel{= \begin{vmatrix} 0 & 1 \\ -3 & 0 \end{vmatrix}}$ $= \begin{vmatrix} -3 & 0 \\ 5 & -2 \end{vmatrix}$

$\cancel{= 0-(-3)}$ $\cancel{= 0-(1-3)}$ $= 6-0$

$\cancel{= 0+3}$ $\cancel{= 0-}$ $= 6$ Ans

$\cancel{= 3}$ ~~Ans~~

Q10. Find the co-factor of element 7 if $A = \begin{vmatrix} -3 & 1 & 2 \\ 7 & 3 & -4 \\ -5 & 6 & -1 \end{vmatrix}$

$\Rightarrow \therefore$ co-factor of element 7 is

$= (-1)^{i+j} \begin{vmatrix} 1 & 2 \\ 6 & -1 \end{vmatrix}$ $\quad i = R = 2$, $j = C = 1$

$= (-1)^{2+1} \times (-1-12)$

$= (-1)^{3} \times (-13)$

$= (-1) \times (-13)$

$= 13$ Ans.

Q11. Find the co-factor of element 6 ~~if A~~ of the determinant $\begin{vmatrix} 2 & -2 & 3 \\ 1 & 5 & 6 \\ 2 & 7 & 3 \end{vmatrix}$

$\Rightarrow \therefore$ co-factor of element 6 is

$= (-1)^{i+j} \begin{vmatrix} 2 & -2 \\ 2 & 7 \end{vmatrix}$ $\quad i = R = 2$, $j = C = 3$

$= (-1)^{2+3} \times (14 - (-4))$

$= (-1)^{5} \times (14+4)$

$= -1 \times 18$

$= -18$ Ans.

Q12. If $\Delta = \begin{vmatrix} 1 & 2 & 3 \\ -3 & 0 & -1 \\ 5 & -6 & 7 \end{vmatrix}$ then the cofactor of the element 2 = ?

⇒ ∵ co-factor of element 2 is

$= (-1)^{i+j} \begin{vmatrix} -3 & -1 \\ 5 & 7 \end{vmatrix}$ $\quad i = R = 1$, $j = C = 2$

$= (-1)^{1+2} \times (-21 - (-5))$

$= (-1)^3 \times (-21+5)$

$= -1 \times -16$

$= 16$ Ans.