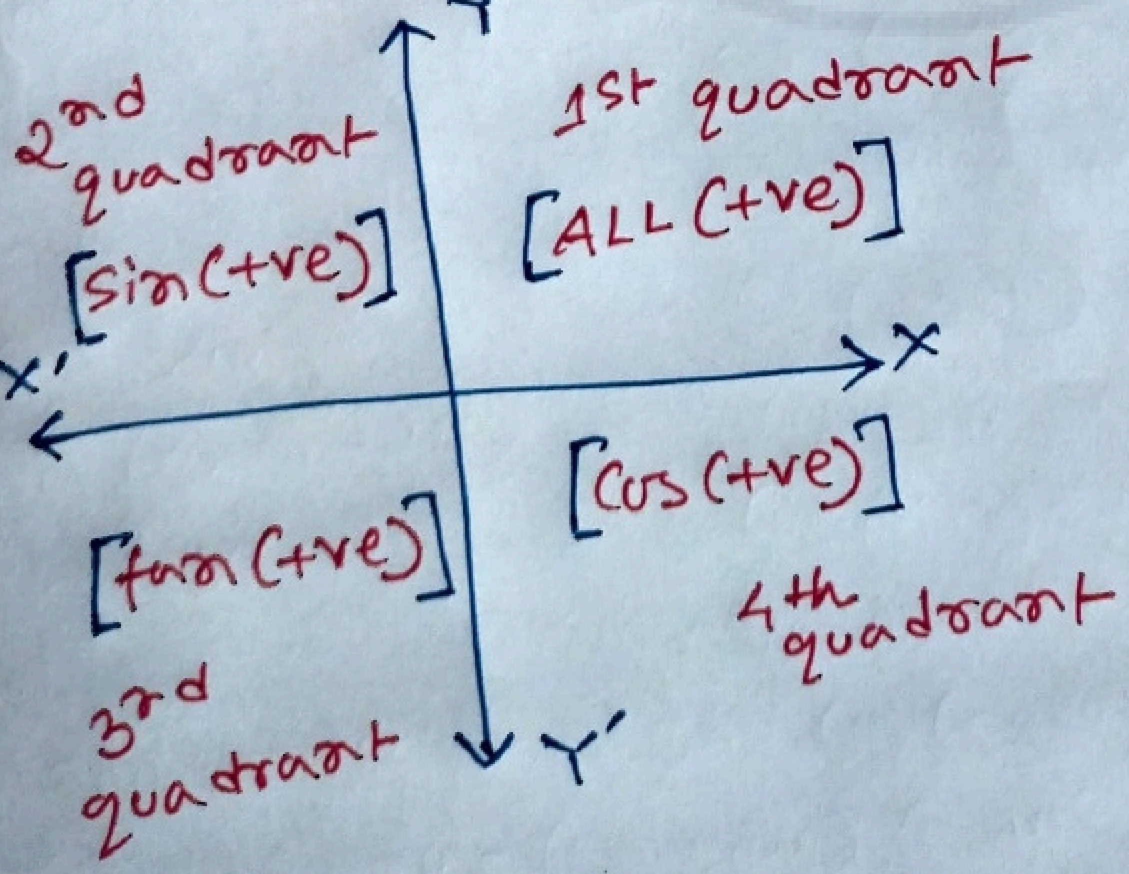
TRIGONOMETRIC RATIO
(FORMULAS)

1. $\sin^2\theta + \cos^2\theta = 1$
2. $\sec^2\theta - \tan^2\theta = 1$
3. $\operatorname{cosec}^2\theta - \cot^2\theta = 1$
4. $\sin\theta = 1/\operatorname{cosec}\theta$ or $\operatorname{cosec}\theta = 1/\sin\theta$
5. $\cos\theta = 1/\sec\theta$ or $\sec\theta = 1/\cos\theta$
6. $\tan\theta = 1/\cot\theta$ or, $\cot\theta = 1/\tan\theta$
7. $\sin\theta \times \operatorname{cosec}\theta = 1$
8. $\cos\theta \times \sec\theta = 1$
9. $\tan\theta \times \cot\theta = 1$
10. $\tan\theta = \sin\theta/\cos\theta$
11. $\cot\theta = \cos\theta/\sin\theta$
12. $\sin(-\theta) = -\sin\theta$
    $\operatorname{cosec}(-\theta) = -\operatorname{cosec}\theta$
    $\cos(-\theta) = \cos\theta$
    $\sec(-\theta) = \sec\theta$
    $\tan(-\theta) = -\tan\theta$
    $\cot(-\theta) = -\cot\theta$
13.

	0°	30°	45°	60°	90°
sin	0	$1/2$	$1/\sqrt{2}$	$\sqrt{3}/2$	1
cos	1	$\sqrt{3}/2$	$1/\sqrt{2}$	$1/2$	0
tan	0	$1/\sqrt{3}$	1	$\sqrt{3}$	∞

14.



TRIGONOMETRIC RATIO OF ASSOCIATE

① Evalute $\cot(-1575^\circ)$

$\Rightarrow \cot(-1575^\circ)$

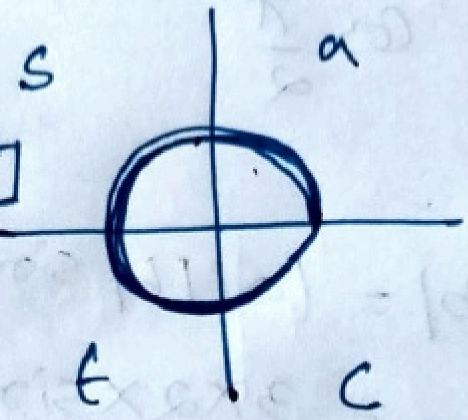
$= -\cot 1575^\circ \;[\because \cot(-\theta) = -\cot\theta]$

$= -\cot(17\times 90^\circ + 45^\circ)$

$= -(-\tan 45^\circ)$

$= \tan 45^\circ$

$= 1$ An



② Evaluate $\cos(-1170^\circ)$

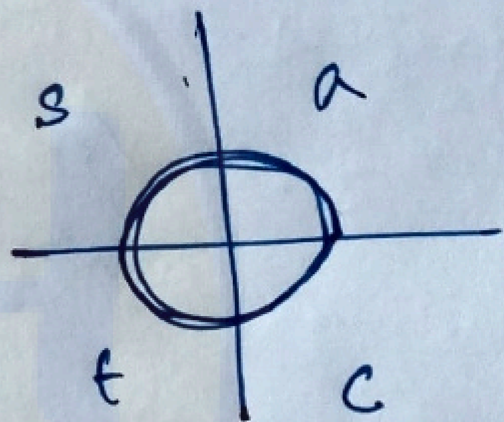
$\Rightarrow \cos(-1170^\circ)$

$= \cos 1170^\circ \;[\because \cos(-\theta) = \cos\theta]$

$= \cos(12\times 90^\circ + 90^\circ)$

$= \cos 90^\circ$

$= 0$ An



③ $\sin\theta + \operatorname{cosec}\theta = 2$, then $\sin^n\theta + \operatorname{cosec}^n\theta = ?$

$\Rightarrow \because \sin\theta + \operatorname{cosec}\theta = 2$

or, $\sin\theta + \dfrac{1}{\sin\theta} = 2$

or, $\sin^2\theta + 1 = 2\sin\theta$

or, $\sin^2\theta - 2\sin\theta + 1 = 0$

or, $(\sin\theta - 1)^2 = 0$

$\therefore \sin\theta = 1 = \sin\dfrac{\pi}{2}$

$\therefore \theta = \dfrac{\pi}{2}$

$\therefore \sin^n\theta + \operatorname{cosec}^n\theta$

$= (\sin\theta)^n + (\operatorname{cosec}\theta)^n$

$= \left(\sin\dfrac{\pi}{2}\right)^n + \left(\operatorname{cosec}\dfrac{\pi}{2}\right)^n$

$= 1^n + 1^n$

$= 1 + 1$

$= 2$ An

④ If $\tan 25^\circ = p$, then prove that $\frac{\tan 155^\circ - \tan 115^\circ}{1 + \tan 155^\circ \tan 115^\circ} = \frac{1-p^2}{2p}$

$\Rightarrow \because \tan 155^\circ = \tan(2\times 90^\circ - 25^\circ)$

$= -\tan 25^\circ = -p$

$\because \tan 115^\circ = \tan(1\times 90^\circ + 25^\circ)$

$= -\cot 25^\circ$

$= -\frac{1}{\tan 25^\circ} = -\frac{1}{p}$

$\therefore$ L.H.S.

$= \frac{\tan 155^\circ - \tan 115^\circ}{1+\tan 155^\circ \tan 115^\circ}$

$= \frac{-p+\frac{1}{p}}{1+(\cancel{-p})\times(-\frac{1}{\cancel{p}})}$

$= \frac{(-p^2+1)/p}{1+1}$

$= \frac{-p^2+1}{2p}$

$= \frac{1-p^2}{2p}$ (proved)

⑤ prove that $\sqrt{\frac{1+\sin\theta}{1-\sin\theta}} = \tan\theta + \sec\theta$

$\Rightarrow$ L.H.S.

$= \sqrt{\frac{1+\sin\theta}{1-\sin\theta}} = \sqrt{\frac{(1+\sin\theta)(1+\sin\theta)}{(1-\sin\theta)(1+\sin\theta)}}$

$= \sqrt{\frac{(1+\sin\theta)^2}{1-\sin^2\theta}}$

$= \sqrt{\frac{(1+\sin\theta)^2}{\cos^2\theta}}$

$= \frac{1+\sin\theta}{\cos\theta} = \frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta}$

$= \sec\theta + \tan\theta$

$= \tan\theta + \sec\theta$

= R.H.S. (proved)

⑥ prove that $1+\frac{\tan^2\alpha}{1+\sec\alpha}=\sec\alpha$

$\Rightarrow$ L.H.S.

$$=1+\frac{\tan^2\alpha}{1+\sec\alpha}$$

$$=\frac{1+\sec\alpha+\tan^2\alpha}{1+\sec\alpha}$$

$$=\frac{\sec^2\alpha-\cancel{\tan^2\alpha}+\sec\alpha+\cancel{\tan^2\alpha}}{1+\sec\alpha}$$

$$=\frac{\sec\alpha(\sec\alpha+1)}{(\sec\alpha+1)}$$

$=\sec\alpha$

$=$ R.H.S. (proved)

⑦ $\cos 210^\circ \sec(-930^\circ)-\sin(-45^\circ)\sin 945^\circ$

$\Rightarrow$ ~~cos 210~~

$\therefore \cos 210^\circ$

$=\cos(2\times 90^\circ+30^\circ)$

$=-\cos 30^\circ$

$=-\frac{\sqrt{3}}{2}$

$\therefore \sec(-930^\circ)$

$=\sec 930^\circ$

$=\sec(10\times 90^\circ+30^\circ)$

$=-\sec 30^\circ$

$=-2/\sqrt{3}$

$\therefore \sin(-45^\circ)$

$=-\sin 45^\circ$

$=-\frac{1}{\sqrt{2}}$

$\therefore \sin 945^\circ$

$=\sin(10\times 90^\circ+45^\circ)$

$=-\sin 45^\circ$

$=-\frac{1}{\sqrt{2}}$

$\therefore \cos 210^\circ \sec(-930^\circ) - \sin(-45^\circ) \cdot \sin 945^\circ$

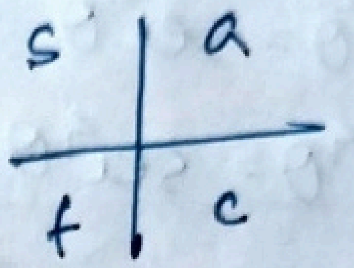
$= \left(-\frac{\sqrt{3}}{2}\right) \times \left(-\frac{2}{\sqrt{3}}\right) - \left(-\frac{1}{\sqrt{2}}\right) \times \left(-\frac{1}{\sqrt{2}}\right)$

$= \frac{\sqrt{3}}{2} \times \frac{2}{\sqrt{3}} - \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}}$

$= 1 - \frac{1}{2}$

$= \frac{1}{2}$ Ans

(8) prove that $\tan 9^\circ \tan 27^\circ \tan 63^\circ \tan 81^\circ = 1$



$\Rightarrow$ L.H.S.

$\cancel{\tan 27}$

$= \tan 9^\circ \tan 27^\circ \tan 63^\circ \tan 81^\circ$

$= \tan 9^\circ \tan 27^\circ \tan(1 \times 90^\circ - 27^\circ) \tan(1 \times 90^\circ - 9^\circ)$

$= \tan 9^\circ \tan 27^\circ \cdot \cot 27^\circ \cdot \cot 9^\circ$

$= \tan 9^\circ \cdot \cot 9^\circ \cdot \tan 27^\circ \cdot \cot 27^\circ$

$= 1 \times 1$ $[\because \tan\theta \times \cot\theta = 1]$

$= 1$

(9) If $\cos^4\theta + \cos^2\theta = 1$, then show that $\sec^4\theta - \sec^2\theta = 1$

$\Rightarrow$ Given,

$\cos^4\theta + \cos^2\theta = 1$

$\therefore \cos^4\theta = 1 - \cos^2\theta$

$\therefore$ L.S.

$= \sec^4\theta - \sec^2\theta$

$= \frac{1}{\cos^4\theta} - \frac{1}{\cos^2\theta}$

$= \frac{1 - \cos^2\theta}{\cos^4\theta}$

$= \frac{\cos^4\theta}{\cos^4\theta}$

$= 1 =$ R.H.S. (proved)

(10) If $\cos^4\theta + \cos^2\theta = 1$, then show that $\cot^4\theta - \cot^2\theta = 1$

$\Rightarrow \because \cos^4\theta + \cos^2\theta = 1$

$\therefore \cos^4\theta = 1 - \cos^2\theta = \sin^2\theta$

$\therefore \cos^2\theta \cdot \cos^2\theta = \sin^2\theta$

$\therefore \dfrac{\cos^2\theta}{\sin^2\theta} = \dfrac{1}{\cos^2\theta} = \sec^2\theta$

$\therefore \cot^2\theta = \sec^2\theta$

$\therefore \cot^4\theta = \sec^4\theta$

$\therefore$ L.H.S.

$= \cot^4\theta - \cot^2\theta$

$= \sec^4\theta - \sec^2\theta$

$= \dfrac{1 - \cos^2\theta}{\cos^4\theta}$

$= \dfrac{\cos^4\theta}{\cos^4\theta}$

$= 1$ = R.H.S. (proved)

(11) prove that $\dfrac{1}{\operatorname{cosec}\alpha + \cot\alpha} + \dfrac{1}{\operatorname{cosec}\alpha - \cot\alpha} = 2\operatorname{cosec}\alpha$

$\Rightarrow$ L.H.S.

$= \dfrac{1}{\cos\alpha + \cot\alpha} + \dfrac{1}{\operatorname{cosec}\alpha - \cot\alpha}$

$= \dfrac{\operatorname{cosec}\alpha - \cancel{\cot\alpha} + \operatorname{cosec}\alpha + \cancel{\cot\alpha}}{\operatorname{cosec}^2\alpha - \cot^2\alpha}$

$= \dfrac{2\operatorname{cosec}\alpha}{1}$ $[\because \operatorname{cosec}^2\theta - \cot^2\theta = 1]$

$= 2\operatorname{cosec}\alpha$

(12) If $\cos\theta + \sec\theta = \sqrt{3}$, then prove that $\cos^3\theta + \sec^3\theta = 0$

$\Rightarrow \because \cos\theta + \sec\theta = \sqrt{3}$

or, $(\cos\theta + \sec\theta)^3 = (\sqrt{3})^3$

or, $\cos^3\theta + \sec^3\theta + 3 \cdot \cos\theta \cdot \sec\theta\,(\cos\theta + \sec\theta) = 3\sqrt{3}$

or, $\cos^3\theta + \sec^3\theta + 3 \times 1 \times \sqrt{3} = 3\sqrt{3}$

or, $\cos^3\theta + \sec^3\theta = \cancel{3\sqrt{3}} - \cancel{3\sqrt{3}}$

$\therefore \cos^3\theta + \sec^3\theta = 0$ (proved)

(13) If $\cos\alpha + \sec\alpha = 2$, then show that $\cos^3\alpha + \sec^3\alpha = 2$

$\Rightarrow \because \cos\alpha + \sec\alpha = 2$

or, $(\cos\alpha + \sec\alpha)^3 = (2)^3$

or, $\cos^3\alpha + \sec^3\alpha + 3 \cdot \cos\alpha \cdot \sec\alpha\,(\cos\alpha + \sec\alpha) = 8$

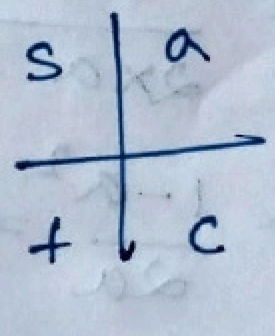
or, $\cos^3\alpha + \sec^3\alpha + 3 \times 1 \times 2 = 8$

or, $\cos^3\alpha + \sec^3\alpha + 6 = 8$

or, $\cos^3\alpha + \sec^3\alpha = 8 - 6$

$\therefore \cos^3\alpha + \sec^3\alpha = 2$ (proved)

(14) $\cos 200^\circ \sin 160^\circ - \sin 340^\circ \cos(-380^\circ) = ?$



$\Rightarrow \cos 200^\circ$

$= \cos(2 \times 90^\circ + 20^\circ)$

$= -\cos 20^\circ$

$\therefore \sin 160^\circ$

$= \sin(2 \times 90^\circ - 20^\circ)$

$= \sin 20^\circ$

$\therefore \sin 340^\circ$

$= \sin(4 \times 90^\circ - 20^\circ)$

$= -\sin 20^\circ$

$\therefore \cos(-380^\circ)$

$= \cos 380^\circ$

$= \cos(4 \times 90^\circ + 20^\circ)$

$= \cos 20^\circ$

$\therefore \cos 200^\circ \sin 160^\circ - \sin 340^\circ \cos(-380^\circ)$

$= (-\cos 20^\circ)\sin 20^\circ - (-\sin 20^\circ)\cos 20^\circ$

$= -\cancel{\sin 20^\circ \cos 20^\circ} + \cancel{\sin 20^\circ \cos 20^\circ} = 0$ Ans

(15) If $\tan 35^\circ = a$, then show that $\dfrac{\tan 145^\circ - \tan 125^\circ}{1 + \tan 145^\circ \tan 125^\circ} = \dfrac{1-a^2}{2a}$

$\Rightarrow \tan 145^\circ$

$= \tan(2\times 90^\circ - 35^\circ)$

$= -\tan 35^\circ$

$= -a$

$\therefore \tan 125^\circ$

$= \tan(1\times 90^\circ + 35^\circ)$

$= -\cot 35^\circ$

$= -\dfrac{1}{\tan 35^\circ}$

$= -\dfrac{1}{a}$

$\therefore$ L.H.S.

$= \dfrac{\tan 145^\circ - \tan 125^\circ}{1 + \tan 145^\circ \tan 125^\circ}$

$= \dfrac{-a + \frac{1}{a}}{1 + (-a)\times(-\frac{1}{a})}$

$= \dfrac{(-a^2+1)/a}{1+1}$

$= \dfrac{-a^2+1}{2\times a}$

$= \dfrac{1-a^2}{2a} =$ R.H.S. (proved)