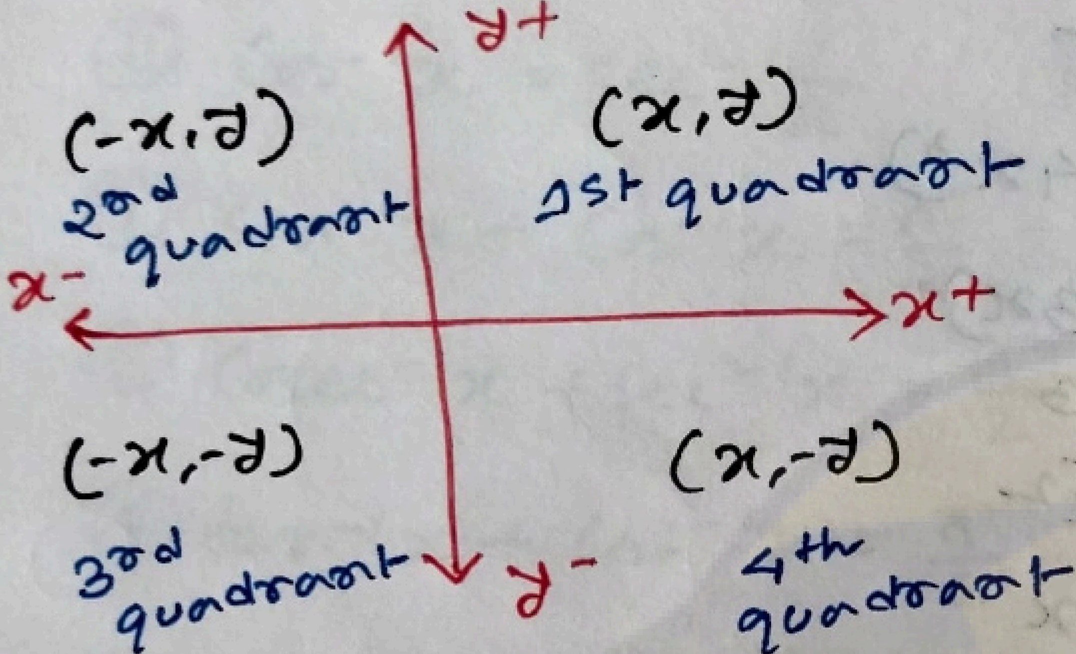
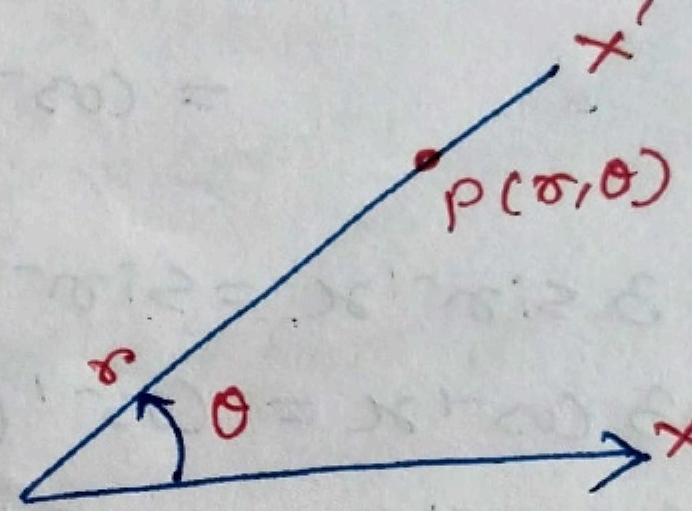
CO-ORDINATE SYSTEM
(formulars)

① Co-ordinates
↳ Cartesian [Ex. (x, y)]
↳ Polar [Ex. (r, θ)]





② Relation between Cartesian & Polar co-ordinate

ⓐ Cartesian → Polar

$$r = \sqrt{x^2+y^2}$$

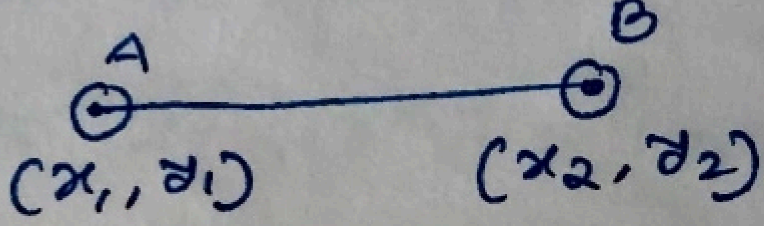
$$\theta = \tan^{-1}\left(\frac{y}{x}\right)$$

ⓑ Polar to Cartesian

$$x = r\cos\theta$$

$$y = r\sin\theta$$

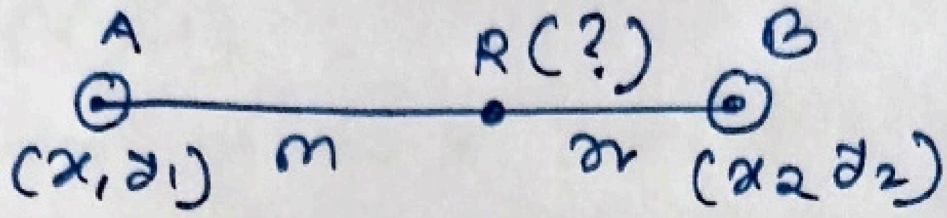
③ Distance between two points



∴ Distance (AB)

$$= \sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$$

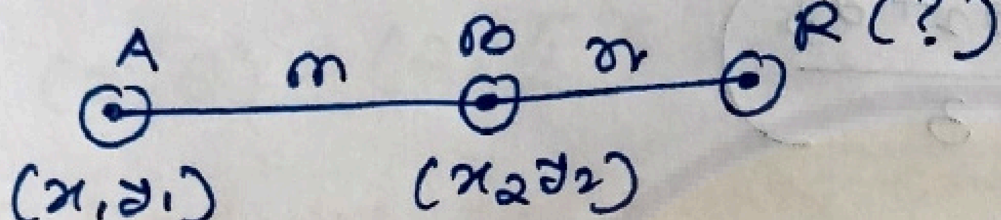
④ Internal Division



$\therefore$ Co-ordinates of R

$$= \left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n} \right)$$

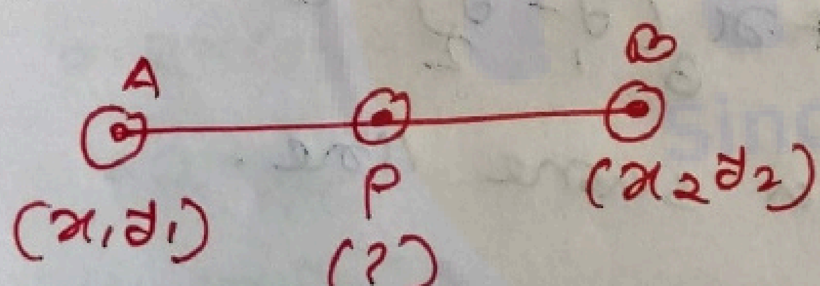
⑤ External Division



$\therefore$ Co-ordinates of R

$$= \left(\frac{mx_2 - nx_1}{m-n}, \frac{my_2 - ny_1}{m-n} \right)$$

⑥ Co-ordinates of a middle point



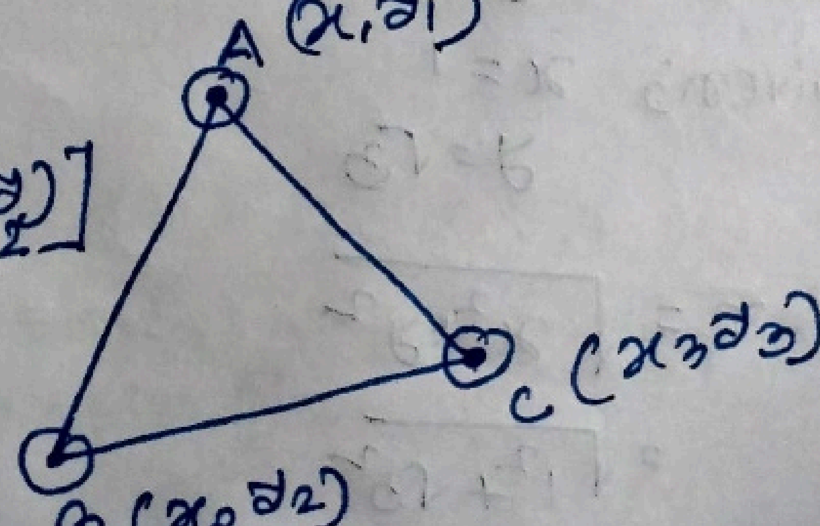
$\therefore$ Co-ordinates of P

$$= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

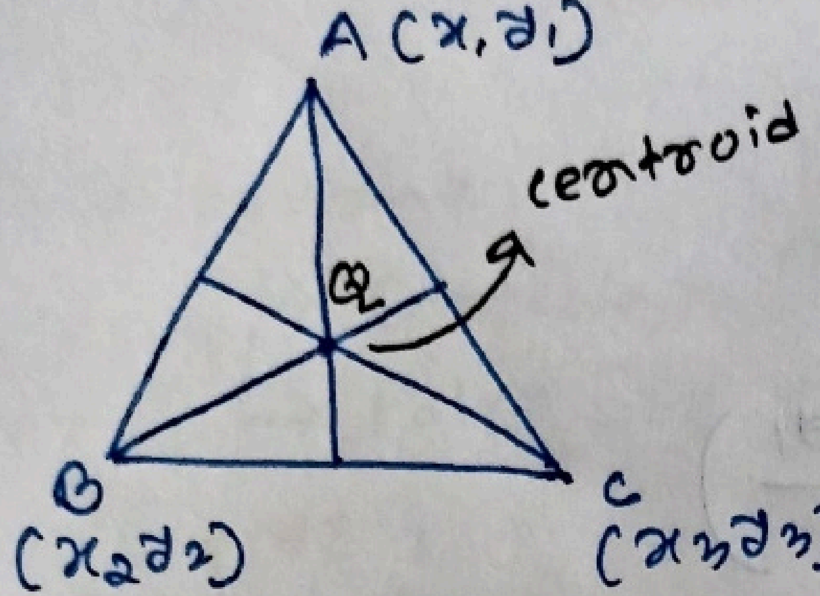
⑦ Area of a traingle

$\therefore$ ΔABC area

$$= \frac{1}{2} \left[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) \right]$$



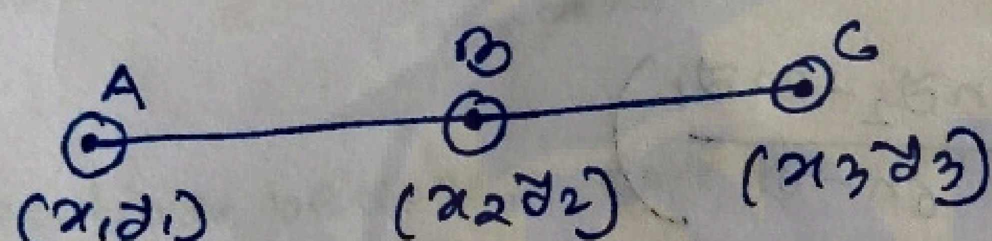
⑧ Co-ordinates of a centroid of a triangle



$\therefore$ Co-ordinates of G

$$= \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3} \right)$$

⑨ Condition of collinearity of three points



If

$$x_1 (y_2 - y_3) + x_2 (y_3 - y_1) + x_3 (y_1 - y_2) = 0$$

then those points lies on a same line.

Numericals =

① Find the polar co-ordinates of the given cartesian co-ordinates.

ⓐ $(1, \sqrt{3})$

Given, $x = 1$

$y = \sqrt{3}$

$\therefore r = \sqrt{x^2 + y^2}$

$= \sqrt{1^2 + \sqrt{3}^2}$

$= \sqrt{1+3} = \sqrt{4} = 2$

$\therefore \theta = \tan^{-1}\left(\frac{y}{x}\right)$

$= \tan^{-1}\left(\frac{\sqrt{3}}{1}\right)$

$= \tan^{-1}\sqrt{3}$

$= 60° = \pi/3$

∴ The polar co-ordinates are $(2, 60°)$ or $(2, \frac{\pi}{3})$ Ans

ⓑ $(\sqrt{3}, 1)$

⇒ Given; $x = \sqrt{3}$ & $y = 1$

$\therefore r = \sqrt{x^2 + y^2}$
$= \sqrt{\sqrt{3}^2 + 1^2}$
$= \sqrt{3+1}$
$= \sqrt{4}$
$= 2$

$\therefore \theta = \tan^{-1}\left(\frac{y}{x}\right)$
$= \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$
$= \tan^{-1}(\tan 30°)$
$= 30° = \frac{\pi}{6}$

$\therefore (r, \theta) = (2, \frac{\pi}{6})$ Ans

② Find the cartesian co-ordinates of the given polar co-ordinates:

ⓐ $(2, \frac{\pi}{4})$

Given; $r = 2$ & $\theta = \frac{\pi}{4} = 45°$

$\therefore x = r\cos\theta$
$= 2 \times \cos 45°$
$= 2 \times \frac{1}{\sqrt{2}}$
$= \frac{2 \times \sqrt{2}}{\sqrt{2} \times \sqrt{2}}$
$= \frac{2\sqrt{2}}{2}$
$= \sqrt{2}$

$\therefore y = r\sin\theta$
$= 2 \times \sin 45°$
$= 2 \times \frac{1}{\sqrt{2}}$
$= \sqrt{2}$

$\therefore (x, y) = (\sqrt{2}, \sqrt{2})$ Ans

ⓑ $(\sqrt{3}, \frac{\pi}{3})$

Given; $r = \sqrt{3}$ & $\theta = \frac{\pi}{3}$

$\therefore x = r\cos\theta$
$= \sqrt{3} \times \cos\frac{\pi}{3}$
$= \sqrt{3} \times \frac{1}{2} = \frac{\sqrt{3}}{2}$

$\therefore y = r\sin\theta$
$= r \times \sin\frac{\pi}{3}$
$= \sqrt{3} \times \frac{\sqrt{3}}{2}$
$= 3/2$

$\therefore (x, y) = \left(\frac{\sqrt{3}}{2}, \frac{3}{2}\right)$ Ans

③ A point divides internally the line segment joining the points (8, 9) & (−7, 4) in the ratio 2:3. Find the co-ordinates of the point.

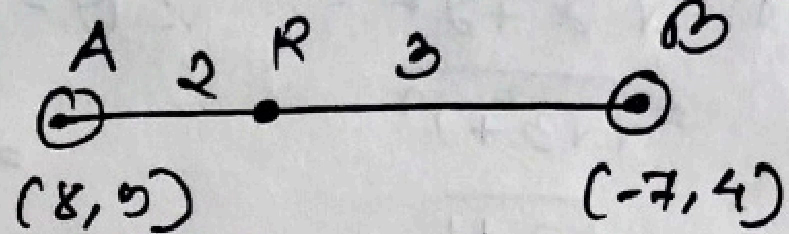
⇒ Given;

$A(x_1, y_1) = (8, 9)$

$B(x_2 y_2) = (-7, 4)$

$m : n = 2 : 3$

& Internally divided



∴ Co-ordinates of R

$$= \left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right)$$

$$= \left(\frac{2 \times (-7) + 3 \times 8}{2+3}, \frac{2 \times 4 + 3 \times 9}{2+3}\right)$$

$$= \left(\frac{-14 + 24}{5}, \frac{8 + 27}{5}\right)$$

$$= \left(\frac{10}{5}, \frac{35}{5}\right)$$

$$= (2, 7) \text{ Ans}$$

④ If A & B be the point (1, 5) & (−4, 7), then find the point P which divides AB internally in the ratio of 2:3

⇒ Given;

$A(x_1, y_1) = (1, 5)$

$B(x_2 y_2) = (-4, 7)$

$m : n = 2 : 3$ (Internally)

∴ co-ordinates of P

$$= \left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}\right)$$

$$= \left(\frac{2\times(-4)+3\times 1}{2+3}, \frac{2\times 7+3\times 5}{2+3}\right)$$

$$= \left(\frac{-8+3}{5}, \frac{14+15}{5}\right)$$

$$= \left(\frac{-5}{5}, \frac{29}{5}\right)$$

$$= \left(-1, \frac{29}{5}\right)$$ Ans

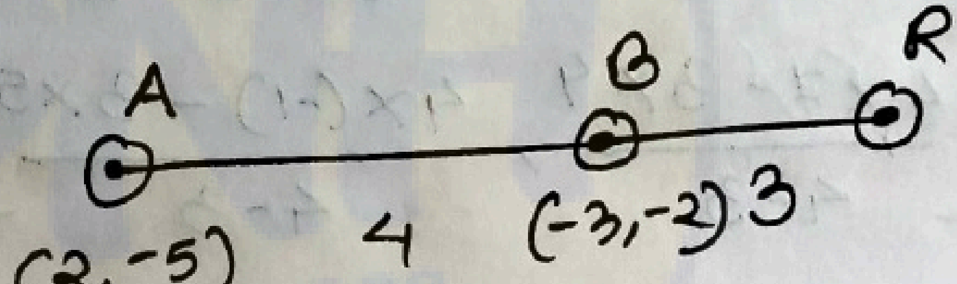
⑤ Find the co-ordinates of the point which divides the line segment joining the points (2, −5) & (−3, −2) externally in the ratio 4 : 3

⇒ Given;

$A(x_1, y_1) = (2, -5)$

$B(x_2 y_2) = (-3, -2)$

$m : n = 4 : 3$ (Externally)



∴ The co-ordinates of R

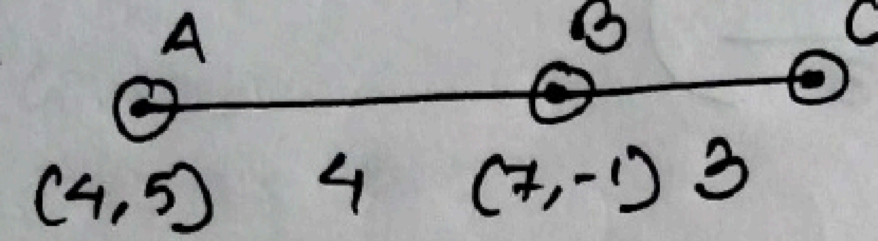
$$= \left(\frac{mx_2-nx_1}{m-n}, \frac{my_2-ny_1}{m-n}\right)$$

$$= \left(\frac{4\times(-3)-3\times 2}{4-3}, \frac{4\times(-2)-3\times(-5)}{4-3}\right)$$

$$= \left(\frac{-12-6}{1}, \frac{-8+15}{1}\right)$$

$$= (-18, 7)$$ Ans

⑥ $A(4,5)$ & $B(7,-1)$ are two given points & the point C divides the line segment AB externally in the ratio 4:3. Find the co-ordinates of C.

⇒ 

→ Given;

$A(x_1, y_1) = (4,5)$

$B(x_2 y_2) = (7,-1)$

$m:n = 4:3$ [externally]

∴ The co-ordinates of C

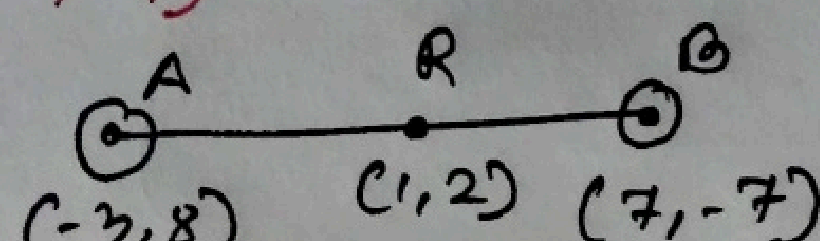
$$= \left(\frac{mx_2 - nx_1}{m-n}, \frac{my_2 - ny_1}{m-n}\right)$$

$$= \left(\frac{4\times 7 - 3\times 4}{4-3}, \frac{4\times(-1) - 3\times 5}{4-3}\right)$$

$$= \left(\frac{28-12}{1}, \frac{(-4)-15}{1}\right)$$

$$= (16, -19)$$ Ans

⑦ Find the ratio in which the point $(1,2)$ divides the line segment joining the points $(-3,8)$ & $(7,-7)$

⇒ 

Given; $A(-3,8) = (x_1, y_1)$

$B(7,-7) = (x_2 y_2)$

∴ As per formula

$$\therefore \frac{mx_2+nx_1}{m+n} = 1$$

or, $\frac{m\times 7+n\times(-3)}{m+n} = 1$

or, $7m-3n = m+n$

or, $7m-m = 3n+n$

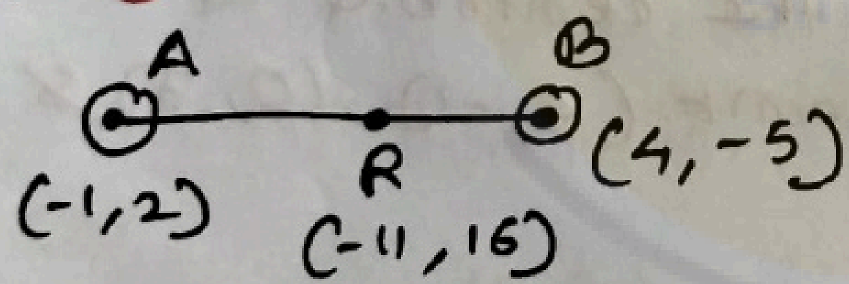
or, $6m = 4n$

or, $\frac{m}{n} = \frac{4}{6}$

∴ $m:n = 2:3$ Ans [internally]

As the ratio comes with (+) sign, so the point is present Internally

⑧ Find the ratio in which the point $(-11, 16)$ divides the line-segment joining the points $(-1, 2)$ & $(4, -5)$

⇒ 

Given; $A(x_1, y_1) = (-1, 2)$
$B(x_2, y_2) = (4, -5)$

∴ As per formula

$$\therefore \frac{mx_2+nx_1}{m+n} = -11$$

or, $\frac{m\times 4+n\times(-1)}{m+n} = -11$

or, $4m-n = -11m-11n$

or, $4m+11m = -11n+n$

or, $15m = -10n$

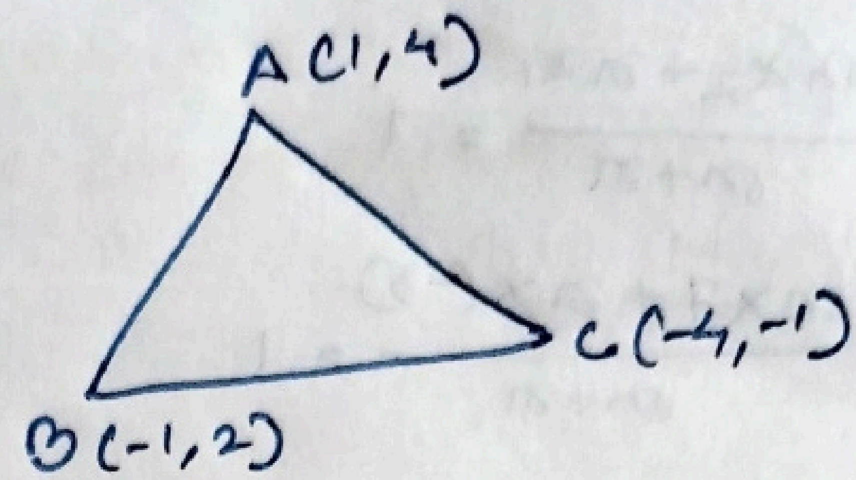
or, $\frac{m}{n} = -\frac{\cancel{10}\,2}{\cancel{15}\,3}$

∴ $m:n = -2:3$

As the sign is negative (-), so divides Externally.

∴ $m:n = -2:3$ [Externally]

⑨ Find the area of a traingle having vertices (1,4), (-1,2) & (-4,-1)



⇒ Given;

$A(x_1, y_1) = (1,4)$

$B(x_2 y_2) = (-1,2)$

$C(x_3 y_3) = (-4,-1)$

∴ ΔABC traingle area

$= \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

$\Rightarrow \frac{1}{2}[1(2+1) + (-1)(-1-4) - 4(4-2)]$

$\Rightarrow \frac{1}{2}[3+5-8]$

$\Rightarrow \frac{1}{2}[8-8]$

$= 0$ unit^2 An

So, the points are co-llinear.

⑩ Find the co-ordinates of the centroid of a traingle formed by the point (4,-1), (0,3) & (-4,-2)

⇒ ∴ Co-ordinates of the centroid -

$$= \left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$$

$$= \left(\frac{4+0-4}{3}, \frac{-1+3-2}{3}\right)$$

$$= (0/3, 0/3)$$

$$= (0,0)$$ An

∴ Centroid is on the origin An

(11) Find the area of a triangle having vertices $(-4, 1), (1, 2)$ & $(4, -3)$

$\Rightarrow$ Given; $A(x_1, y_1) = (-4, 1)$

$B(x_2 y_2) = (1, 2)$

~~C(4, -3) =~~

$C(x_3 y_3) = (4, -3)$

$\therefore$ The area of a triangle

$$= \frac{1}{2}\left[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right]$$

$$= \frac{1}{2}\left[-4(2+3) + 1(-3-1) + 4(1-2)\right]$$

$$= \frac{1}{2}\left[-20 - 4 - 4\right]$$

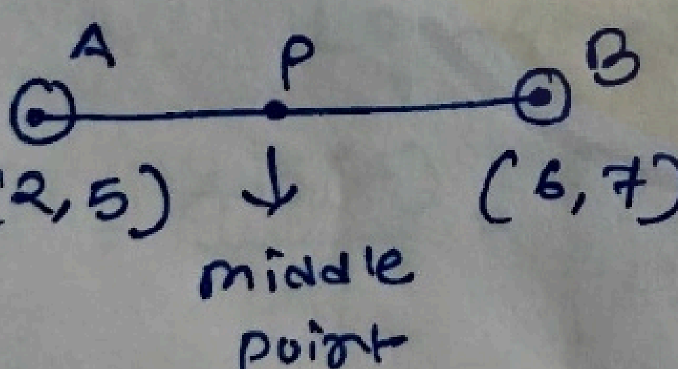
$$= \frac{1}{2}\left[-28\right]$$

$= -14$ sq. unit Ans

$\therefore$ The area of a triangle is 14 sq. unit Ans

(12) Find the co-ordinates of a middle point of the line joining two points $(2, 5)$ & $(6, 7)$

$\Rightarrow$



$\therefore$ The co-ordinates of P

$$= \left(\frac{2+6}{2}, \frac{5+7}{2}\right)$$

$$= \left(\frac{8}{2}, \frac{12}{2}\right)$$

$= (4, 6)$ Ans

(13) The area of a traingle formed by joining the points (2,7), (5,1) & (x,3) is 18 sq. unit. Find x.

⇒ Given, $A = (2,7)$ $[x_1, y_1]$

$B = (5,1)$ $[x_2 y_2]$

$C = (x,3)$ $[x_3 y_3]$

∴ As per formula –

$$\therefore \frac{1}{2}[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)] = 18$$

$$\text{or, } \frac{1}{2}[2(1-3)+5(3-7)+x(7-1)] = 18$$

$$\text{or, } \frac{1}{2}[-4-20+6x] = 18$$

$$\text{or, } -24+6x = 36$$

$$\text{or, } 6x = 36+24$$

$$\text{or, } 6x = 60$$

$$\text{or, } x = 60/6$$

$$\therefore x = 10 \text{ Ans}$$

(14) Find the distance between these points (3,5) & (8,10)

⇒ Given, $(x_1, y_1) = (3,5)$ & $(x_2 y_2) = (8,10)$

∴ Distance

$$= \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$$= \sqrt{(8-3)^2+(10-5)^2}$$

$$= \sqrt{5^2+5^2}$$

$$= \sqrt{50}$$

$$= 5\sqrt{2} \text{ units Ans}$$