1) If $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 4 & 5 \\ 5 & 6 & 7 \end{bmatrix}$ then show that $A + A^T$ symmetric matrix & $A - A^T$ = skew symmetric matrix.

→ Given; $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 4 & 5 \\ 5 & 6 & 7 \end{bmatrix}$

$\therefore A^T = \begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \\ 3 & 5 & 7 \end{bmatrix}$

$\therefore A + A^T = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 4 & 5 \\ 5 & 6 & 7 \end{bmatrix} + \begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \\ 3 & 5 & 7 \end{bmatrix}$

$= \begin{bmatrix} 2 & 5 & 8 \\ 5 & 8 & 11 \\ 8 & 11 & 14 \end{bmatrix}$

$\therefore A + A^T$ is a symmetric matrix. (Proved)

$\therefore A - A^T = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 4 & 5 \\ 5 & 6 & 7 \end{bmatrix} - \begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \\ 3 & 5 & 7 \end{bmatrix}$

$= \begin{bmatrix} 0 & -1 & -2 \\ 1 & 0 & -1 \\ 2 & 1 & 0 \end{bmatrix}$

$\therefore A - A^T$ is a skew symmetric matrix.

2) If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ then show that $A^2 - 5A - 2I = 0$

$\rightarrow$ $A^2 = A \times A$

$= \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \times \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

$= \begin{bmatrix} 1\times1 + 2\times3 & 1\times2 + 2\times4 \\ 3\times1 + 4\times3 & 3\times2 + 4\times4 \end{bmatrix}$

$= \begin{bmatrix} 1+6 & 2+8 \\ 3+12 & 6+16 \end{bmatrix}$

$= \begin{bmatrix} 7 & 10 \\ 15 & 22 \end{bmatrix}$

$\therefore A^2 - 5A - 2I = \begin{bmatrix} 7 & 10 \\ 15 & 22 \end{bmatrix} - 5\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} - 2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 7 & 10 \\ 15 & 22 \end{bmatrix} - \begin{bmatrix} 5 & 10 \\ 15 & 20 \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$

$= \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$

$= 0$ (Proved)

3) If $A = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$ then show that $A^2 - 7A + 6I = O$

$\rightarrow$ $A^2 = A \times A$

$$= \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix} \times \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 25+4 & 20+8 \\ 5+2 & 4+4 \end{bmatrix}$$

$$= \begin{bmatrix} 29 & 28 \\ 7 & 8 \end{bmatrix}$$

$$\therefore A^2 - 7A + 6I = \begin{bmatrix} 29 & 28 \\ 7 & 8 \end{bmatrix} - 7\begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix} + 6\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 29 & 28 \\ 7 & 8 \end{bmatrix} - \begin{bmatrix} 35 & 28 \\ 7 & 14 \end{bmatrix} + \begin{bmatrix} 6 & 0 \\ 0 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} -6 & 0 \\ 0 & -6 \end{bmatrix} + \begin{bmatrix} 6 & 0 \\ 0 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} \cancel{-6}+\cancel{6} & 0+0 \\ 0+0 & \cancel{-6}+\cancel{6} \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$= O$ (proved)

4) Verify $A^2 + I = O$, where $A = \begin{bmatrix} 1 & 2 \\ -1 & -1 \end{bmatrix}$

→ Given $A = \begin{bmatrix} 1 & 2 \\ -1 & -1 \end{bmatrix}$

$\therefore A^2 = A \times A \quad \begin{bmatrix} 1 & 2 \\ -1 & -1 \end{bmatrix} \times \begin{bmatrix} 1 & 2 \\ -1 & -1 \end{bmatrix}$

$$= \begin{bmatrix} 1 + 2 & 2 - 2 \\ -1 + 1 & -2 + 1 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

$\therefore A^2 + I = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$$= \begin{bmatrix} \cancel{-1} + \cancel{1} & 0 + 0 \\ 0 + 0 & \cancel{-1} + \cancel{1} \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$= O$ (Proved)

5) If $A = \begin{bmatrix} 2 & 1 & 0 \\ 3 & -1 & 4 \\ 4 & 2 & 1 \end{bmatrix}$ show that $A + A^T$ is a symmetric matrix, and $A - A^T$ is a skew symmetric matrix.

→ Given $A = \begin{bmatrix} 2 & 1 & 0 \\ 3 & -1 & 4 \\ 4 & 2 & 1 \end{bmatrix}$

$\therefore A^T = \begin{bmatrix} 2 & 3 & 4 \\ 1 & -1 & 2 \\ 0 & 4 & 1 \end{bmatrix}$

$\therefore A + A^T = \begin{bmatrix} 2 & 1 & 0 \\ 3 & -1 & 4 \\ 4 & 2 & 1 \end{bmatrix} + \begin{bmatrix} 2 & 3 & 4 \\ 1 & -1 & 2 \\ 0 & 4 & 1 \end{bmatrix}$

$= \begin{bmatrix} 4 & 4 & 4 \\ 4 & -2 & 6 \\ 4 & 6 & 2 \end{bmatrix}$

$\therefore A + A^T$ is a symmetric matrix (Proved)

$\therefore A - A^T = \begin{bmatrix} 2 & 1 & 0 \\ 3 & -1 & 4 \\ 4 & 2 & 1 \end{bmatrix} - \begin{bmatrix} 2 & 3 & 4 \\ 1 & -1 & 2 \\ 0 & 4 & 1 \end{bmatrix}$

$= \begin{bmatrix} 0 & -2 & -4 \\ 2 & -2 & 2 \\ 4 & -2 & 0 \end{bmatrix}$

$\therefore A - A^T$ is a skew symmetric matrix (Proved)

6) If $A = \begin{bmatrix} 1 & 0 & 0 \\ -1 & -2 & -1 \\ -2 & 3 & -2 \end{bmatrix}$, then show that $A + A^T$ is symmetric and $A - A^T$ is skew symmetric and then express A as the sum of a symmetric and a skew symmetric matrix.

→ Given $A = \begin{bmatrix} 1 & 0 & 0 \\ -1 & -2 & -1 \\ -2 & 3 & -2 \end{bmatrix}$

$\therefore A^T = \begin{bmatrix} 1 & -1 & -2 \\ 0 & -2 & 3 \\ 0 & -1 & -2 \end{bmatrix}$

$0 + (-1)$
$0 - 1$
-1

$0 + (-2)$
$0 - 2$
-2

$-1 + 0$
$= -1$

$\therefore A + A^T = \begin{bmatrix} 1 & 0 & 0 \\ -1 & -2 & -1 \\ -2 & 3 & -2 \end{bmatrix} + \begin{bmatrix} 1 & -1 & -2 \\ 0 & -2 & 3 \\ 0 & -1 & -2 \end{bmatrix}$

$= \begin{bmatrix} 2 & -1 & -2 \\ -1 & -4 & 2 \\ -2 & 2 & -4 \end{bmatrix}$ $\therefore A + A^T$ is a symmetric (Proved)

$\therefore A - A^T = \begin{bmatrix} 1 & 0 & 0 \\ -1 & -2 & -1 \\ -2 & 3 & -2 \end{bmatrix} - \begin{bmatrix} 1 & -1 & -2 \\ 0 & -2 & 3 \\ 0 & -1 & -2 \end{bmatrix}$

$= \begin{bmatrix} 0 & 1 & 2 \\ -1 & 0 & -4 \\ -2 & 4 & 0 \end{bmatrix}$

$\therefore A - A^T$ is a skew symmetric (Proved)