Chapter Name: Binomial Theorm

Short Questions

Formulas are at the Last Page

1

Which one of the following is a polynomial —

(a) x^7

(b) $x^3+2x^2+\sqrt{x}+1$

(c) $x^{2/3}+x^2$

(d) $\log x$

[WBSCTE – 2013]

Solution: Hence the correct answer is (a).

2

The number of terms in the expansion of $(1+x)^8$ is—

(a) 9

(b) 8

(c) 7

(d) 10

Solution:

Here $n=8$ (even number).

So, the number of terms in the expansion of $(1+x)^8$ is $(8+1)$ i.e., 9.

Hence the correct answer is (a).

3

The number of terms in the expansion of $(1-x)^{-3/2}$ is—

(a) 3

(b) 2

(c) 1

(d) none of these

Solution:

Here $n=-\frac{3}{2}$ (negative fraction).

As power is negative fraction, the number of terms in the expansion of $(1-x)^{-3/2}$ is infinite.

Hence the correct answer is (d).

4

The number of terms in the expansion of $(1+2x)^{-5}$ is—

(a) 5

(b) 6

(c) 4

(d) none of these

Solution:

Here $n=-5$ (negative integer).

Since power is negative integer, the number of terms in the expansion of $(1+2x)^{-5}$ is infinite.

Hence the correct answer is (d).

5

Number of middle terms in the expansion of $\left(1-\frac{1}{x}\right)^{13}$ is—

(a) 1

(b) 2

(c) 3

(d) none of these

Solution:

Here $n = 13$.

Number of terms in the expansion of $\left(1 - \frac{1}{x}\right)^{13}$ is 14 (even number).

So, the number of middle terms is 2.

Hence the correct answer is (b).

6

Expand $(a + 2b)^5$

Solution :

$= a^5 + 5a^4 \cdot (2b) + 5a^3(2b)^2 + 5a^2(2b)^3 + 5a(2b)^4 + (2b)^5$

Now : 5C1 = 5, 5C2 = 10 ($\because$ nCr = nC(n–r))

and 5C4 = 5C1 = 5

$\therefore (a + 2b)^5$

$= a^5 + 10a^4b + 40a^3b^2 + 80a^2b^3 + 80ab^4 + 32b^5$

7

Expand $(a - 1/a)^6$

Solution :

$(a - 1/a)^6$

$= a^6 - 6a^5(1/a) + 15a^4(1/a)^2 - 20a^3(1/a)^3 + 15a^2(1/a)^4 - 6a(1/a)^5 + (1/a)^6$

$= a^6 - 6a^4 + 15a^2 - 20 + 15/a^2 - 6/a^4 + 1/a^6$

8

The co-efficient of middle term in the expansion of $(1-x)^8$ is—
(a) ${}^8C_4$, (b) ${}^8C_{5}\_3$, (c) ${}^8C_{2}\_6$, (d) none of these.

Solution : Here $n=8$ (even no.)

So, middle term is $\left(\frac{n}{2}+1\right)^{\mathrm{th}}$ term i.e. 5th term.

$\therefore t_5 = {}^8C_4(-x)^4$

$\therefore$ Co-efficient of middle term $= {}^8C_4 = \boxed{{}^8C_4}$

Hence the correct answer is (a).

9

The middle term in the expansion of $\left(x-\frac{1}{x}\right)^6$ is—

(a) $20x$, (b) $20x^2$, (c) -20, (d) none of these.

Solution : Here $n=6$ (even number)

So, middle term is $\left(\frac{n}{2}+1\right)^{\mathrm{th}}$ term i.e. 4th term.

$\therefore$ Middle term in the expansion of $\left(x-\frac{1}{x}\right)^6$

$= {}^6C_3 x^{6-3}\left(-\frac{1}{x}\right)^3$

$$= 20x^3 \cdot \left(-\frac{1}{x^3}\right)$$
$$= -20$$

Hence the correct answer is (c).

10

Co-efficient of the Middle term in the expansion of $\left(a - \frac{1}{2a}\right)^6$ is—

(a) $\frac{5}{2}a$, (b) $\frac{5}{2}$, (c) $-\frac{5}{2}$, (d) none of these.

Solution : Here total no. of terms in the expansion = 7 (odd)

Hence the middle term is $\left(\frac{7+1}{2}\right)^{\text{th}}$ term i.e. 4th term = $(r+1)^{\text{th}}$ term.

$\therefore$ Middle term $= {}^6C_3 \cdot a^{(6-3)} \cdot \left(-\frac{1}{2a}\right)^3 = \frac{6 \times 5 \times 4}{3 \times 2} a^3 \cdot \left(-\frac{1}{8a^3}\right) = -\frac{5}{2}$

Hence the correct answer is (c).

Broad Questions

1

Find the coefficient of x in $\left(x^2 + \frac{a^2}{x}\right)^5$.

General term

$$T_{k+1} = \binom{5}{k} (x^2)^{5-k} \left(\frac{a^2}{x}\right)^k$$

Term becomes

$$\binom{5}{k} a^{2k} x^{10-3k}$$

For coefficient of x^1:

$$10 - 3k = 1 \Rightarrow k = 3$$

Coefficient

$$= \binom{5}{3} a^6 = 10a^6$$

2

Find the coefficient of x in $(1 - 2x^3 + 3x^5)\left(1 + \frac{1}{x}\right)^{10}$.

$$\left(1 + \frac{1}{x}\right)^{10} = \sum_{r=0}^{10} \binom{10}{r} x^{-r}$$

From term $-2x^3$: need power

$$3 - r = 1 \Rightarrow r = 2$$

Contribution

$$-2\binom{10}{2} = -2(45) = -90$$

From term $3x^5$: need power

$$5 - r = 1 \Rightarrow r = 4$$

Contribution

$$3\binom{10}{4} = 3(210) = 630$$

Total coefficient

$$-90 + 630 = 540$$

3

Find the term independent of x in $\left(x^2 + \frac{1}{x}\right)^{12}$.

General term in binomial expansion:

$$T_{k+1} = \binom{12}{k} (x^2)^{12-k} \left(\frac{1}{x}\right)^k$$

Simplifying the powers of x:

$$(x^2)^{12-k} = x^{24-2k}$$

$$\left(\frac{1}{x}\right)^k = x^{-k}$$

So,

$$T_{k+1} = \binom{12}{k} x^{24-2k-k} = \binom{12}{k} x^{24-3k}$$

Term independent of x means:

power of $x = 0$

So,

$$24 - 3k = 0$$

$$3k = 24$$

$$k = 8$$

This value of k gives the independent term.

Now the independent term:

$$T_{8+1} = T_9 = \binom{12}{8}$$

$$\binom{12}{8} = \binom{12}{4} = 495$$

Final answer (independent term)

495

4

Find the coefficient of x^{16} in the expansion of $(x^2 - 2x)^{10}$.

For $(x^2 - 2x)^{10}$:

General term

$$T_{k+1} = \binom{10}{k}(x^2)^{10-k}(-2x)^k$$

Simplify powers of x:

$(x^2)^{10-k} = x^{20-2k}$, $(-2x)^k = (-2)^k x^k$

So

$$T_{k+1} = \binom{10}{k}(-2)^k x^{20-k}$$

Require power x^{16}:

$20 - k = 16 \Rightarrow k = 4$

Coefficient $= \binom{10}{4}(-2)^4 = \ 210 \times 16 = 3360$

6

Find the coefficient of x^{10} in the expansion of $\left(x^2 - \frac{1}{x^3}\right)^{10}$.

For $\left(x^2 - \dfrac{1}{x^3}\right)^{10}$:

General term

$$T_{k+1} = \binom{10}{k} (x^2)^{10-k} \left(-\frac{1}{x^3}\right)^k$$

Simplify powers of x:

$(x^2)^{10-k} = x^{20-2k}$, $\left(-\frac{1}{x^3}\right)^k = (-1)^k x^{-3k}$

So

$$T_{k+1} = \binom{10}{k} (-1)^k x^{20-5k}$$

Require power x^{10}:

$20 - 5k = 10 \Rightarrow k = 2$

Coefficient $= \binom{10}{2}(-1)^2 = 45 \times 1 = 45$

7

If the coefficient of x^7 in the expansion of $\left(px^2 + \frac{1}{qx}\right)^{11}$ be equal to the coefficient of x^{-7} in the expansion of $\left(px - \frac{1}{qx^2}\right)^{11}$, prove that $pq = 1$.

General term of $\left(px^2 + \frac{1}{qx}\right)^{11}$:

$$T_{k+1} = \binom{11}{k}(px^2)^{11-k}\left(\frac{1}{qx}\right)^k$$

$$= \binom{11}{k}p^{11-k}q^{-k}x^{2(11-k)-k} = \binom{11}{k}p^{11-k}q^{-k}x^{22-3k}$$

For x^7:

$$22 - 3k = 7 \Rightarrow 3k = 15 \Rightarrow k = 5$$

Coefficient of x^7 is

$$\binom{11}{5}p^6q^{-5}$$

General term of $\left(px - \frac{1}{qx^2}\right)^{11}$:

$$U_{r+1} = \binom{11}{r}(px)^{11-r}\left(-\frac{1}{qx^2}\right)^r$$

$$= \binom{11}{r}p^{11-r}(-1)^rq^{-r}x^{11-r-2r} = \binom{11}{r}p^{11-r}(-1)^rq^{-r}x^{11-3r}$$

For x^{-7}:

$$11 - 3r = -7 \Rightarrow 3r = 18 \Rightarrow r = 6$$

Coefficient of x^{-7} is

$$\binom{11}{6}p^5(-1)^6q^{-6} = \binom{11}{6}p^5q^{-6}$$

Given these two coefficients are equal.

$$p^6q^{-5} = p^5q^{-6}$$

Divide both sides by p^5q^{-6}:

$$pq = 1$$

Thus proved: $pq = 1$.

8

Find the middle term in the expansion of:

(i) $(a + 3b)^6$

(ii) $(ax - \frac{1}{ax})^8$

(iii) $(3x - \frac{1}{2x})^8$

i

$(a + 3b)^6$

Total terms = 7

Middle term = 4th term

$T_4 = C(6,3) \cdot a^3 \cdot (3b)^3$

$T_4 = 20 \cdot a^3 \cdot 27b^3$

$T_4 = 540a^3b^3$

Middle term = $\mathbf{540a^3b^3}$

ii

$(ax - 1/ax)^8$

Total terms = 9

Middle term = 5th term

$T_5 = C(8,4) \cdot (ax)^4 \cdot (-1/ax)^4$

$T_5 = 70 \cdot a^4x^4 \cdot 1/(a^4x^4)$

$T_5 = 70$

Middle term = **70**

iii

$(3x - 1/(2x))^8$

Total terms = 9

Middle term = 5th term

$T_5 = C(8,4) \cdot (3x)^4 \cdot (-1/(2x))^4$

$T_5 = 70 \cdot 81x^4 \cdot 1/(16x^4)$

$T_5 = 70 \cdot 81/16$

$T_5 = 2835/8$

Middle term = **2835/8**

Formulas

A. If n be the positive integer then,

$$(a+x)^n = a^n + {}^nc_1\, a^{n-1}x + {}^nc_2 a^{n-2} x^2 + \ldots + {}^nc_r\, a^{n-r}x^r + \ldots + x^n.$$

B. $t_{r+1} = {}^nc_r\, a^{n-r} \,.\, x^r$

C. (i) If n is even positive integer then, there is one middle term and $\left(\frac{n}{2}+1\right)$th term is the middle term.

(ii) If n is odd positive integer then there are two middle terms and they are $\left(\frac{n-1}{2}+1\right)$th term and $\left(\frac{n+1}{2}+1\right)$th term.

D. (i) $(1-x)^{-1} = 1 + x + x^2 + x^3 + \ldots \infty$

(ii) $(1+x)^{-1} = 1 - x + x^2 - x^3 + \ldots \infty$

(iii) $(1-x)^{-2} = 1 + 2x + 3x^2 + 4x^3 + \ldots \alpha$

(iv) $(1+x)^{-2} = 1 - 2x + 3x^2 - 4x^3 + \cdots \infty$