Chapter Name: Limit

1

The value of

$$\lim_{x\to 0} \frac{\sin\frac{x}{3}}{x}$$

(a) 3

(b) $\frac{1}{3}$

(c) 0

(d) none of these

Solution:

$$\lim_{x\to 0} \frac{\sin\frac{x}{3}}{x}$$

$$= \lim_{x\to 0} \frac{\sin\frac{x}{3}}{\frac{x}{3}} \times \frac{\frac{x}{3}}{x}$$

$$= 1 \times \frac{1}{3}$$

$$= \frac{1}{3}$$

2

The value of

$$\lim_{x\to 0} \frac{\tan x}{x}$$

(a) −1

(b) 1

(c) 0

(d) none of these

Solution:

$$\lim_{x\to 0} \frac{\tan x}{x}$$

$$= \lim_{x\to 0} \frac{\sin x}{x\cos x}$$

$$= 1 \times 1$$

$$= 1$$

Formulas are at the Last Page

3

The value of

$$\lim_{y \to 1} \frac{y-1}{\sqrt{y}-1}$$

(a) 1

(b) 0

(c) 2

(d) none of these

Solution:

$$\lim_{y \to 1} \frac{y-1}{\sqrt{y}-1}$$

$$= \lim_{y \to 1} \frac{(y-1)(\sqrt{y}+1)}{(y-1)}$$

$$= \lim_{y \to 1} (\sqrt{y}+1)$$

$$= 1 + 1$$

$$= 2$$

4

The value of

$$\lim_{x \to 0} \frac{x}{\sqrt{1+x}-1}$$

(a) $\frac{1}{2}$

(b) 0

(c) 1

(d) none of these

Solution:

$$\lim_{x \to 0} \frac{x}{\sqrt{1+x}-1}$$

$$= \lim_{x \to 0} \frac{x(\sqrt{1+x}+1)}{(1+x)-1}$$

$$= \lim_{x \to 0} (\sqrt{1+x}+1)$$

$$= 1 + 1$$

$$= 2$$

5

The value of

$$\lim_{x \to \frac{\pi}{2}} \frac{\cos^2 x}{1-\sin x}$$

Solution:

$$\lim_{x \to \frac{\pi}{2}} \frac{\cos^2 x}{1 - \sin x}$$

$$= \lim_{x \to \frac{\pi}{2}} \frac{1-\sin^2 x}{1-\sin x}$$

$$= \lim_{x \to \frac{\pi}{2}} \frac{(1+\sin x)(1-\sin x)}{1-\sin x}$$

$$= \lim_{x \to \frac{\pi}{2}} (1 + \sin x)$$

Now at $x = \frac{\pi}{2}$:

$\sin \frac{\pi}{2} = 1$

So, $1 + 1 = 2$

6

The value of

$$\lim_{y \to 0} \frac{e^{3y} - 1}{\log(1 + 5y)}$$

(a) 0

(b) $\frac{5}{3}$

(c) $\frac{3}{5}$

(d) none of these

Solution:

$$\lim_{y\to 0} \frac{e^{3y}-1}{\log(1+5y)}$$

$$= \lim_{y\to 0} \frac{e^{3y}-1}{y} \cdot \frac{y}{\log(1+5y)}$$

$$= 3 \cdot \lim_{y\to 0} \frac{e^{3y}-1}{3y} \cdot \frac{1}{5} \cdot \lim_{y\to 0} \frac{5y}{\log(1+5y)}$$

$$\frac{e^{3y}-1}{3y} \to 1$$

$$\frac{5y}{\log(1+5y)} \to 1$$

So the limit becomes:

$$\frac{3}{5} \times 1 \times 1 = \frac{3}{5}$$

7

The value of

$$\lim_{x\to 0} \frac{3\sin 3x}{5\sin 2x}$$

Solution:

$$\lim_{x \to 0} \frac{3 \sin 3x}{5 \sin 2x}$$

$$= \frac{3}{5} \cdot \lim_{x \to 0} \frac{\sin 3x}{\sin 2x}$$

$$= \frac{3}{5} \cdot \lim_{x \to 0} \frac{\sin 3x}{3x} \cdot \frac{2x}{\sin 2x} \cdot \frac{3x}{2x}$$

$$\frac{\sin 3x}{3x} \to 1$$
$$\frac{2x}{\sin 2x} \to 1$$

So the limit becomes:

$$\frac{3}{5} \cdot 1 \cdot 1 = \frac{3}{5}$$

8

The value of

$$\lim_{x \to 0} \frac{1 - \cos x}{x^2}$$

(a) $\frac{1}{2}$

(b) 1

(c) 2

(d) none of these

Solution:

$$\lim_{x\to 0} \frac{1-\cos x}{x^2}$$

Use identity:

$$1-\cos x = 2\sin^2\tfrac{x}{2}$$

So:

$$\frac{1-\cos x}{x^2} = \frac{2\sin^2(x/2)}{x^2} = 2\cdot\frac{\sin^2(x/2)}{(x/2)^2}\cdot\frac{1}{4}$$

$$\frac{\sin(x/2)}{x/2} \to 1$$

So limit becomes:

$$2\cdot 1\cdot\frac{1}{4} = \frac{1}{2}$$

9

The value of

$$\lim_{x\to 0} \frac{e^{2x}-1}{\sin 3x}$$

Solution:

$$\lim_{x\to 0} \frac{e^{2x}-1}{\sin 3x}$$

$$= \lim_{x\to 0} \frac{e^{2x}-1}{2x} \cdot \frac{2x}{3x} \cdot \frac{3x}{\sin 3x}$$

$$\frac{e^{2x}-1}{2x} \to 1$$

$$\frac{3x}{\sin 3x} \to 1$$

So the limit becomes:

$$1 \cdot \frac{2}{3} \cdot 1 = \frac{2}{3}$$

10

$$\lim_{x\to 0} \frac{e^{\sin x}-1}{\log_e(1+3x)}$$

$$\lim_{x \to 0} \frac{e^{\sin x} - 1}{\log_e(1 + 3x)}$$

$$= \lim_{x \to 0} \frac{e^{\sin x} - 1}{\sin x} \cdot \frac{\sin x}{x} \cdot \frac{x}{\log_e(1 + 3x)}$$

$$\frac{e^{\sin x} - 1}{\sin x} \to 1$$

$$\frac{\sin x}{x} \to 1$$

$$\frac{x}{\log_e(1 + 3x)} = \frac{1}{\frac{\log_e(1+3x)}{x}}$$

$$\lim_{x \to 0} \frac{\log_e(1 + 3x)}{x} = 3$$

$$\frac{x}{\log_e(1 + 3x)} \to \frac{1}{3}$$

$$1 \cdot 1 \cdot \frac{1}{3} = \frac{1}{3}$$

11

$$\lim_{x \to y} \frac{x^{7/2} - y^{7/2}}{x^{3/2} - y^{3/2}}$$

$$\lim_{x \to y} \frac{x^{1/2}\, x^3 - y^{1/2}\, y^3}{x^{3/2} - y^{3/2}}$$

$$= \lim_{x \to y} \frac{x^{1/2}\, x^3 - y^{1/2}\, y^3}{x^{1/2}\, x - y^{1/2}\, y}$$

$$= \lim_{x \to y} \frac{x^{7/2} - y^{7/2}}{x^{3/2} - y^{3/2}}$$

$$[\text{Since } x \to y,\ x - y \neq 0]$$

$$= \lim_{x \to y} \frac{x^{7/2} - y^{7/2}}{x - y} \cdot \frac{x - y}{x^{3/2} - y^{3/2}}$$

$$\left[\lim_{x \to a} \frac{x^n - a^n}{x - a} = n\, a^{n-1}\right]$$

$$= \frac{7}{2} y^{5/2} \div \left(\frac{3}{2} y^{1/2}\right)$$

$$= \frac{7}{2} y^{5/2} \times \frac{2}{3} y^{-1/2}$$

$$= \frac{7}{3} y^2$$

Formulas

- $\lim_{x \to a} c = c$
- $\lim_{x \to a} x = a$
- $\lim_{x \to a} [f(x) + g(x)] = \lim_{x \to a} f(x) + \lim_{x \to a} g(x)$
- $\lim_{x \to a} [f(x) - g(x)] = \lim_{x \to a} f(x) - \lim_{x \to a} g(x)$
- $\lim_{x \to a} [f(x) \cdot g(x)] = \left(\lim_{x \to a} f(x)\right)\left(\lim_{x \to a} g(x)\right)$
- $\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)}$ (if denominator ≠ 0)
- $\lim_{x \to a} x^n = a^n$
- $\lim_{x \to a} \frac{x^n - a^n}{x - a} = na^{n-1}$
- $\lim_{x \to 0} \frac{\sin x}{x} = 1$
- $\lim_{x \to 0} \frac{\tan x}{x} = 1$
- $\lim_{x \to 0} \frac{1 - \cos x}{x^2} = \frac{1}{2}$
- $\lim_{x \to 0} \frac{\ln(1 + x)}{x} = 1$
- $\lim_{x \to \infty} \left(1 + \frac{1}{x}\right)^x = e$
- $\lim_{x \to 0} (1 + x)^{1/x} = e$
- $\lim_{x \to 0} \frac{a^x - 1}{x} = \ln a$