22.02.2

1) Given $A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$ then $A^{-1} = ?$

$\therefore A_{11} = \begin{vmatrix} -3 & 4 \\ -1 & 1 \end{vmatrix} = -3+4 = 1$

$\therefore A_{12} = \begin{vmatrix} 2 & 4 \\ 0 & 1 \end{vmatrix} = 2-0 = 2$

$\therefore A_{13} = \begin{vmatrix} 2 & -3 \\ 0 & -1 \end{vmatrix} = -2-0 = -2$

$\therefore A_{21} = \begin{vmatrix} -3 & 4 \\ -1 & 1 \end{vmatrix} = -3+4 = 1$

$\therefore A_{22} = \begin{vmatrix} 3 & 4 \\ 0 & 1 \end{vmatrix} = 3-0 = 3$

$\therefore A_{23} = \begin{vmatrix} 3 & -3 \\ 0 & -1 \end{vmatrix} = -3-0 = -3$

$\therefore A_{31} = \begin{vmatrix} -3 & 4 \\ -3 & 4 \end{vmatrix} = -12+12 = 0$

$\therefore A_{32} = \begin{vmatrix} 3 & 4 \\ 2 & 4 \end{vmatrix} = 12-8 = 4$

$\therefore A_{33} = \begin{vmatrix} 3 & -3 \\ 2 & -3 \end{vmatrix} = -9+6 = -3$

$\therefore$ Adj. (A)

$$= \begin{bmatrix} A_{11} & -A_{12} & A_{13} \\ -A_{21} & A_{22} & -A_{23} \\ A_{31} & -A_{32} & A_{33} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -2 & -2 \\ -1 & 3 & 3 \\ 0 & -4 & -3 \end{bmatrix}$$

$$\therefore |A| = \begin{vmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{vmatrix}$$

$$= 3 \begin{vmatrix} -3 & 4 \\ -1 & 1 \end{vmatrix} + 3 \begin{vmatrix} 2 & 4 \\ 0 & 1 \end{vmatrix} + 4 \begin{vmatrix} 2 & -3 \\ 0 & -1 \end{vmatrix}$$

$= 3(-3+4) + 3(2-0) + 4(-2+0)$

$= 3 \times 1 + 3 \times 2 + 4 \times (-2)$

$= 3 + 6 - 8$

$= 9 - 8$

$= 1$

$\therefore$ Inverse of A

$$\therefore A^{-1} = \frac{\text{Adj}(A)}{|A|}$$

$$= \frac{\begin{bmatrix} 1 & -2 & -2 \\ -1 & 3 & 3 \\ 0 & -4 & -3 \end{bmatrix}}{1}$$

$$= \begin{bmatrix} 1 & -2 & -2 \\ -1 & 3 & 3 \\ 0 & -4 & -3 \end{bmatrix} \quad \text{Ans.}$$

2) Find the inverse of the matrix $\begin{bmatrix} 0 & 1 & 3 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$

$\therefore A_{11} = \begin{vmatrix} 2 & 3 \\ 1 & 1 \end{vmatrix} = 2-3 = -1$

$\therefore A_{12} = \begin{vmatrix} 1 & 3 \\ 3 & 1 \end{vmatrix} = 1-9 = -8$



$\therefore A_{13} = \begin{vmatrix} 1 & 2 \\ 3 & 1 \end{vmatrix} = 1-6 = -5$

$\therefore A_{21} = \begin{vmatrix} 1 & 3 \\ 1 & 1 \end{vmatrix} = 1-3 = -2$

$\therefore A_{22} = \begin{vmatrix} 0 & 3 \\ 3 & 1 \end{vmatrix} = 0-9 = -9$

$\therefore A_{23} = \begin{bmatrix} 0 & 1 \\ 3 & 1 \end{bmatrix} = 0-3 = -3$

$\therefore A_{31} = \begin{bmatrix} 1 & 3 \\ 2 & 3 \end{bmatrix} = 3-6 = -3$

$\therefore A_{32} = \begin{bmatrix} 0 & 3 \\ 1 & 3 \end{bmatrix} = 0-3 = -3$

$\therefore A_{33} = \begin{bmatrix} 0 & 1 \\ 1 & 2 \end{bmatrix} = 0-1 = -1$

$\therefore$ Adj, (A)

$$= \begin{bmatrix} A_{11} & -A_{12} & A_{13} \\ -A_{21} & A_{22} & -A_{23} \\ A_{31} & -A_{32} & A_{33} \end{bmatrix}$$

$$= \begin{bmatrix} -1 & +8 & -5 \\ +2 & -9 & +3 \\ -3 & +3 & -1 \end{bmatrix}$$

$|A|$

$=$



$$\therefore |A| = \begin{vmatrix} 0 & 1 & 3 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{vmatrix}$$

$$= 0\begin{vmatrix} 2 & 3 \\ 1 & 1 \end{vmatrix} - 1\begin{vmatrix} 1 & 3 \\ 3 & 1 \end{vmatrix} + 3\begin{vmatrix} 1 & 2 \\ 3 & 1 \end{vmatrix}$$

$$= 0(2-3) - 1(1-9) + 3(1-6)$$

$$= 0(-1) - 1(-8) + 3(-5)$$

$$= \cancel{0 + 8 - 21} \quad = 0 + 8 - 15$$

$$\cancel{= 1 - 13} \quad = -7$$

$$\cancel{= -14}$$

$\therefore$ Inverse of A

$$\therefore A^{-1} = \frac{Adj(A)}{|A|} = \frac{\begin{bmatrix} -1 & +8 & -5 \\ +2 & -9 & +3 \\ -3 & +3 & -1 \end{bmatrix}}{-7}$$

$$= \frac{1}{-7}\begin{bmatrix} 1 & -2 & 3 \\ -8 & 9 & -3 \\ 5 & -3 & 1 \end{bmatrix}$$

5) Find adj(A) if $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & -4 & 5 \\ 6 & 1 & 0 \end{bmatrix}$

$\therefore A_{11} = \begin{vmatrix} -4 & 5 \\ 1 & 0 \end{vmatrix} = -0 - 5 = -5$

$\therefore A_{12} = \begin{vmatrix} 2 & 5 \\ 6 & 0 \end{vmatrix} = 0 - 30 = -30$

$\therefore A_{13} = \begin{vmatrix} 2 & -4 \\ 6 & 1 \end{vmatrix} = 2 + 24 = 26$

$\therefore A_{21} = \begin{vmatrix} 2 & 3 \\ 1 & 0 \end{vmatrix} = 0 - 3 = -3$

$\therefore A_{22} = \begin{vmatrix} 1 & 3 \\ 6 & 0 \end{vmatrix} = 0 - 18 = -18$

$\therefore A_{23} = \begin{vmatrix} 1 & 2 \\ 6 & 1 \end{vmatrix} = 1 - 12 = -11$

$\therefore A_{31} = \begin{vmatrix} 2 & 3 \\ -4 & 5 \end{vmatrix} = 10 + 12 = 22$

$\therefore A_{32} = \begin{vmatrix} 1 & 3 \\ 2 & 5 \end{vmatrix} = 5 - 6 = -1$

$\therefore A_{33} = \begin{vmatrix} \cancel{A}1 & 2 \\ 2 & -4 \end{vmatrix} = -4 - 4 = -8$

$\therefore Adj(A) = \begin{bmatrix} A_{11} & -A_{12} & A_{13} \\ -A_{21} & A_{22} & -A_{23} \\ A_{31} & -A_{32} & A_{33} \end{bmatrix}$

$= \begin{bmatrix} -9 & +30 & -26 \\ +3 & +18 & -12 \\ -22 & -1 & +8 \end{bmatrix}$

$= \begin{bmatrix} -5 & 30 & 26 \\ 3 & -18 & 11 \\ 22 & 1 & -8 \end{bmatrix}$



4) Find adj(A) if $A = \begin{bmatrix} 1 & -2 & 3 \\ 0 & 2 & -1 \\ -4 & 5 & 2 \end{bmatrix}$

$\therefore A_{11} = \begin{vmatrix} 2 & -1 \\ 5 & 2 \end{vmatrix} = 4 + 5 = 9$

$\therefore A_{12} = \begin{vmatrix} 0 & -1 \\ -4 & 2 \end{vmatrix} = 0 - 4 = -4$

$\therefore A_{13} = \begin{vmatrix} 0 & 2 \\ -4 & 5 \end{vmatrix} = 0 + 8 = 8$

$\therefore A_{21} = \begin{vmatrix} -2 & 3 \\ 5 & 2 \end{vmatrix} = -4-15 = -19$

$\therefore A_{22} = \begin{vmatrix} 1 & 3 \\ -4 & 2 \end{vmatrix} = 2+12 = 14$

$\therefore A_{23} = \begin{vmatrix} 1 & -2 \\ -4 & 5 \end{vmatrix} = 5-8 = -3$

$\therefore A_{31} = \begin{vmatrix} -2 & 3 \\ 2 & -1 \end{vmatrix} = 2-6 = -4$

$\therefore A_{32} = \begin{vmatrix} 1 & 3 \\ 0 & -1 \end{vmatrix} = -1-0 = -1$

$\therefore A_{33} = \begin{vmatrix} 1 & -2 \\ 0 & 2 \end{vmatrix} = 2+0 = 2$



$$\therefore \text{Adj}(A) = \begin{bmatrix} A_{11} & -A_{12} & A_{13} \\ -A_{21} & A_{22} & -A_{23} \\ A_{31} & -A_{32} & A_{33} \end{bmatrix}$$

$$= \begin{bmatrix} 9 & 4 & 8 \\ 19 & 14 & 3 \\ -4 & 1 & 2 \end{bmatrix}$$