Chapter Name: Rotational Motion

Translational Motion

Translational motion is the type of motion in which every point of a body moves through the same distance in the same direction and in the same time interval.

In this motion, the object as a whole changes its position from one place to another, without any rotation about its own axis.

The entire body moves as a single unit, maintaining the same orientation throughout the motion.

Translational motion can be of two types:

- **Rectilinear motion:** When the path followed by the object is a straight line.
  Example: A car moving on a straight road.

- **Curvilinear motion:** When the path followed by the object is curved.
  Example: The motion of a football kicked into the air following a curved path.

Examples of Translational Motion

- A car moving on a straight road.
- A train moving along the track.
- A ball rolling (without rotation) across the floor.
- A person walking or an airplane flying in a straight path.

Rotational Motion

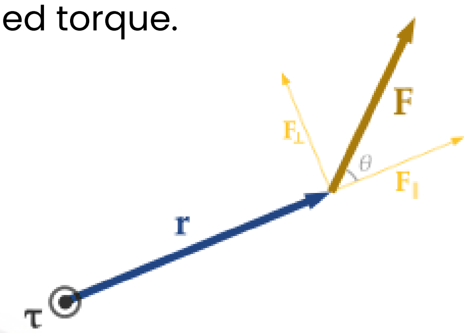
Rotational motion is the type of motion in which a body spins or rotates about a fixed axis, such that every point of the body moves in a circular path around that axis, and the axis itself remains fixed (or stationary).

In this motion, different parts of the body move through different distances in the same time, depending on their distance from the axis of rotation. The greater the distance from the axis, the larger the circular path covered by that point.

Torque (τ)

Torque is the rotational equivalent of force. It is the tendency of a force to rotate an object about a fixed point or axis.

When a force is applied at some distance from the axis of rotation, it produces a turning effect, and that effect is called torque.



Mathematically:

$$\tau = r \times F = rF\sin\theta$$

Where:

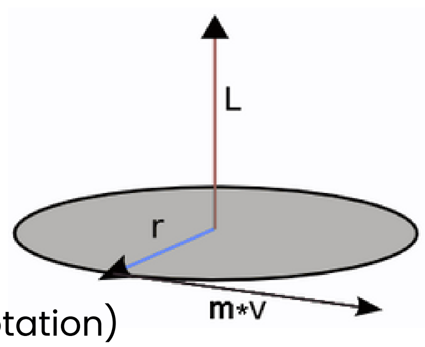
- τ = Torque
- r = Perpendicular distance between the axis of rotation and the point of application of force (called moment arm)
- F = Force applied
- θ = Angle between r and F

Unit: Newton-metre (N·m)

Example:
When you use a spanner to turn a bolt, the force applied at a distance from the bolt creates torque, causing the bolt to rotate.

Angular Momentum (L)

Angular momentum is the rotational analogue of linear momentum. It is the quantity of rotational motion possessed by a rotating body about a fixed axis. It depends on both the moment of inertia (I) of the body and its angular velocity (ω).



Mathematically:

$$L = I\omega$$

Where:

- L = Angular momentum
- I = Moment of inertia (resistance to change in rotation)
- ω = Angular velocity

Unit: $kg \cdot m^2/s$

Example:
A spinning ice skater who pulls her arms inward increases her angular speed — but her angular momentum remains constant (conservation of angular momentum).

Relation Between Torque and Angular Momentum

Torque and angular momentum are closely related, just as force and linear momentum are related in linear motion.

The relation is:

$$\tau = \frac{dL}{dt}$$

Meaning:
The torque acting on a body is equal to the rate of change of its angular momentum with respect to time.

👉 If no external torque acts on a system, then

$$\frac{dL}{dt} = 0 \quad \Rightarrow \quad L = \text{constant}$$

That means angular momentum is conserved.

Example:
When a figure skater pulls in her arms while spinning, the moment of inertia decreases, so angular velocity increases — keeping angular momentum constant because no external torque acts.

Examples of Torque:

- **Opening a door** – Force at the edge makes it easier to turn.
- **Pedaling a bicycle** – Force on the pedal creates torque to move the wheel.

Examples of Angular Momentum

- **Rotating fan** – Blades have angular momentum while spinning
- **Earth's rotation** – Earth keeps spinning due to its angular momentum.

In short:

Concept	Description	Formula	Example
Torque (τ)	Turning effect of a force	$\tau = rF\sin\theta$	Using a wrench to tighten a bolt
Angular Momentum (L)	Measure of rotational motion	$L = I\omega$	A spinning ice skater
Relation	Torque = rate of change of angular momentum	$\tau = \frac{dL}{dt}$	More torque → faster change in rotation

Conservation of Angular Momentum (Quantitative)

Statement:

If no external torque acts on a system, the total angular momentum of the system remains constant.

$$L = I\omega = \text{constant}$$

or, mathematically,

$$I_1\omega_1 = I_2\omega_2$$

Where:
I_1 & I_2 = moments of inertia before and after
ω_1 & ω_2 = angular velocities before and after

✅ This means:
If the moment of inertia of a rotating body changes, its angular velocity changes **inversely** — so that the product Iω remains constant.

Examples / Applications

Spinning Ice Skater

- When a skater pulls in her arms, III decreases → ω\omegaω increases.
- She spins faster without any external torque.

Diver or Gymnast in Air

- A diver tucks in to spin faster and stretches out to slow down before entering the water.

Planetary Motion

- As a planet moves closer to the Sun in its orbit, its speed increases (Kepler's 2nd law) — an example of conservation of angular momentum.

Neutron Star Formation

- When a massive star collapses, its radius decreases drastically, so its rotation rate increases rapidly.

Spinning Top or Wheel

- A freely spinning top maintains its angular momentum until external torque (like friction) slows it down.

Moment of Inertia (I)

The moment of inertia of a body is the measure of its resistance to change in rotational motion about a given axis.

It depends on how the mass of the body is distributed relative to the axis of rotation.

$$I = \sum m_i r_i^2 \quad \text{or} \quad I = \int r^2 \, dm$$

Where:

- mi= mass of each particle
- ri = perpendicular distance of the particle from the axis of rotation

Unit: kg·m²
Physical quantity type: Scalar

Example:

- A solid sphere and a ring of same mass and radius roll down an incline. The sphere (smaller I) reaches the bottom first because it offers less resistance to rotation.

Physical Significance of Moment of Inertia

Rotational Inertia:

- It tells how difficult it is to rotate an object about an axis.
- Larger I ⇒ harder to start or stop rotation.

Dependence on Mass and Shape:

- I increases with mass and with how far the mass is from the axis.
- Example: A ring (mass far from the axis) has more I than a disc of the same mass and radius.

Analogue of Mass in Linear Motion:

- Just as mass resists linear acceleration, moment of inertia resists angular acceleration.

Determines Rotational Kinetic Energy:

- Rotational kinetic energy = Iw^2/2
- Greater I → more energy needed for the same angular speed.

Radius of Gyration (K) for a Rigid Body

The radius of gyration of a rigid body about a given axis is the distance from the axis at which, if the whole mass of the body were concentrated, it would have the same moment of inertia as the actual distribution of mass.

$$K = \sqrt{\frac{I}{M}}$$

Where:
K = radius of gyration
I = moment of inertia of the body about the axis
M = total mass of the body

Physical Meaning:

It gives a measure of how far the mass is spread from the axis of rotation.

- Larger K → mass is distributed farther from the axis.
- Smaller k → mass is closer to the axis.

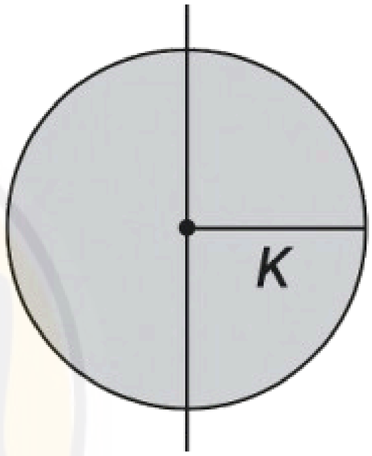
Example:

- For a **thin ring** of radius R:

$$I = MR^2 \Rightarrow K = R$$

- For a **solid disc** of radius R:

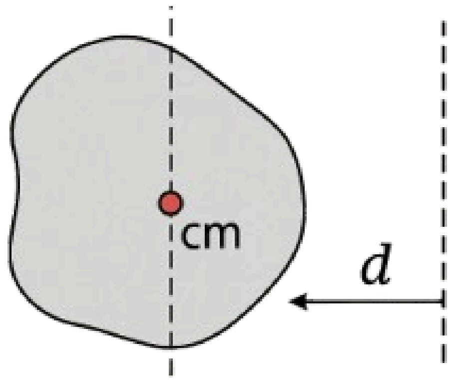
$$I = \frac{1}{2}MR^2 \Rightarrow K = \frac{R}{\sqrt{2}}$$



Parallel Axis Theorem (Statement)

The moment of inertia of a body about any axis **parallel** to its **center of mass axis** is equal to the sum of the moment of inertia about the center of mass axis and the product of the mass of the body and the square of the distance between the two axes.

$$I = I_{cm} + Md^2$$



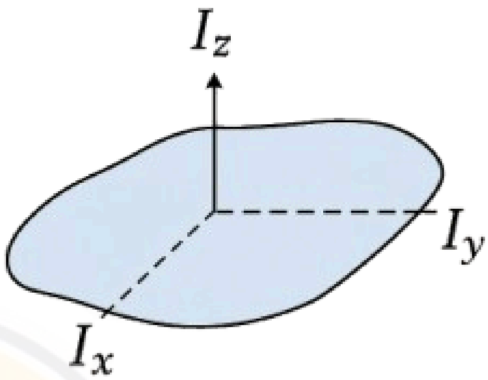
Where;

- I → Moment of inertia about any axis parallel to the center of mass axis.
- Icm → Moment of inertia about the axis through the center of mass.
- M → Total mass of the body.
- d → Perpendicular distance between the two parallel axes.

Perpendicular Axis Theorem (Statement)

The moment of inertia of a plane lamina about an **axis perpendicular** to its plane is equal to the **sum of its moments of inertia** about two mutually perpendicular axes lying in the plane and intersecting at the same point.

$$I_z = I_x + I_y$$



Where;

- Iz → Moment of inertia about the axis perpendicular to the plane.
- Ix → Moment of inertia about the X-axis in the plane.
- Iy → Moment of inertia about the Y-axis in the plane.

Moment of Inertia of Common Rigid Bodies

1. Uniform Rod

- About center (axis perpendicular to length):

$$I = \frac{1}{12}ML^2$$

- About one end (axis perpendicular to length):

$$I = \frac{1}{3}ML^2$$

2. Circular Disc

- About center (axis perpendicular to plane):

$$I = \frac{1}{2}MR^2$$

- About a diameter:

$$I = \frac{1}{4}MR^2$$

✅ ***Symbols:***

- M → Mass of the body
- R → Radius
- L → Length

3. Ring

- About central axis (perpendicular to plane):

$$I = MR^2$$

- About a diameter:

$$I = \frac{1}{2}MR^2$$

4. Solid Sphere

- About any diameter:

$$I = \frac{2}{5}MR^2$$

5. Hollow Sphere (Thin Spherical Shell)

- About any diameter:

$$I = \frac{2}{3}MR^2$$