TRIGONOMETRIC MULTIPLE & SUB-MULTIPLE ANGLES (FORMULAS)

(1) $\sin(A+B) = \sin A\cos B + \cos A\sin B$

(2) $\cancel{\sin}\cos(A-B) = \cos A\cos B - \sin A\sin B$

(3) $\sin(A-B) = \sin A\cos B - \cos A\sin B$

(4) $\cos(A-B) = \cos A\cos B + \sin A\sin B$

(5) $\sin(A+B)\sin(A-B) = \sin^2 A - \sin^2 B = \cos^2 B - \cos^2 A$

(6) $\cos(A+B)\cos(A-B) = \cos^2 A - \sin^2 B = \cos^2 B - \sin^2 A$

(7) $\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A\tan B}$ (8) $\tan(A-B) = \dfrac{\tan A - \tan B}{1 + \tan A\tan B}$

(9) $\cot(A+B) = \dfrac{\cot A\cot B - 1}{\cot B + \cot A}$ (10) $\cot(A-B) = \dfrac{\cot A\cot B + 1}{\cot B - \cot A}$

(11) $2\sin A\cos B = \sin(A+B) + \sin(A-B)$

(12) $2\cos A\sin B = \sin(A+B) - \sin(A-B)$

(13) $2\cos A\cos B = \cos(A+B) + \cos(A-B)$

(14) $2\sin A\sin B = \cos(A-B) - \cos(A+B)$

(15) $\sin A + \sin B = 2\sin\dfrac{A+B}{2}\cos\dfrac{A-B}{2}$

(16) $\sin A - \sin B = 2\cos\dfrac{A+B}{2}\sin\dfrac{A-B}{2}$

(17) $\cos A + \cos B = 2\cos\dfrac{A+B}{2}\cos\dfrac{A-B}{2}$

(18) $\cos A - \cos B = 2\sin\dfrac{A+B}{2}\sin\dfrac{B-A}{2}$

(19) $\sin 2\theta = 2\sin\theta\cos\theta$

(20) $\cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta = \dfrac{1-\tan^2\theta}{1+\tan^2\theta}$

(21) $\tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta}$ (22) $\cot 2\theta = \dfrac{\cot^2\theta - 1}{2\cot\theta}$

(23) $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$ (24) $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$

(24) $\tan 3\theta = \dfrac{3\tan\theta - \tan^3\theta}{1-3\tan^2\theta}$

(25) $\cot 3\theta = \dfrac{\cot^3\theta - 3\cot\theta}{3\cot^2\theta - 1}$

TRIGONOMETRI MULTIPLE & SUB-MULTIPLE ANGLES

① $\sin 15^\circ \sin 75^\circ = ?$

$= \sin 15^\circ \sin 75^\circ$

$= \frac{1}{2}(2\sin 15^\circ \sin 75^\circ)$

$= \frac{1}{2}[\cos 60^\circ - \cos 90^\circ]$

$= \frac{1}{2}\cos 60^\circ - \frac{1}{2}\cos 90^\circ$

$= \frac{1}{2}\times\frac{1}{2} - \frac{1}{2}\times 0$

$= \frac{1}{4}$ Ans

② $\sin 105^\circ + \cos 105^\circ$

$= \sin(90^\circ + 15^\circ) + \cos 105^\circ$

$= \cos 15^\circ + \cos 105^\circ$

$= 2\cos\frac{105+15}{2}\cos\frac{105-15}{2}$

$= 2\cos 60^\circ \cos 45^\circ$

$= 2\times\frac{1}{2}\times\frac{1}{\sqrt{2}}$

$= \frac{1}{\sqrt{2}}$ Ans

③ Prove that $\frac{1}{2\sin 10^\circ} - \frac{\sqrt{3}}{2\cos 10^\circ}$

L.H.S.

$= \frac{1}{2\sin 10^\circ} - \frac{\sqrt{3}}{2\cos 10^\circ}$

$= \frac{\cos 10^\circ - \sqrt{3}\sin 10^\circ}{2\sin 10^\circ \cos 10^\circ}$

$= \frac{2\left(\frac{1}{2}\cos 10^\circ - \frac{\sqrt{3}}{2}\sin 10^\circ\right)}{\sin(2\times 10^\circ)}$

$= \frac{2(\sin 30^\circ \cos 10^\circ - \cos 30^\circ \sin 10^\circ)}{\sin 20^\circ}$

$= \frac{2\sin(30^\circ - 10^\circ)}{\sin 20^\circ} = \frac{2\cancel{\sin 20^\circ}}{\cancel{\sin 20^\circ}} = 2 = R.H.S.$ (Proved)

④ prove that $\frac{\cos 8^\circ + \sin 8^\circ}{\cos 8^\circ - \sin 8^\circ} = \tan 53^\circ$

$\Rightarrow$ L.H.S.

$$= \frac{\cos 8^\circ + \sin 8^\circ}{\cos 8^\circ - \sin 8^\circ}$$

$$= \frac{1 + \tan 8^\circ}{1 - \tan 8^\circ}$$

$$= \frac{\tan 45^\circ + \tan 8^\circ}{1 - \tan 45^\circ \cdot \tan 8^\circ}$$

$$= \tan(45+8) \quad \left[\because \tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}\right]$$

$= \tan 53^\circ =$ R.H.S. (proved)

⑤ prove that $\frac{\cos 10^\circ - \sin 10^\circ}{\cos 10^\circ + \sin 10^\circ} = \tan 35^\circ$

$\Rightarrow$ L.H.S.

$$= \frac{\cos 10^\circ - \sin 10^\circ}{\cos 10^\circ + \sin 10^\circ}$$

$$= \frac{1 - \tan 10^\circ}{1 + \tan 10^\circ}$$

$$= \frac{\tan 45^\circ - \tan 10^\circ}{1 + \tan 45^\circ \tan 10^\circ}$$

$$= \tan(45^\circ - 10^\circ) \quad \left[\because \tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}\right]$$

$= \tan 35^\circ =$ R.H.S. (proved)

⑥ Show that $\frac{1}{\sin 10^\circ} - \frac{\sqrt{3}}{\cos 10^\circ} = 4$

$\Rightarrow$ L.H.S.

$$= \frac{1}{\sin 10^\circ} - \frac{\sqrt{3}}{\cos 10^\circ}$$

$$= \frac{\cos 10^\circ - \sqrt{3}\sin 10^\circ}{\sin 10^\circ \cos 10^\circ}$$

$$= \frac{\left(\frac{1}{2}\cos 10^\circ - \frac{\sqrt{3}}{2}\sin 10^\circ\right)\times 2}{\sin 10^\circ \cos 10^\circ}$$

$$= \frac{(\sin 30^\circ \cos 10^\circ - \cos 30^\circ \sin 10^\circ)\times 2}{\sin 10^\circ \cos 10^\circ}$$

$$= \frac{\sin(30^\circ - 10^\circ)\times 2}{\sin 10^\circ \cos 10^\circ} \quad [\because \sin(A-B) = \sin A\cos B - \cos A\sin B]$$

$$= \frac{2\sin 20^\circ}{\sin 10^\circ \cos 10^\circ}$$

$$= \frac{2\times 2\sin 20^\circ}{2\sin 10^\circ \cos 10^\circ}$$

$$= \frac{4\cancel{\sin 20^\circ}}{\cancel{\sin 20^\circ}} \quad [\because 2\sin\theta\cos\theta = \sin 2\theta]$$

$= 4 =$ R.H.S. (proved)

(7) Show that $(1+\tan A)(\tan B+1) = 2$ when $A+B = \pi/4$

$\Rightarrow \tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$

or, $\tan\dfrac{\pi}{4} = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$

or, $1 = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$

or, $1 - \tan A \tan B = \tan A + \tan B$

or, $\tan A + \tan B + \tan A \tan B = 1$

or, $\tan A + \tan B + \tan A \tan B + 1 = 2$

[Add '1' both sides]

or, $\tan A(\tan B + 1) + 1(\tan B + 1) = 2$

or, $(1+\tan A)(1+\tan B) = 2$

$\therefore$ R.H.S (proved)

(8) prove ~~to~~ that $\cos 20^\circ \cos 40^\circ \cos 60^\circ \cos 80^\circ = \dfrac{1}{16}$

$\Rightarrow$ L.H.S.

$= \cos 20^\circ \cos 40^\circ \cos 60^\circ \cos 80^\circ$

$= \dfrac{1}{2} \cos 20^\circ \cos 40^\circ \cos 80^\circ$

$= \dfrac{1}{2} \times \dfrac{1}{2} (2\cos 20^\circ \cos 40^\circ) \cos 80^\circ$

$= \dfrac{1}{4} (\cos 60^\circ + \cos 20^\circ) \cos 80^\circ$

$= \dfrac{1}{4} \left(\dfrac{1}{2} + \cos 20^\circ\right) \cos 80^\circ$

$= \dfrac{1}{8} \cos 80^\circ + \dfrac{1}{4} \times \dfrac{1}{2}\, 2\cos 80^\circ \cos 20^\circ$

$= \frac{1}{8}\cos 80^\circ + \frac{1}{8}(\cos 100^\circ + \cos 60^\circ)$

$= \frac{1}{8}\cos 80^\circ + \frac{1}{8}\cos 100^\circ + \frac{1}{8} \times \frac{1}{2}$

$= \frac{1}{8}\cos 80^\circ + \frac{1}{8}\cos(180^\circ - 80^\circ) + \frac{1}{16}$

$= \cancel{\frac{1}{8}\cos 80^\circ} - \cancel{\frac{1}{8}\cos 80^\circ} + \frac{1}{16}$

$= \frac{1}{16}$ = R.H.S. (proved)

(5) If $A+B+C=\pi$ & $\cos A = \cos B \cdot \cos C$ then show that $2\cot B \cot C = 1$

=> Given,

$\because A+B+C = \pi$

or, $B+C = \pi - A$

or, $\cos(B+C) = \cos(\pi - A)$

or, $\cos B \cos C - \sin B \sin C = -\cos A$

or, $\cos B \cos C = \sin B \sin C - \cos B \cos C$

or, $2\cos B \cos C = \sin B \sin C$

or, $\frac{\cos B \cos C}{\sin B \sin C} = \frac{1}{2}$

or, $\cot B \cdot \cot C = \frac{1}{2}$

$\therefore 2\cot B \cdot \cot C = 1$ = R.H.S. (proved)

(10) If $A+B+C=\pi$ & $\cos A = \cos B \cdot \cos C$, then show that $\tan A = \tan B + \tan C$

$\Rightarrow$ Given: $A+B+C=\pi$

$\therefore A = \pi - (B+C)$

$\because \tan A = \dfrac{\sin A}{\cos A}$

or, $\tan A = \dfrac{\sin[\pi-(B+C)]}{\cos A}$

or, $\tan A = \dfrac{\sin(B+C)}{\cos A}$

$\therefore \tan A = \dfrac{\sin B\cos C + \cos B \sin C}{\cos A}$

$= \dfrac{\sin B\cos C + \cos B\sin C}{\cos B\cos C}$

$= \dfrac{\sin B}{\cos B} + \dfrac{\sin C}{\cos C}$

$= \tan B + \tan C$

$\therefore \tan A = \tan B + \tan C$ (proved)

(11) If $A+B = 45^\circ$, then show that $(\cot A - 1)(\cot B - 1) = 2$

$\Rightarrow \because \cot(A+B) = \dfrac{\cot A\cot B - 1}{\cot B + \cot A}$

or, $\cot 45^\circ = \dfrac{\cot A\cot B - 1}{\cot B + \cot A}$

or, $1 = \dfrac{\cot A\cot B - 1}{\cot B + \cot A}$

or, $\cot B + \cot A = \cot A \cdot \cot B - 1$

or, $1 = \cot A \cot B - \cot B - \cot A$

or, $2 = 1 + \cot A \cot B - \cot B - \cot A$

or, $2 = 1 - \cot A + \cot B (\cot A - 1)$

or, $2 = (1 - \cot A) - \cot B (1 - \cot A)$

or, $2 = (1 - \cot A)(1 - \cot B)$

$\therefore (\cot A - 1)(\cot B - 1) = 2$ (proved)

(12) If $A+B+C = \pi$, prove that $\tan B \tan C + \tan C \tan A + \tan A \tan B = 1$

$\Rightarrow \because A+B+C = \pi$

$\therefore A+B = \pi - C$

$\therefore \tan(A+B) = \tan(\pi - C)$

or, $\dfrac{\tan A + \tan B}{1 - \tan A \tan B} = -\tan C$

or, $-\tan C + \tan A \tan B \tan C = \tan A + \tan B$

or, $\tan A + \tan B + \tan C = \tan A \tan B \tan C$

or, $\tan A + \tan B + \tan C - \tan A \tan B \tan C = 0$

or, $\tan(A+B+C) \cdot (1 - \tan A \tan B - \tan C \tan A - \tan B \tan C) = 0$

or, $1 - \tan A \tan B - \tan C \tan A - \tan B \tan C = 0$

$\therefore \tan B \tan C + \tan C \tan A + \tan A \tan B = 1$ (proved)

(13) If $A+B+C=\pi$, show that

$\cot B \cot C + \cot C \cdot \cot A + \cot A \cdot \cot B = 1$

$\Rightarrow$ Given: $A+B+C=\pi$

$\therefore A+B = \pi - C$

$\therefore \cot(A+B) = \cot(\pi - C)$

or, $\dfrac{\cot A \cot B - 1}{\cot B + \cot A} = -\cot C$

or, $-\cot B \cot C - \cot A \cot C = \cot A \cot B - 1$

or, $-\cot A \cot B - \cot B \cot C - \cot A \cot C = -1$

or, $-(\cot B \cot C + \cot C \cdot \cot A + \cot A \cot B) = -1$

$\therefore \cot B \cot C + \cot C \cot A + \cot A \cot B = 1$ (proved)

(14) If $\tan\frac{\theta}{2} = \tan^3\frac{\phi}{2}$ & $\tan\phi = 2\tan\alpha$, show that

$\theta + \phi = 2\alpha$

$\Rightarrow \because \tan\left(\frac{\theta}{2} + \frac{\phi}{2}\right)$

$= \dfrac{\tan\theta/2 + \tan\phi/2}{1 - \tan\theta/2 \tan\phi/2}$

$= \dfrac{\tan^3\phi/2 + \tan\phi/2}{1 - \tan^4\phi/2}$ $[\because \tan\theta/2 = \tan^3\phi/2]$

$= \dfrac{\tan\frac{\phi}{2}\cancel{\left(1+\tan^2\frac{\phi}{2}\right)}}{\cancel{\left(1+\tan^2\frac{\phi}{2}\right)}\left(1-\tan^2\frac{\phi}{2}\right)}$

$$= \frac{\tan\frac{\phi}{2}}{1-\tan^2\frac{\phi}{2}}$$

$$= \frac{1}{2}\tan\phi \quad \left[\because \tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}\right]$$

$$= \frac{1}{2} \times 2\tan\alpha \quad [\because \tan\phi = 2\tan\alpha]$$

$$= \tan\alpha$$

$\therefore \tan\left(\frac{\theta}{2}+\frac{\phi}{2}\right) = \tan\alpha$

or, $\frac{\theta}{2}+\frac{\phi}{2} = \alpha$

or, $2\left(\frac{\theta}{2}+\frac{\phi}{2}\right) = 2\alpha$

$\therefore \theta+\phi = 2\alpha$ (proved)

(15) If $\cos^2\theta - \sin^2\theta = \tan^2\alpha$, show that $\cos^2\alpha - \sin^2\alpha = \tan^2\theta$

$\Rightarrow$ $\cos^2\theta - \sin^2\theta = \tan^2\alpha$

$\therefore \cos^2\theta - \sin^2\theta = \frac{\sin^2\alpha}{\cos^2\alpha}$

or, $1 - 2\sin^2\theta = \frac{\sin^2\alpha}{\cos^2\alpha}$

$\therefore \frac{1-2\sin^2\theta+1}{1-2\sin^2\theta-1} = \frac{\sin^2\alpha+\cos^2\alpha}{\sin^2\alpha-\cos^2\alpha}$

or, $\frac{2(1-\sin^2\theta)}{-2\sin^2\theta} = \frac{1}{\sin^2\alpha-\cos^2\alpha}$

or, $-(\cos^2\alpha-\sin^2\alpha) = \frac{-\sin^2\theta}{1-\sin^2\theta}$

$\therefore \cos^2\alpha - \sin^2\alpha = \frac{\sin^2\theta}{\cos^2\theta} = \tan^2\theta$ (proved)

(16) Show that $\sec\alpha + \tan\alpha = \tan\left(\frac{\pi}{4}+\frac{\alpha}{2}\right)$

$\Rightarrow$ L.H.S.

$= \sec\alpha + \tan\alpha$

$= \frac{1}{\cos\alpha} + \frac{\sin\alpha}{\cos\alpha}$

$= \frac{1+\sin\alpha}{\cos\alpha}$

$= \frac{\sin^2\frac{\alpha}{2} + \cos^2\frac{\alpha}{2} + 2\cdot\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}}{\cos^2\frac{\alpha}{2} - \sin^2\frac{\alpha}{2}}$

$= \frac{\left(\sin\frac{\alpha}{2}+\cos\frac{\alpha}{2}\right)^2}{\left(\cos\frac{\alpha}{2}+\sin\frac{\alpha}{2}\right)\left(\cos\frac{\alpha}{2}-\sin\frac{\alpha}{2}\right)}$

$= \frac{\cos\frac{\alpha}{2}+\sin\frac{\alpha}{2}}{\cos\frac{\alpha}{2}-\sin\frac{\alpha}{2}}$

$= \frac{1+\tan\frac{\alpha}{2}}{1-\tan\frac{\alpha}{2}}$

$= \frac{\tan\frac{\pi}{4}+\tan\frac{\alpha}{2}}{1-\tan\frac{\pi}{4}\tan\frac{\alpha}{2}} = \tan\left(\frac{\pi}{4}+\frac{\alpha}{2}\right)$

= R.H.S. (proved)

(17) prove that $\sin 16^\circ + \cos 16^\circ = \frac{1}{\sqrt{2}}(\sin 1^\circ + \sqrt{3}\cos 1^\circ)$

$\Rightarrow$ L.H.S.

$= \sin 16^\circ + \cos 16^\circ$

$= \sin(90^\circ - 74^\circ) + \cos 16^\circ$

$= \cos 74^\circ + \cos 16^\circ$

$= 2\cos\frac{74+16}{2}\cos\frac{74-16}{2}$

$= 2\cos 45^\circ \cos 29^\circ$

$= 2 \times \frac{1}{\sqrt{2}} \cos 29^\circ$

$= \frac{2}{\sqrt{2}} \cos(30^\circ - 1^\circ)$

$= \frac{2\times\sqrt{2}}{\sqrt{2}\times\sqrt{2}}[\cos 30^\circ \cos 1^\circ + \sin 30^\circ \sin 1^\circ]$

$= \sqrt{2}\left(\frac{\sqrt{3}}{2}\cos 1^\circ + \frac{1}{2}\sin 1^\circ\right)$

$= \frac{\sqrt{2}}{2}(\sqrt{3}\cos 1^\circ + \sin 1^\circ)$

$= \frac{1}{\sqrt{2}}(\sin 1^\circ + \sqrt{3}\cos 1^\circ)$

= R.H.S. (proved)

(18) prove that $\cos\alpha \cos 2\alpha \cos 4\alpha \cos 7\alpha = \frac{1}{16}$ if $\alpha = \frac{2\pi}{15}$

$\Rightarrow$ L.H.S.

$= \cos\alpha \cos 2\alpha \cos 4\alpha \cos 7\alpha$

$= \frac{1}{2\sin\alpha}(2\sin\alpha \cos\alpha)\cos 2\alpha \cos 4\alpha \cos 7\alpha$

$= \frac{1}{2\sin\alpha}\sin 2\alpha \cos 2\alpha \cos 4\alpha \cos 7\alpha$

$= \frac{1}{4\sin\alpha}(2\sin 2\alpha \cos 2\alpha)\cos 4\alpha \cos 7\alpha$

$= \frac{1}{4\sin\alpha} \sin 4\alpha \cos 4\alpha \cos 7\alpha$

$= \frac{1}{8\sin\alpha} (2\sin 4\alpha \cos 4\alpha) \cos 7\alpha$

$= \frac{1}{8\sin\alpha} \sin 8\alpha \cos 7\alpha$

Now,
$\therefore \alpha = \frac{2\pi}{15}$

$\therefore 15\alpha = 2\pi$

$\therefore 7\alpha + 8\alpha = 2\pi$

$\therefore 8\alpha = 2\pi - 7\alpha$

Also

$\therefore 14\alpha + \alpha = 2\pi$

$\therefore 14\alpha = 2\pi - \alpha$

$= \frac{1}{8\sin\alpha} \sin(2\pi - 7\alpha) \cos 7\alpha$

$= \frac{-\sin 7\alpha \cos 7\alpha}{8\sin\alpha}$

$= \frac{-2\sin 7\alpha \cos 7\alpha}{16\sin\alpha}$

$= \frac{-\sin 14\alpha}{16\sin\alpha}$

$= \frac{-\sin(2\pi - \alpha)}{16\sin\alpha}$

$= \frac{-(-\sin\alpha)}{16\sin\alpha}$

$= \frac{\sin\alpha}{16\sin\alpha}$

$= \frac{1}{16}$

$=$ R.H.S. (proved)

(15) If $2\tan\alpha = 3\tan\beta$, show that

$$\tan(\alpha-\beta) = \frac{\sin 2\beta}{5-\cos 2\beta}$$

$\Rightarrow$ $\because 2\tan\alpha = 3\tan\beta$

$\therefore \tan\alpha = \frac{3}{2}\tan\beta$

$$\therefore \tan(\alpha-\beta) = \frac{\tan\alpha - \tan\beta}{1+\tan\alpha\tan\beta}$$

$$= \frac{3/2\tan\beta - \tan\beta}{1+\frac{3}{2}\tan\beta\tan\beta}$$

$$= \frac{3\tan\beta - 2\tan\beta}{2+3\tan^2\beta}$$

$$= \frac{\tan\beta}{2+3\tan^2\beta}$$

$$= \frac{\sin\beta/\cos\beta}{2+3\cdot\frac{\sin^2\beta}{\cos^2\beta}}$$

$$= \frac{\sin\beta\cos\beta}{2\cos^2\beta+3\sin^2\beta}$$

$$= \frac{2\sin\beta\cos\beta}{2(2\cos^2\beta)+3(2\sin^2\beta)}$$

$$= \frac{\sin 2\beta}{2(1+\cos 2\beta)+3(1-\cos 2\beta)}$$

$$= \frac{\sin 2\beta}{2+2\cos 2\beta+3-3\cos 2\beta} = \frac{\sin 2\beta}{5-\cos 2\beta}$$

(20) If $\operatorname{cosec} A + \sec A = \operatorname{cosec} B + \sec B$, show that

$\tan A \cdot \tan B = \cot \frac{A+B}{2}$

$\Rightarrow \because \operatorname{cosec} A + \sec A = \operatorname{cosec} B + \sec B$

or, $\operatorname{cosec} A - \operatorname{cosec} B = \sec B - \sec A$

or, $\frac{1}{\sin A} - \frac{1}{\sin B} = \frac{1}{\cos B} - \frac{1}{\cos A}$

or, $\frac{\sin B - \sin A}{\sin A \sin B} = \frac{\cos A - \cos B}{\cos A \cos B}$

or, $\frac{\sin B - \sin A}{\cos A - \cos B} = \frac{\sin A \sin B}{\cos A \cos B}$

or, $\frac{\cancel{2} \cos \frac{B+A}{2} \cancel{\sin \frac{B-A}{2}}}{\cancel{2} \sin \frac{A+B}{2} \cancel{\sin \frac{B-A}{2}}} = \tan A \tan B$

$\therefore \tan A \tan B = \cot \frac{B+A}{2}$ or $\cot \frac{A+B}{2}$ (proved)

(21) Prove that $\sec\theta - \tan\theta = \tan\left(\frac{\pi}{4} - \frac{\theta}{2}\right)$

$\Rightarrow$ L.H.S.

$= \sec\theta - \tan\theta$

$= \frac{1 - \sin\theta}{\cos\theta}$

$= \frac{\sin\frac{\theta}{2}^2 + \cos\frac{\theta}{2}^2 - 2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{\cos\frac{\theta}{2}^2 - \sin^2\frac{\theta}{2}}$

$\left[\because \sin^2\theta + \cos^2\theta = 1\right.$

$\sin 2\theta = 2\sin\theta\cos\theta$

$\left.\cos 2\theta = \cos^2\theta - \sin^2\theta\right]$

$$= \frac{(\sin\frac{\theta}{2} - \cos\frac{\theta}{2})^2}{(\cos\frac{\theta}{2} + \sin\frac{\theta}{2})(\cos\frac{\theta}{2} - \sin\frac{\theta}{2})}$$

$$= \frac{(\cos\frac{\theta}{2} - \sin\frac{\theta}{2})^2}{(\cos\theta/2 + \sin\theta/2)(\cos\theta/2 - \sin\theta/2)}$$

$$= \frac{\cos\theta/2 - \sin\theta/2}{\cos\theta/2 + \sin\theta/2}$$

$$= \frac{1 - \tan\theta/2}{1 + \tan\theta/2}$$

$$= \frac{\tan\pi/4 - \tan\theta/2}{1 + \tan\frac{\pi}{4}\tan\theta/2}$$ $\left[\because 1 = \tan\frac{\pi}{4} \text{ and } \tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}\right]$

$$= \tan\left(\frac{\pi}{4} - \theta/2\right)$$

= R.H.S. (proved)

(22) Show that $\frac{1}{2\sin 10^\circ} - 2\sin 70^\circ = 1$

$\Rightarrow$ L.H.S.

$$= \frac{1}{2\sin 10^\circ} - 2\sin 70^\circ$$

$$= \frac{1 - 2.2.\sin 70^\circ \sin 10^\circ}{2\sin 10^\circ}$$

$$= \frac{1 - 2[\cos 60^\circ - \cos 80^\circ]}{2\sin 10^\circ}$$ $[2\sin A \sin B = \cos(A-B) - \cos(A+B)]$

$$= \frac{1 - 2\times\frac{1}{2} + 2\cos 80^\circ}{2\sin 10^\circ} = \frac{1 - 1 + 2\cos 80^\circ}{2\sin 10^\circ} = \frac{\cos 80^\circ}{\sin 10^\circ}$$

$$= \frac{\cos(1\times 90^\circ - 10^\circ)}{\sin 10^\circ}$$

$$= \frac{\sin 10^\circ}{\sin 10^\circ} = 1 = \text{R.H.S.}$$ (proved)