

April 2022

MATHEMATICS - I

Time Allowed: 3 Hours

Full Marks: 60

Answer the following questions from Group A, B & C as directed.

GROUP A

1. Choose the correct alternatives (any ten):

1x10

- i. The value of $\log_b a \times \log_c b \times \log_d c$ is- (a) $\log_c a$ ~~(b) $\log_d c$~~ ~~(c) $\log_b a$~~ ~~(d) $\log_d a$~~
- ii. $z^{\log_a a}$ is equal to- (a) a^2 ~~(b) a^3~~ ~~(c) a~~ (d) z^a
- iii. The equation $\log_a x + \log_a(1+x) = 0$ can be written as- (a) $x^2 + x + 1 = 0$ (b) $x^2 - x + 1 = 0$ ~~(c) $x^2 + x - 1 = 0$~~ (d) None of these
- iv. If one root of the equation $x^2 + 6x + m = 0$ is 1 then the value of m is- (a) 7 ~~(b) -7~~ (c) 6 ~~(d) None~~
- v. In the expansion of $(1+px)^{10}$ the coefficient of x^2 is- (a) $^{10}C_2$ (b) $^{10}C_2 p$ ~~(c) $^{10}C_2 p^2$~~ (d) None
- vi. Square root of (-7) is- (a) 49 ~~(b) $7i$~~ ~~(c) $\sqrt{7}i$~~ (d) None
- vii. Which one of the following is impossible?- (a) $0 < \cos \theta < 1$ ~~(b) $\cos^2 \theta < 1$~~ (c) $-1 \leq \sin \theta \leq 1$ (d) $\sin \theta = 0$
- viii. If $\tan A = \frac{7}{5}$ and $\tan B = \frac{5}{7}$ then the value of $\cot(A-B)$ is- (a) 0 ~~(b) $\frac{25}{12}$~~ (c) $\frac{5}{12}$ ~~(d) None~~
- ix. If $\frac{\pi}{2} < \theta < \frac{3\pi}{4}$ and $\tan 2\theta = \frac{3}{4}$ then the value of $\tan \theta$ is- (a) 3 ~~(b) -3~~ (c) 2 ~~(d) None~~
- x. Which of the following does not exist? (a) $\sin^{-1}(0.6)$ (b) $\operatorname{cosec}^{-1}(6.6)$ ~~(c) $\cot^{-1}(10)$~~ (d) $\sec^{-1}(0.3)$
- xi. Which of the following is true? ~~(a) $\sin x$ is a periodic function of period π~~ (b) $\cos x$ is an odd function (c) x^3 is an even function (d) $\tan x$ is a periodic function of period π
- xii. $x \rightarrow 2$ does imply ~~(a) $x < 2$~~ (b) $x > 2$ ~~(c) $x = 2$~~ (d) x is nearer to 2
- xiii. $\frac{d}{dx}(\log_{10} \sin x)$ is equal to- ~~(a) $\cot x$~~ (b) $\frac{\cot x}{\log_{10} e}$ ~~(c) $\frac{\cot x}{\log_e 10}$~~ (d) None

- xiv. Which of the following is not a vector? (a) Velocity (b) Displacement (c) Mass (d) none of these
- xv. $\vec{a} \times \vec{b}$ is a vector- (a) parallel to $\vec{a}$ (b) perpendicular to both $\vec{a}$ & $\vec{b}$ (c) parallel to $\vec{b}$ (d) perpendicular to $\vec{a}$

1x10

2. Fill in the blanks (any ten):

- i. If $\log_{10} a + \log_{10} b = \log_{10}(a+b)$ then b is equal to _____.
- ii. If one root of the equation $x^2 - 8x + m = 0$ is 2 then the other root is _____.
- iii. The number of terms in the expansion of $(a+b)^n$, n is a positive integer, is _____.
- iv. Conjugate of the complex number 5 is _____.
- v. If z is purely imaginary, then $z + \bar{z}$ is _____.
- vi. The value of $\cos 1^\circ \cos 2^\circ \cos 3^\circ \dots \cos 123^\circ \cos 124^\circ \cos 125^\circ$ is _____.
- vii. Minimum value of $(\sin \theta + \cos \theta)$ is _____.
- viii. If $\tan \theta = 2$, then the value of $\cos 2\theta$ is _____.
- ix. $\cos(\sin^{-1} \frac{1}{2} + \sec^{-1} 2)$ is _____.
- x. Value of $\lim_{x \rightarrow 0} \frac{x}{3x^2 - 2x}$ is _____.
- xi. If $y = \tan^{-1} \sqrt{\frac{1 - \cos 2x}{1 + \cos 2x}}$ then $\frac{dy}{dx}$ is _____.
- xii. If $y = (\tan^{-1} x)^2$ & $\frac{dy}{dx} = 2 \tan^{-1} x \cdot f(x)$ then $f(x)$ is _____.
- xiii. If $\vec{a} = -\hat{i} + 2\hat{j}$, $\vec{b} = 3\hat{i} - 2\hat{j}$ and $\vec{c} = 5\hat{j}$ then $|\vec{a} + \vec{b} - 2\vec{c}|$ is _____.
- xiv. If the position vector of two points A and B are respectively $\hat{i} + 2\hat{j} - 3\hat{k}$ and $4\hat{i} - 5\hat{j} + 6\hat{k}$ then the unit vector in the direction of $\vec{AB}$ is _____.
- xv. The angle between the vectors $2\hat{i} + 3\hat{j} + \hat{k}$ and $2\hat{i} - \hat{j} - \hat{k}$ is _____.

1x10

3. Answer the following questions (any ten):

- i. If $\log_{10}^a = r$, then find the value of $a^{\frac{3}{r}}$.
- ii. If $(4 - 3i)$ be a root of the equation $px^2 + qx + 1$, where p, q are real, find the values of p and q.

- iii. If the roots of the equation $x^2 - px + q = 0$ be $1:2$, then find the relation between p and q .
- iv. Find the middle term of the expansion of $(2x^2 - \frac{1}{x})^8$.
- v. What is the amplitude of the complex number (-10) .
- vi. If $\cot 3x \cot 5x = 1$, then find the value of $\cot 4x$.
- vii. Find the value of $\tan 27^\circ + \tan 18^\circ + \tan 27^\circ \tan 18^\circ$.
- viii. If $\cos(x - y) + 1 = 0$, then find the value of $(\cos x + \cos y)$.
- ix. What is the value of $\sec^{-1}(-\sqrt{2})$.
- x. Evaluate $\lim_{x \rightarrow 1} \frac{\sqrt{x} - 1}{x - 1}$.
- xi. $f(x) = \frac{2x^2 - 8}{x - 2}$ is undefined at $x = 2$, then find the value of $f(2)$.
- xii. If $y = \cot^{-1} x + \cot^{-1} \frac{1}{x}$, find the value of $\frac{dy}{dx}$.
- xiii. Force $\vec{F} = 2i - j - k$ is applied to an object making a displacement $\vec{d} = 3i + 2j - 5k$, find the work done.
- xiv. A force $\vec{F} = 2i + j - k$ acts at a point whose position vector is $2i - j$, find the moment of $\vec{F}$ about the origin.
- xv. If $\vec{a} = 2i - j + \lambda k$ and $\vec{b} = 4i - 2j + 3k$ are collinear, then find the value of λ .

GROUP B

4. Answer the following questions (any six):

2x6

- i. Evaluate: $\log_x^x \times \log_x^y \times \log_x^z$.
- ii. The roots α and β of the equation $x^2 - px + q = 0$, are such that $2\beta + \alpha = 0$. Then find the relation between p and q .
- iii. If x, y are real and $x + 3i$ and $-2 + iy$ are conjugate to each other, find the values of x and y .
- iv. If $|z - 5| = 5$ where $z = x + 3i$, x and y being real, then find the locus of the point (x, y) .

- v. Find the value of $\tan 1^\circ \tan 2^\circ \tan 3^\circ \dots \tan 89^\circ$.
- vi. Prove that $\tan A + \frac{1}{\tan A} = \operatorname{cosec}^2 A$.
- vii. If $180^\circ < \theta < 270^\circ$ and $\sin \theta = -\frac{3}{5}$ then find the value of $\cos \theta$.
- viii. Evaluate $\lim_{x \rightarrow 0} \frac{\sin^{-1} x}{x}$.
- ix. If $f(x) = \frac{1}{1+x^{b-c}+x^{a-c}} + \frac{1}{1+x^{c-b}+x^{a-b}} + \frac{1}{1+x^{c-a}+x^{b-a}}$ and $f'(0)$.
- x. If the diagonals of a parallelogram are represented by the vectors $3i + j - 2k$ and $i + 3j - 4k$, then find its area.

GROUP C

5. Answer the following question (any one): 1x6
- i) If one root of the equation $x^2 + rx - s = 0$ is square of other, prove that $r^3 + s^2 + 3rs - s = 0$.
- ii) Form a quadratic equation with real co-efficients, whose one root is $(2 - 3i)$.
6. Answer the following question (any one): 1x6
- i) If $x + iy = \sqrt{\frac{a+ib}{c+id}}$, then prove that $(x^2 + y^2)^2 = \frac{a^2 + b^2}{c^2 + d^2}$, where x, y, a, b, c, d all are real.
- ii) If $z_1 = 3 + 5i, z_2 = 1 - i$, verify the relation $\left| \frac{z_1}{z_2} \right| = \frac{|z_1|}{|z_2|}$.
7. Answer the following question (any one): 1x6
- i) If the 4th term in the expansion of $(px + \frac{1}{x})^n$ is independent of x , find the value of n . Also calculate p if the 4th term be $\frac{5}{2}$.
- ii) Prove that $\frac{1}{\log_a^{bc} + 1} + \frac{1}{\log_b^{ca} + 1} + \frac{1}{\log_c^{ab} + 1} = 1$.