

Chapter Name: Limit



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The value of

$$\lim_{x o 0}rac{\sinrac{x}{3}}{x}$$

- (a) 3
- (b) $\frac{1}{3}$
- (c) 0
- (d) none of these

Solution:

$$egin{aligned} &\lim_{x o 0} rac{\sinrac{x}{3}}{x} \ &= \lim_{x o 0} rac{\sinrac{x}{3}}{rac{x}{3}} imes rac{rac{x}{3}}{rac{x}{3}} imes rac{x}{3} \ &= 1 imes rac{1}{3} \ &= rac{1}{3} \end{aligned}$$

The value of

$$\lim_{x o 0} rac{ an x}{x}$$

- (a) -1
- (b) 1
- (c) 0
- (d) none of these

Solution:

$$\lim_{x \to 0} \frac{\tan x}{x}$$

$$= \lim_{x \to 0} \frac{\sin x}{x \cos x}$$

$$= 1 \times 1$$

$$= 1$$

Formulas are at the Last Page

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The value of

$$\lim_{y o 1}rac{y-1}{\sqrt{y}-1}$$

- (a) 1
- (b) 0
- (c) 2
- (d) none of these

Solution:

$$egin{aligned} & \lim_{y o 1} rac{y-1}{\sqrt{y}-1} \ & = \lim_{y o 1} rac{(y-1)(\sqrt{y}+1)}{(y-1)} \ & = \lim_{y o 1} (\sqrt{y}+1) \ & = 1+1 \ & = 2 \end{aligned}$$

The value of

$$\lim_{x o rac{\pi}{2}}rac{\cos^2x}{1-\sin x}$$

The value of

$$\lim_{x o 0}rac{x}{\sqrt{1+x}-1}$$

- (a) $\frac{1}{2}$
- (b) 0
- (c) 1
- (d) none of these

Solution:

$$egin{array}{lll} \lim_{y o 1} rac{y}{\sqrt{y}-1} & \lim_{x o 0} rac{x}{\sqrt{1+x}-1} \ = \lim_{y o 1} rac{(y-1)(\sqrt{y}+1)}{(y-1)} & = \lim_{x o 0} rac{x(\sqrt{1+x}+1)}{(1+x)-1} \ = \lim_{x o 0} (\sqrt{1+x}+1) \ = \lim_{x o 0} (\sqrt{1+x}+1) \ = \lim_{x o 0} (\sqrt{1+x}+1) \ = 1+1 \ = 2 \ = 2 \end{array}$$

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Solution:

$$\lim_{x o rac{\pi}{2}}rac{\cos^2x}{1-\sin x}$$

$$=\lim_{x o \frac{\pi}{2}} \frac{1-\sin^2 x}{1-\sin x}$$

$$=\lim_{x o rac{\pi}{2}}rac{(1+\sin x)(1-\sin x)}{1-\sin x}$$

$$=\lim_{x o rac{\pi}{2}}(1+\sin x)$$

Now at $x=\frac{\pi}{2}$:

$$\sin \frac{\pi}{2} = 1$$

So,
$$1+1=2$$

The value of

$$\lim_{y o 0}rac{e^{3y}-1}{\log(1+5y)}$$

- (a) 0
- (b) $\frac{5}{3}$
- (c) $\frac{3}{5}$
- (d) none of these

$$\lim_{y o 0}rac{e^{3y}-1}{\log(1+5y)}$$

$$=\lim_{y o 0}rac{e^{3y}-1}{y}\cdotrac{y}{\log(1+5y)}$$

$$= 3 \cdot \lim_{y o 0} rac{e^{3y}-1}{3y} \cdot rac{1}{5} \cdot \lim_{y o 0} rac{5y}{\log(1+5y)}$$

$$rac{e^{3y}-1}{3y}
ightarrow 1$$

$$rac{5y}{\log(1+5y)}
ightarrow 1$$

So the limit becomes:

$$rac{3}{5} imes 1 imes 1 = rac{3}{5}$$

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The value of

$$\lim_{x o 0} rac{3\sin 3x}{5\sin 2x}$$

$$\lim_{x o 0}rac{3\sin 3x}{5\sin 2x}$$

$$=rac{3}{5}\cdot \lim_{x o 0}rac{\sin 3x}{\sin 2x}$$

$$= \frac{3}{5} \cdot \lim_{x \to 0} \frac{\sin 3x}{3x} \cdot \frac{2x}{\sin 2x} \cdot \frac{3x}{2x}$$

$$rac{\sin 3x}{3x}
ightarrow 1 \ rac{2x}{\sin 2x}
ightarrow 1$$

So the limit becomes:

$$\frac{3}{5} \cdot 1 \cdot 1 = \frac{3}{5}$$

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The value of

$$\lim_{x o 0}rac{1-\cos x}{x^2}$$

- (a) $\frac{1}{2}$
- (b) 1
- (c) 2

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(d) none of these

$$\lim_{x o 0}rac{1-\cos x}{x^2}$$

Use identity:

$$1 - \cos x = 2\sin^2\frac{x}{2}$$

So:

$$\frac{1-\cos x}{x^2} = \frac{2\sin^2(x/2)}{x^2} = 2 \cdot \frac{\sin^2(x/2)}{(x/2)^2} \cdot \frac{1}{4}$$

$$rac{\sin(x/2)}{x/2}
ightarrow 1$$

So limit becomes:

$$2\cdot 1\cdot \frac{1}{4}=\frac{1}{2}$$

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The value of

$$\lim_{x o 0} rac{e^{2x}-1}{\sin 3x}$$

$$\lim_{x o 0}rac{e^{2x}-1}{\sin 3x}$$

$$=\lim_{x\to 0} \frac{e^{2x}-1}{2x}\cdot \frac{2x}{3x}\cdot \frac{3x}{\sin 3x}$$

$$rac{e^{2x}-1}{2x}
ightarrow 1$$

$$rac{3x}{\sin 3x}
ightarrow 1$$

So the limit becomes:

$$1\cdot\frac{2}{3}\cdot 1=\frac{2}{3}$$

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$$\lim_{x o 0}rac{e^{\sin x}-1}{\log_e(1+3x)}$$

$$egin{aligned} &\lim_{x o 0} rac{e^{\sin x}-1}{\log_e(1+3x)} \ &= \lim_{x o 0} rac{e^{\sin x}-1}{\sin x} \cdot rac{\sin x}{x} \cdot rac{x}{\log_e(1+3x)} \ &rac{e^{\sin x}-1}{\sin x} o 1 \ &rac{\sin x}{x} o 1 \ &rac{\cos(1+3x)}{x} o 1 \end{aligned}$$

$$rac{x}{\log_e(1+3x)}
ightarrowrac{1}{3}$$

$$1 \cdot 1 \cdot \frac{1}{3} = \frac{1}{3}$$

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$$\lim_{x o y}rac{x^{7/2}-y^{7/2}}{x^{3/2}-y^{3/2}}$$

$$\lim_{x \to y} \frac{x^{1/2} \, x^3 - y^{1/2} \, y^3}{x^{3/2} - y^{3/2}}$$

$$=\lim_{x o y}rac{x^{1/2}\,x^3-y^{1/2}\,y^3}{x^{1/2}\,x-y^{1/2}\,y}$$

$$=\lim_{x o y}rac{x^{7/2}-y^{7/2}}{x^{3/2}-y^{3/2}}$$

[Since $x \to y, \ x - y \neq 0$]

$$=\lim_{x o y}rac{x^{7/2}-y^{7/2}}{x-y}\cdotrac{x-y}{x^{3/2}-y^{3/2}}$$

$$\left[\lim_{x o a}rac{x^n-a^n}{x-a}=n\,a^{\,n-1}
ight]$$

$$=rac{7}{2}\,y^{5/2}\div\left(rac{3}{2}\,y^{1/2}
ight)$$

$$=rac{7}{2}y^{5/2} imesrac{2}{3}y^{-1/2}$$

$$=rac{7}{3}y^2$$





- $\lim c = c$
- $\lim x = a$
- $\lim_{x o a}[f(x)+g(x)]=\lim_{x o a}f(x)+\lim_{x o a}g(x)$
- $egin{aligned} &\lim_{x o a}[f(x)-g(x)]=\lim_{x o a}f(x)-\lim_{x o a}g(x)\ &\lim_{x o a}[f(x)\cdot g(x)]=\left(\lim_{x o a}f(x)
 ight)\left(\lim_{x o a}g(x)
 ight) \end{aligned}$
- $\lim_{x o a} rac{f(x)}{g(x)} = rac{\lim_{x o a} f(x)}{\lim_{x o a} g(x)}$ (if denominator eq 0)
- $\lim x^n = a^n$
- $\lim^{x \to a} \frac{x^n a^n}{n} = na^{n-1}$ $x \rightarrow a \quad x - a$
- $\lim \frac{\sin x}{x} = 1$ $x \rightarrow 0$ x
- $\lim \frac{\tan x}{x} = 1$ x
- $\lim_{x \to 0} \frac{1 \cos x}{x^2} = \frac{1}{2}$
- $\lim_{x o 0} rac{\ln(ilde{1} + x)}{x} = 1$
- $\lim_{x \to 0} (1+x)^{1/x} = e$
- $\lim_{x \to 0} \frac{a^x 1}{x} = \ln a$



