11.02.25

Matrix Algebra

Raw ~~Colu~~

column

Format of matrix

$$\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix} = \begin{Bmatrix} 2 & 3 \\ 4 & 5 \end{Bmatrix} = \begin{pmatrix} 2 & 3 \\ 4 & 5 \end{pmatrix}$$

2 × 2

Raw ← → Column

Ex.

$$\begin{bmatrix} 2 & 3 \\ 4 & 5 \\ 6 & 7 \end{bmatrix} \qquad \begin{bmatrix} 4 & 6 & 0 \\ 5 & 7 & -1 \end{bmatrix}$$

3×2 → order of the matrix.

$$\begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}_{3\times 1} \qquad \begin{bmatrix} 0 & 1 & 2 \end{bmatrix}_{1\times 3}$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & -2 & 4 \\ 5 & 6 & 7 \end{bmatrix} \quad 3\times 3$$

Types of Matrices :-

① Raw matrix

$\boxed{R = 1}$

$[1 \quad 2 \quad 3]$

② column matrix

$\boxed{C = 1}$

$$\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

③ Single element matrix

$\boxed{R = C = 1}$

$[10]$ or $[5]$

④ Rectangular matrix

→ vertical matrix

$$\begin{bmatrix} 2 & 3 \\ 4 & 5 \\ 6 & 7 \end{bmatrix} \quad \boxed{R > C}$$

→ Horizontal matrix

$$\begin{bmatrix} 2 & 4 & 6 \\ 3 & 5 & 7 \end{bmatrix} \quad \boxed{C > R}$$

⑤ Square matrix

$\boxed{R = C}$

$$\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix} \text{ or } \begin{bmatrix} 7 & 8 & 9 \\ -1 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

⑥ Null or zero matrix

$[0]$ or $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ (Here only zero)

⑦ Diagonal matrix

$$\begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix} \text{ or } \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

⑧ Scalar matrix

$$\begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix} \text{ or } \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} \text{ or } \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

⑨ Unit or Identity matrix

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \text{ or } \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{ or } \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Example :-

I_2, I_3, I_4

$$I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \Rightarrow 2I_2 = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow 5I_3 = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

$I_3 + 2I_3$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix} = 3I_3$$

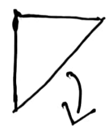
(10) Triangular matrix

- **Upper**

$$\begin{bmatrix} 0 & 5 & 6 \\ 0 & 7 & -1 \\ 0 & 0 & 5 \end{bmatrix}$$

- **Lower**

$$\begin{bmatrix} 0 & 0 & -1 \\ 0 & 5 & 6 \\ 8 & 9 & -2 \end{bmatrix}$$



(11) Transpose of a matrix (A^T)

$\boxed{R \rightarrow C}$ or $C \rightarrow R$

Let,

$$A = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$$

$$\therefore A^T = \begin{bmatrix} 5 & 7 \\ 6 & 8 \end{bmatrix} \text{ Ans}$$

Let, $A = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$

$$\therefore A^T = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \text{ Ans}$$

Let, $A = \begin{bmatrix} 0 & 1 & 2 \\ 5 & 6 & 9 \\ -1 & -2 & -3 \end{bmatrix}$

$$A^T = \begin{bmatrix} 0 & 5 & -1 \\ 1 & 6 & -2 \\ 2 & 9 & -3 \end{bmatrix} \text{ Ans}$$

(12) Orthogonal matrix

$$A \cdot A^T = I$$

(13) Singular matrix

$$|A| = 0$$

(14) Non-singular matrix

$$|A| \neq 0$$

(15) Idempotent matrix

$$A^2 = A$$

(16) Involutory matrix

$$A^2 = I$$

(17) Nilpotent matrix

$$A^2 = 0$$

Symetric matrix

$$\begin{bmatrix} 2 & 5 & 8 \\ 5 & 7 & -1 \\ 8 & -1 & 0 \end{bmatrix}$$

Skew symmentric matrix

$$\begin{bmatrix} 2 & 5 & -8 \\ -5 & 7 & -1 \\ 8 & 1 & 0 \end{bmatrix}$$

Q1.) If $A = \cancel{2-1} \begin{bmatrix} 2 & -1 & 0 \\ 0 & 2 & 4 \end{bmatrix}$ then find A^T ?

$$A^T = \begin{bmatrix} 2 & 0 \\ -1 & 2 \\ 0 & 4 \end{bmatrix}$$

Q3.) If $A = \begin{bmatrix} 3 & -1 & 4 \\ 2 & 5 & 6 \\ 1 & 2 & 3 \end{bmatrix}$ then $A + A^T = ?$

$$\therefore A^T = \begin{bmatrix} 3 & 2 & 1 \\ -1 & 5 & 2 \\ 4 & 6 & 3 \end{bmatrix}$$

$$\therefore A + A^T = \begin{bmatrix} 3 & -1 & 4 \\ 2 & 5 & 6 \\ 1 & 2 & 3 \end{bmatrix} + \begin{bmatrix} 3 & 2 & 1 \\ -1 & 5 & 2 \\ 4 & 6 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & 1 & 5 \\ 1 & 10 & 8 \\ 5 & 8 & 6 \end{bmatrix} \text{ Ans}$$

$\therefore$ It is a symmetric matrix. Ans