VECTOR (DOT PRODUCT)

Formula

① If $\vec{a} = a_1 i + a_2 j + a_3 K$

$\vec{b} = b_1 i + b_2 j + b_3 K$

$\therefore \vec{a} \cdot \vec{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$

② The angle $(\theta) = \cos^{-1} \dfrac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|}$

③ If $\vec{a} = a_1 i + a_2 j + a_3 K$

$\therefore |\vec{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$

④ Work Done $(W) = \vec{F} \cdot \vec{d}$

where; F = Resultant Force

d = Displacement

⑤ $i \cdot i = 1,\ j \cdot j = 1,\ K \cdot K = 1$
$i \cdot j = 0,\ j \cdot K = 0,\ K \cdot i = 0$ } unit vectors DOT product

⑥ Projection of $\vec{A}$ on $\vec{B} = \dfrac{\vec{A} \cdot \vec{B}}{|\vec{B}|}$

⑦ Parallel vectors : $\vec{A} \cdot \vec{B} = |\vec{A}||\vec{B}|$

⑧ Perpendicular vectors : $\vec{A} \cdot \vec{B} = 0$

⑨ Anti-parallel vectors : $\vec{A} \cdot \vec{B} = -|\vec{A}||\vec{B}|$

DOT Product

① If $a = 2i - 2j + k$ & $b = 2i - 3j - k$ then $\vec{a} \cdot \vec{b} = ?$

⇒ Given; $a = 2i - 2j + k$

$b = 2i - 3j - k$

$\therefore \vec{a} \cdot \vec{b} = (2 \times 2) + (-2 \times -3) + 1 \times (-1)$

$= 4 + 6 - 1$

$= 10 - 1$

$= 9$ Ans

② If $a = 2i - 2j + k$ & $b = 3i + 2j + k$, then what is the value of $\vec{a} \cdot \vec{b}$

⇒ Given; $a = 2i - 2j + k$

$b = 3i + 2j + k$

$\therefore \vec{a} \cdot \vec{b} = (2 \times 3) + (-2 + 2) + (1 \times 1)$

$= 6 - 4 + 1$

$= 2 + 1$

$= 3$ Ans

③ What will the projection of vector $\vec{A} = i - 2j + k$ on the vector $\vec{B} = 4i - 4j + 7k$

⇒ Given; $\vec{A} = i - 2j + k$

$\vec{B} = 4i - 4j + 7k$

∴ The projection of $\vec{A}$ on $\vec{B}$

$$= \frac{\vec{A}\cdot\vec{B}}{|\vec{B}|}$$

$$= \frac{(i-2j+k)\cdot(4i-4j+7k)}{\sqrt{4^2+(-4)^2+7^2}}$$

$$= \frac{1\times4+(-2)\times(-4)+1\times7}{\sqrt{16+16+49}}$$

$$= \frac{4+8+7}{9}$$

$= 19/9$ Ans

(4) If $(mi-3j+4k)\cdot(2i+4j+mk)=0$, then $m=?$

$\Rightarrow$ ∴ $(mi-3j+4k)\cdot(2i+4j+mk)=0$

or, $m\times2+(-3)\times4+4\times m=0$

or, $2m-12+4m=0$

or, $6m-12=0$

or, $6m=12$

∴ $m=12/6=2$ Ans

(5) Find the angle between the vectors $(2i+4j-7k)$ & $(3i+2j+2k)$

$\Rightarrow$ Let; $a = 2i+4j-7k$

& $b = 3i+2j+2k$

$\therefore a \cdot b = 2\times3 + 4\times2 + (-7)\times2$

$= 6+8-14$

$= 14-14$

$= 0$

$\therefore |\vec{a}| = \sqrt{2^2+4^2+(-7)^2} = \sqrt{69}$

$\therefore |\vec{b}| = \sqrt{3^2+2^2+2^2} = \sqrt{17}$

$\therefore$ The angle is

$\therefore \cos\theta = \dfrac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|} = \dfrac{0}{\sqrt{69}\cdot\sqrt{17}} = 0$

$\therefore \cos\theta = 0$

$\therefore \theta = \cos^{-1}(0)$

$\therefore \theta = 90^\circ$

$\therefore \theta = \pi/2$ Ans

(6) Find the angle between the vectors $4i+2j-7k$ & $3i+2j+2k$

$\Rightarrow$ Let's $a = 4i+2j-7K$.

& $b = 3i+2j+2K$

$\therefore \vec{a}\cdot\vec{b} = 4\times3+2\times2+(-7)\times2$

$= 12+4-14$

$= 16-14$

$= 2$

$\therefore |\vec{a}| = \sqrt{4^2+2^2+(-7)^2} = \sqrt{69}$

$\therefore |\vec{b}| = \sqrt{3^2+2^2+2^2} = \sqrt{17}$

$\therefore$ The angle is

$\therefore \theta = \cos^{-1}\left\{\frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}\right\}$

$= \cos^{-1}\left(\frac{2}{\sqrt{69}\cdot\sqrt{17}}\right)$

$= \cos^{-1}\left(\frac{2}{\sqrt{1173}}\right)$ Ans

(7) If the vector $(2i+j-K)$ is perpendicular to the vector $(i-4j+\lambda K)$ then $\lambda = ?$

$\Rightarrow$ As both of the vector are perpendicular to each other, so

$\therefore (2i+j-K)\cdot(i-4j+\lambda K) = 0$

or, $2\times1+1\times(-4)+(-1)\times\lambda = 0$

or, $2-4-\lambda = 0$

or, $-2-\lambda = 0$

or, $-\lambda = -2$

$\therefore \lambda = 2$ Ans

(8) If $a = 2i + 2j + K$ & $b = i + j - K$, then the projection of $\vec{a}$ on $\vec{b}$ is = ?

$\Rightarrow$ Given; $a = 2i + 2j + K$

$b = i + j - K$

$\therefore \vec{a} \cdot \vec{b} = 2 \times 1 + 2 \times 1 + 1 \times (-1)$

$= 2 + 2 - 1$

$= 4 - 1$

$= 3$

$\therefore |\vec{b}| = \sqrt{1^2 + 1^2 + (-1)^2}$

$= \sqrt{1 + 1 + 1}$

$= \sqrt{3}$

$\therefore$ Projection of $\vec{a}$ on $\vec{b}$ $= \dfrac{\vec{a} \cdot \vec{b}}{|\vec{b}|} = \dfrac{3}{\sqrt{3}}$ Ans

(9) If $\vec{A} = 2i + j + K$, $\vec{B} = i - 2j + 2K$, $\vec{C} = 3i - 4j + 2K$ then the projection of $(\vec{A} + \vec{C})$ in the direction of $\vec{B}$ = ?

$\Rightarrow \therefore \vec{A} + \vec{C} = (2i + j + K) + (3i - 4j + 2K)$

$= 5i - 3j + 3K$

$\therefore (\vec{A} + \vec{C}) \cdot \vec{B} = (5i - 3j + 3K) \cdot (i - 2j + 2K)$

$= 5 \times 1 + (-3) \times (-2) + 3 \times 2$

$= 5 + 6 + 6$

$= 11 + 6$

$= 17$

∴ The projection of $(\vec{A}+\vec{C})$ on direction $\vec{B}$

$$= \frac{(\vec{A}+\vec{C}).\vec{B}}{|\vec{B}|}$$

$$= \frac{17}{\sqrt{1^2+(-2)^2+2^2}}$$

$$= \frac{17}{\sqrt{1+4+4}}$$

$$= \frac{17}{\sqrt{9}}$$

$$= \frac{17}{3}$$ Ans

(10) A particle acted on by constant forces $2i+3j-K$ & $3i-j+5K$ is displaced from the point $A(1,3,2)$ to the point $B(4,5,-1)$. Find the work done by the force.

⟹ Given; $F_1 = 2i+3j-K$ & $F_2 = 3i-j+5K$

∴ Resultant force ⟹ $F = F_1+F_2$

$$= 2i+3j-K+3i-j+5K$$

$$= 5i+2j+4K$$

∴ The displacement $\vec{d} = B(4,5,-1) - A(1,3,2)$

$$= (4i+5j-K) - (i+3j+2K)$$

$$= 3i+2j-3K$$

∴ Work done (W)

$$= \vec{F}.\vec{d} = (5i+2j+4K).(3i+2j-3K)$$

$$= 5\times3+2\times2+4\times(-3)$$

$$= 15+4-12 = 19-12 = 7$$ units Ans

(12) A particle acted on by forces $(4i+j-3k)$ & $(3i+j-k)$ is displaed from the point $A(1,2,3)$ to $B(5,4,1)$. So, find the work done.

→ Let; $F_1 = 4i+j-3K$

$F_2 = 3i+j-K$

$\therefore$ Total force $(F) = F_1 + F_2$

$= 4i+j-3K+3i+j-K$

$= 7i+2j-4K$

$\therefore$ The displacement $(d) = B(5,4,1) - A(1,2,3)$

$= (5i+4j+K) - (i+2j+3K)$

$= 4i+2j-2K$

$\therefore$ Work Done $(W) = \vec{F} \cdot \vec{d}$

$= (7i+2j-4K) \cdot (4i+2j-2K)$

$= 7\times4 + 2\times2 + (-4)\times(-2)$

$= 28+4+8$

$= 28+12$

$= 40$ units Ans

(13) What is the work done by the force $\vec{F} = 2i-3j+2K$ is moving a particle from $A(3,4,5)$ to $B(1,2,3)$.

→ Given; $\vec{F} = 2i-3j+2K$

$\therefore \vec{d} = B(1,2,3) - A(3,4,5)$

$= (i+2j+3K) - (3i+4j+5K)$

$= -2i - 2j - 2k$

$\therefore$ Work Done

$(W) = F \cdot d$

$= (2i - 3j + 2k) \cdot (-2i - 2j - 2k)$

$= 2 \times (-2) + (-3) \times (-2) + 2 \times (-2)$

$= -4 + 6 - 4$

$= 2 - 4$

$= -2$ units Ans

(14) If α, β, γ be three unit vectors & $\alpha + \beta + \gamma = 0$ then show that $\alpha \cdot \beta + \beta \cdot \gamma + \gamma \cdot \alpha = -3/2$

$\Rightarrow \because \alpha, \beta$ & γ are the unit vectors then;

$|\alpha| = 1, \ |\beta| = 1,$ & $|\gamma| = 1$

Given;

$\therefore \alpha + \beta + \gamma = 0$

$\therefore |\alpha + \beta + \gamma|^2 = 0$

$\therefore (\alpha + \beta + \gamma)(\alpha + \beta + \gamma) = 0$

$\therefore \alpha \cdot \alpha + \alpha \cdot \beta + \alpha \cdot \gamma + \alpha \cdot \beta + \beta \cdot \beta + \beta \cdot \gamma + \alpha \cdot \gamma + \beta \cdot \gamma + \gamma \cdot \gamma = 0$

$\therefore 1 + 1 + 1 + 2(\alpha \cdot \beta + \beta \cdot \gamma + \gamma \cdot \alpha) = 0$

$\therefore 3 + 2(\alpha \cdot \beta + \beta \cdot \gamma + \gamma \cdot \alpha) = 0$

$\therefore \alpha \cdot \beta + \beta \cdot \gamma + \gamma \cdot \alpha = -3/2.$ Ans

(15) If $|A-B| = 30$ & $|A+B| = 40$ & $|A| = 17$, so then $|B| = ?$

$\Rightarrow$ Given;

$\therefore |A-B| = 30$

or, $|A-B||A-B| = 900$

or, $|A|^2 + |B|^2 - 2.|A|.|B| = 900$ — (i)

~~or~~, and

$\because |A+B| = 40$

or, $|A+B||A+B| = 1600$

or, $|A|^2 + |B|^2 + 2.|A|.|B| = 1600$ — (ii)

so, by adding (i) & (ii), we get

$\therefore 2(|A|^2 + |B|^2) = 900 + 1600 = 2500$

or, $|A|^2 + |B|^2 = 2500/2 = 1250$

$\therefore |B|^2 = 1250 - |A|^2$

$= 1250 - 17^2$

$= 961$

$\therefore |B| = \sqrt{961} = 31$ Ans

CROSS PRODUCT

Formula

① If $a = a_1 i + a_2 j + a_3 k$

$b = b_1 i + b_2 j + b_3 k$

$$\therefore a \times b = \begin{vmatrix} i & j & k \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

② vector perpendicular a & b both

$= \pm (a \times b)$

③ Unit vector perpendicular $= \pm \frac{(a \times b)}{|a \times b|}$

④ Area of parallelogram $= |a \times b|$

⑤ vector area of " $= a \times b$

⑥ Area of traingle $= \frac{1}{2} |a \times b|$

⑦ vector area " " $= \frac{1}{2} (a \times b)$

⑧ Condition of parallel to each other

$$\boxed{a \times b = 0}$$

① If $\vec{a} = 4i + 2j - 5k$ & $\vec{b} = -12i + 15k - 6j$ then $\vec{a} \times \vec{b} = ?$

$\Rightarrow$ Given; $\vec{a} = 4i + 2j - 5k$ & $\vec{b} = -12i + 15k - 6j$

$$\therefore \vec{a} \times \vec{b} = \begin{vmatrix} i & j & k \\ 4 & 2 & -5 \\ -12 & -6 & 15 \end{vmatrix}$$

$$= i\{2 \times 15 - (-6) \times (-5)\} - j\{4 \times 15 - (-5) \times (-12)\} + k\{4 \times (-6) - (-12) \times 2\}$$

$$= i(\cancel{30} - \cancel{30}) - j(\cancel{60} - \cancel{60}) + k(-\cancel{24} + \cancel{24})$$

$$= 0 \text{ Ans}$$

② If $a = 2i - 3j + k$ & $b = i + 4j - 2k$, find cross product.

$\Rightarrow$ Given; $\vec{a} = 2i - 3j + k$

$\vec{b} = i + 4j - 2k$

$$\therefore \vec{a} \times \vec{b} = \begin{vmatrix} i & j & k \\ 2 & -3 & 1 \\ 1 & 4 & -2 \end{vmatrix}$$

$$= \cancel{i\{2 \times (-2)\}}$$

$$= i\{6 - 4\} - j\{-4 - 1\} + k\{8 + 3\}$$

$$= 2i + 5j + 11k \text{ Ans}$$

③ Find the vector perpendicular to both $2i + 3j - k$ & $3i - j + 2k$.

$\Rightarrow$ Let; $\vec{a} = 2i + 3j - k$ & $\vec{b} = 3i - j + 2k$

$\therefore \vec{a}\times\vec{b} = \begin{vmatrix} i & j & k \\ 2 & 3 & -1 \\ 3 & -1 & 2 \end{vmatrix}$

$= i(6-1) - j(4+3) + k(-2-9)$

$= 5i - 7j - 11k$

$\therefore$ The vector perpendicular to the both vector is

$= \pm(5i - 7j - 11k)$ Ans

(4) If $\vec{A} = i - 3j + k$ & $\vec{B} = -2i + j + k$ then $|\vec{A}\times\vec{B}| = ?$

$\Rightarrow$ Given, $\vec{A} = i - 3j + k$

& $\vec{B} = -2i + j + k$

$\therefore \vec{A}\times\vec{B} = \begin{vmatrix} i & j & k \\ 1 & -3 & 1 \\ -2 & 1 & 1 \end{vmatrix}$

$= i(-3-1) - j(1+2) + k(1-6)$

$= -4i - 3j - 5k$

$\begin{array}{r|l} 2 & 50 \\ 5 & 25 \\ & 5 \end{array}$

$\therefore |\vec{A}\times\vec{B}| = \sqrt{(-4)^2 + (-3)^2 + (-5)^2}$

$= \sqrt{16+9+25}$

$= \sqrt{25+25}$

$= \sqrt{50}$

$= 5\sqrt{2}$ Ans

⑤ Find the vector area of a triangle whose adjacent sides are $-i+2j+k$ & $3i-2j+2k$

$\Rightarrow$ Let; $\vec{a} = -i+2j+k$

$\vec{b} = 3i-2j+2k$

$\therefore$ The vector area of a triangle is

$= \frac{1}{2}(\vec{a}\times\vec{b})$

$= \frac{1}{2}\begin{vmatrix} i & j & k \\ -1 & 2 & 1 \\ 3 & -2 & 2 \end{vmatrix}$

$= \frac{1}{2}[i(4+2) - j(-2-3) + k(2-6)]$

$= \frac{1}{2}(6i+5j-4k)$ Ans

⑥ If $a = 2i-j+2k$ & $b = -3i+j-3k$, find the unit vector perpendicular to the both a & b

$\Rightarrow$ Given; $a = 2i-j+2k$ & $b = -3i+j-3k$

$\therefore \vec{a}\times\vec{b} = \begin{vmatrix} i & j & k \\ 2 & -1 & 2 \\ -3 & 1 & -3 \end{vmatrix}$

$= i(3-2) - j(-6+6) + k(2-3)$

$= i-k$

$\therefore$ perpendicular to the both $= \pm(i-k)$

$\therefore$ Unit vector perpendicular $= \pm\frac{i-k}{\sqrt{1^2+(-1)^2}}$

$= \pm\frac{i-k}{\sqrt{2}}$

$= \pm\frac{1}{\sqrt{2}}(i-k)$ Ans

(7) Find unit vector perpendicular to both the vectors $(3i+j+2k)$ & $(2i-2j+4k)$ and also find the angle between them.

$\Rightarrow$ Given, $a = 3i+j+2k$ & $b = 2i-2j+4k$

$\therefore \vec{a}\times\vec{b} = 8i-8j-8k$ (Like previous method)

$\therefore$ perpendicular to both vector $= \pm(8i-8j-8k)$

$\therefore$ Unit vector perpendicular

$$= \pm \frac{8i-8j-8k}{\sqrt{8^2+(-8)^2+(-8)^2}}$$

$$= \pm \frac{\cancel{8}(i-j-k)}{\cancel{8}\sqrt{3}}$$

$$= \pm \frac{1}{\sqrt{3}}(i-j-k) \text{ Ans}$$

$\therefore$ Angle between them

$$\therefore \theta = \cos^{-1}\left[\frac{(3i+j+2k)\cdot(2i-2j+4k)}{\sqrt{3^2+1^2+2^2}\cdot\sqrt{2^2+(-2)^2+4^2}}\right]$$

$$= \cos^{-1}\left[\frac{3\times2+1\times(-2)+2\times4}{\sqrt{14}\cdot\sqrt{24}}\right]$$

$$= \cos^{-1}\frac{\cancel{12}^{\,3}}{\cancel{4}\sqrt{21}}$$

$$= \cos^{-1}\frac{3}{\sqrt{21}}$$

$$= \cos^{-1}\sqrt{\frac{3}{7}} \text{ Ans}$$

(8) The two adjacent sides are $2i-k$ & $j+2k$ of a traingle then what will be the area of it.

⇒ Let; $\vec{A} = 2i-k$

$\vec{B} = j+2k$

∴ The area of the traingle $= \frac{1}{2}|\vec{A}\times\vec{B}|$

∴ $\vec{A}\times\vec{B} = \begin{vmatrix} i & j & k \\ 2 & 0 & -1 \\ 0 & 1 & 2 \end{vmatrix} = i(0\times2+1) - j(4-0) + k(2-0)$

$= i-4j+2k$

∴ $|\vec{A}\times\vec{B}| = \sqrt{1^2+4^2+2^2} = \sqrt{21}$

∴ The area of the traingle

$= \frac{1}{2}\sqrt{21}$ Ans sq. unit

(9) Find the area of a parallelogram formed by two vectors $3i+2j$ & $2j-4k$.

⇒ Let; $\vec{A} = 3i+2j$

& $\vec{B} = 2j-4k$

∴ The area will be $= |\vec{A}\times\vec{B}|$

∴ The area of the parallelogram

∴ The area

$= \vec{A}\times\vec{B} = \begin{vmatrix} i & j & k \\ 3 & 2 & 0 \\ 0 & 2 & -4 \end{vmatrix}$

$= i(-8-0) - j(-12-0) + k(6-0)$

$= -8i+12j+6k$

∴ $|\vec{A}\times\vec{B}| = \sqrt{(-8)^2+12^2+6^2} = \sqrt{244} = 2\sqrt{61}$ sq. unit Ans

(10) Find the sine angle between the vector $-2i-j-k$ & $3i+4j-k$

$\Rightarrow$ Let, $\vec{A} = -2i-j-k$

& $\vec{B} = 3i+4j-k$

$\therefore \vec{A}\times\vec{B} = \begin{vmatrix} i & j & k \\ -2 & -1 & -1 \\ 3 & 4 & -1 \end{vmatrix}$

$= 5i - 5j - 5k$

$\therefore |\vec{A}\times\vec{B}| = \sqrt{5^2+(-5)^2+(-5)^2} = 5\sqrt{3}$

$\therefore |\vec{A}| = \sqrt{(-2)^2+(-1)^2+(-1)^2} = \sqrt{6}$

$\therefore |\vec{B}| = \sqrt{3^2+4^2+(-1)^2} = \sqrt{26}$

$\therefore \sin\theta = \frac{|A\times B|}{|A|\,|B|} = \frac{5\sqrt{3}}{\sqrt{6}\cdot\sqrt{26}} = \frac{5\sqrt{52}}{52}$ Ans

(11) Find the length of the vector c where $c = a\times b$ & $a = i-j+k$ & $b = 2i+j+3k$

$\Rightarrow \therefore a\times b = \begin{vmatrix} i & j & k \\ 1 & -1 & 1 \\ 2 & 1 & 3 \end{vmatrix}$

$= i(-3-1) - j(3-2) + k(1+2)$

$= -4i - j + 3k$

$\therefore c = -4i - j + 3k$

$\therefore$ Length of $c = |\vec{c}| = \sqrt{(-4)^2+(-1)^2+3^2}$

$= \sqrt{16+1+9}$

$= \sqrt{26}$ unit Ans

(12) If $|a| = 3$, $|b| = 2$ & $a \cdot b = 0$, then $|a \times b| = ?$

$\Rightarrow$ we know that;

$$a \cdot b = |a||b| \cos\theta = 0$$

$\therefore \cos\theta = 0 = \cos\frac{\pi}{2}$

$\therefore \theta = \frac{\pi}{2}$

Now; $|a \times b| = |a||b| \sin\theta$

$= 3 \times 2 \times \sin\frac{\pi}{2}$

$= 3 \times 2 \times 1$

$= 6$ An