March 2023

MATHEMATICS-I

Time Allowed : 2.5 Hours *Full Marks : 60*

Answer to Question No. 1 of Group A is compulsory and to be answered first. This answer is to be made in separate loose script(s) provided for the purpose. Maximum time allowed is 30 minutes, after which the loose answer scripts will be collected and fresh answer scripts for answering the remaining part of the question will be provided. On early submission of answer scripts of Question No. 1, a student will get the remaining script earlier.

Answer any Five (05) Questions from Group B.

Group A

1. **Choose the correct answer from the given alternatives with minimum justification (any twenty) :** 1×20

i) If $\log_2 x^2 = 2$, then value of x is

(a) 2 (b) -2 (c) ± 2 (d) 4.

ii) If $\log_2 3 = x$, then $\log_8 27 =$

(a) x^2 (b) x (c) 2x (d) none of these.

iii) If the roots of the equation $ax^2 - bx + c = 0$ are reciprocal to each other, then

(a) a = b (b) b = c (c) c = a (d) none of these.

iv) If 2 + 3i be a root of the equation $x^2 - px + q = 0$, then the value of p is

(a) 4 (b) 3 (c) 2 (d) none of these.

v) If $z = -3\hat{i}$, then $\text{amp}(z) =$ (a) $\frac{\pi}{2}$ (b) π (c) $-\frac{\pi}{2}$ (d) 0.

vi) If $i = \sqrt{-1}$, then $i^{2020} + i^{2021} + i^{2022} + i^{2023} =$

(a) -i (b) i (c) -1 (d) 0

vii) The number of terms in the expansion of $(1 + 3x + 3x^2 + x^3)^7$ is

(a) 20 (b) 21 (c) 8 (d) 22.

viii) The fifth term in the expansion of $(1+x)^5$ is

(a) $5x^4$ (b) $6x^4$ (c) $7x^4$. (d) none of these.

ix) If the position vector $\vec{a}$ of the point (12, b) is such that $|\vec{a}| = 13$, then the value of b is (a) 5 (b) -5 (c) ±5 (d) none of these.

x) The dot product of the vectors $\hat{i}+\hat{k}$ and $\hat{j}$ is

(a) 0 (b) 1 (c) -1 (d) none of these.

xi) If the vectors $\vec{a}=\hat{i}-\hat{j}+\hat{k}$ and $\vec{b}=\hat{i}+m\hat{j}+\hat{k}$ are collinear, then m =

(a) 1 (b) -1 (c) 0 (d) none of these.

xii) The value of λ for which the vectors $2\vec{i}+\lambda\vec{j}+\vec{k}$ and $4\vec{i}-2\vec{j}-2\vec{k}$ are perpendicular to each other is (a) 2 (b) 3 (c) 4 (d) none of these.

xiii) If $\sec\theta+\cos\theta=\sqrt{3}$, then the value of $\sec^3\theta+\cos^3\theta$ is

(a) 3 (b) 2 (c) −1 (d) 0.

xiv) If $\tan A=\cot B$, then $A+B=$ (a) 90^0 (b) 60^0 (c) 45^0 (d) 0^0.

xv) The value of $\cos 1^0 \cos 2^0 \cos 3^0 ....\cos 98^0 \cos 99^0 \cos 100^0$ is

(a) 0 (b) 1 (c) -1 (d) none of these.

xvi) The value of $\sin(\sin^{-1}x+\cos^{-1}x)$ is

(a) 0 (b) 1 (c) 2 (d) none of these.

xvii) $\tan^{-1}\frac{1}{2}+\tan^{-1}\frac{1}{3}=$ (a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{4}$ (d) none of these.

xviii) In a triangle ABC, if a = 1, b = 2 and $\angle A = 30^0$. Then $\angle B =$

(a) $\frac{\pi}{2}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{4}$ (d) none of these.

xix) The domain of the function $f(x)=\frac{1}{\sqrt{7-x}}$ is

(a) $(-\infty, 7)$ (b) $(-\infty, 7]$ (c) $(7, \infty)$ (d) none of these.

xx) If $f(x-1)=x^2-x+1$, then $f(x)=$

(a) x^2+x+1 (b) x^2-x+1 (c) x^2+x-1 (d) none of these.

xxi) The value of $\lim_{x\to 0}\frac{x}{\sin^{-1}x}$ is (a) -1 (b) 0 (c) 1 (d) none of these.

xxii) If $y=e^{\log_e x}$, then $\frac{dy}{dx}=$ (a) 1 (b) 0 (c) $\frac{1}{x}$ (d) none of these.

xxiii) $\frac{d}{dx}\left(\sin^{-1}\frac{2x}{1+x^2}\right)=$ (a) 2 (b) $\frac{2}{1+x^2}$ (c) $\frac{1}{1+x^2}$ (d) none of these.

xxiv) If $y=\sin^2 x$, then $\left(\frac{d^2y}{dx^2}\right)_{x=0}$ is (a) 1 (b) 2 (c) -2 (d) 0.

xxv) A ball travels s feet in t seconds, where $s=8t+10t^2$. The velocity of the ball when $t=2$ is (a) 8 ft. / sec. (b) 18 ft. / sec. (c) 28 ft. /sec. (d) 48 ft /sec.

Group B

2. a) If $\frac{\log x}{ry-qz}=\frac{\log y}{pz-rx}=\frac{\log z}{qx-py}$, show that $x^p y^q z^r=1$.

b) If α, β be the imaginary cube roots of unity, prove that $\alpha^4+\beta^4+\alpha^{-1}\beta^{-1}=0$.

5+3

3. a) If the ratio of the roots of the equation $ax^2+bx+c=0$ be m : n, prove that $\frac{(m+n)^2}{mn}=\frac{b^2}{ac}$.

b) If for two vectors $\vec{a}$ and $\vec{b}$, $|\vec{a}+\vec{b}|=|\vec{a}-\vec{b}|$, find the angle between the vectors $\vec{a}$ and $\vec{b}$.

4+4

4. a) Find the middle term in the expansion of $\left(ax-\frac{1}{ax}\right)^8$.

b) Prove that $\left(\frac{1+\sqrt{-3}}{2}\right)^6+\left(\frac{1-\sqrt{-3}}{2}\right)^9=0$. 5+3

5. a) Forces $3\hat{i}+2\hat{j}+5\hat{k}$ and $2\hat{i}+\hat{j}-3k$ acting on a particle produces a displacement from the point (2, -1, -3) to (4, -3, 7). Find the work done by the forces.

b) Find the scalar area of the triangle, the position vectors of whose vertices are $3\hat{i}+\hat{j}$, $5\hat{i}+2\hat{j}+\hat{k}$ and $\hat{i}-2\hat{j}+3\hat{k}$. 4+4

6. a) If $\tan A=\frac{1-\cos B}{\sin B}$, prove that $\tan 2A=\tan B$

b) Solve : $\sin\theta+\sqrt{3}\cos\theta=\sqrt{2}$, $0\le\theta\le\pi$. 4+4

7. a) If $\operatorname{cosec} A + \sec A = \operatorname{cosec} B + \sec B$, show that $\tan A \tan B = \cot\left(\frac{A+B}{2}\right)$.

b) If $\tan^{-1} x + \tan^{-1} y + \tan^{-1} z = \frac{\pi}{2}$, show that $xy + yz + zx = 1$. 4+4

8. a) Discuss the continuity at $x = 0$ & $x = 1$ of the following function :

$$f(x) \begin{array}{l} = -x, \text{ when } x \le 0 \\ = x, \text{ when } 0 < x \le 1 \\ = 2x - 1, \text{ when } x > 1. \end{array}$$

b) Find $\frac{dy}{dx}$, where $y = \tan^{-1}\sqrt{\frac{1-\cos x}{1+\cos x}}$. 5+3

9. a) Evaluate : $\lim_{x \to 0} \frac{2^{2x} - 3^{3x}}{x}$.

b) If $y = e^{3\sin^{-1} x}$, prove that $(1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - 9y = 0$. 3+5

10. a) If $g(x) = \frac{x-a}{x} + \frac{x}{x-b}$, prove that $g\left(\frac{a+b}{2}\right) = \frac{4ab}{a^2 - b^2}$.

b) If the area of a circle changes uniformly with respect to time, prove that the rate of change of its circumference varies inversely with its radius. 4+4