Formulas =

# [DETERMINANTS]

① $\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} = a_1 b_2 - b_1 a_2$

(2) (1)

② $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = a_1 \begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} - b_1 \begin{vmatrix} a_2 & c_2 \\ a_3 & c_3 \end{vmatrix} + c_1 \begin{vmatrix} a_2 & b_2 \\ a_3 & b_3 \end{vmatrix}$

③ **Minor element or minor**

Let's $\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ then the minor of $b_2$

, to find the minor of element $b_2$ we need cancel out the row & column of the element and then find the determinant of the rest of the elements.

So, = minor element = $\begin{vmatrix} a_1 & c_1 \\ a_3 & c_3 \end{vmatrix}$

$= a_1 c_3 - a_3 c_1$

④ **Co-factor**

∴ Co-factor = $(-1)^{i+j} \times$ minor element

Here's $i$ = No. of row which cancel out

$j$ = No. of column which cancel out

# DETERMINANTS [$2^{nd}$ Semester] [Math-2]

① Find the value of $\begin{vmatrix} 5 & 6 \\ 9 & -8 \end{vmatrix}$

$= \begin{vmatrix} 5 & 6 \\ 9 & -8 \end{vmatrix} = 5\times(-8) - 6\times 9 = -40-54 = -94$ Ans

② Find the value of $\begin{vmatrix} 1 & 3 & 5 \\ 2 & 3 & 4 \\ 1 & 3 & 5 \end{vmatrix}$

$= \begin{vmatrix} 1 & 3 & 5 \\ 2 & 3 & 4 \\ 1 & 3 & 5 \end{vmatrix}$

$= 1\begin{vmatrix} 3 & 4 \\ 3 & 5 \end{vmatrix} - 3\begin{vmatrix} 2 & 4 \\ 1 & 5 \end{vmatrix} + 5\begin{vmatrix} 2 & 3 \\ 1 & 3 \end{vmatrix}$

$= 1(15-12) - 3(10-4) + 5(6-3)$

$= 1\times 3 - 3\times 6 + 5\times 3$

$= 3 - 18 + 15$

$= -15 + 15$

$= 0$ Ans

③ Find the value of $\begin{vmatrix} 2 & 3 & -1 \\ 1 & 0 & 2 \\ 1 & 2 & 3 \end{vmatrix}$

$= \begin{vmatrix} 2 & 3 & -1 \\ 1 & 0 & 2 \\ 1 & 2 & 3 \end{vmatrix}$

$= 2\begin{vmatrix} 0 & 2 \\ 2 & 3 \end{vmatrix} - 3\begin{vmatrix} 1 & 2 \\ 1 & 3 \end{vmatrix} - 1\begin{vmatrix} 1 & 0 \\ 1 & 2 \end{vmatrix}$

$= 2(0-4) - 3(3-2) - 1(2-0)$

$= 2\times(-4) - 3\times 1 - 1\times 2 = -8-3-2 = -13$ Ans

④ If $\Delta = \begin{vmatrix} 0 & 2 & 4 \\ -3 & 0 & -1 \\ 5 & -2 & 7 \end{vmatrix}$ then the minor of element 2 = ?

$\Rightarrow$ If $\Delta = \begin{vmatrix} 0 & 2 & 4 \\ -3 & 0 & -1 \\ 5 & -2 & 7 \end{vmatrix}$

then the minor of element 2 is

$= \begin{vmatrix} -3 & -1 \\ 5 & 7 \end{vmatrix}$

$= -21+5$

$= -16$ Ans

⑤ If $\Delta = \begin{vmatrix} 0 & 1 & 4 \\ -3 & 0 & -1 \\ 5 & -2 & 7 \end{vmatrix}$ then the minor of element 4 = ?

$\Rightarrow$ If $\Delta = \begin{vmatrix} 0 & 1 & 4 \\ -3 & 0 & -1 \\ 5 & -2 & 7 \end{vmatrix}$

then the minor of element 4 is

$= \begin{vmatrix} -3 & 0 \\ 5 & -2 \end{vmatrix}$

$= 6-0$

$= 6$ Ans

⑥ If $\Delta = \begin{vmatrix} -3 & 1 & 2 \\ 7 & 3 & -4 \\ -5 & 6 & -1 \end{vmatrix}$ then co-factor of element 7 = ?

i = Row

j = column

$\Rightarrow$ ∴ The co-factor of element 7 is

$= (-1)^{i+j} \begin{vmatrix} 1 & 2 \\ 6 & -1 \end{vmatrix} = (-1)^{2+1}[-1-12] = (-1)^3(-13) = 13$ Ans

⑦ Find the co-factor of element 6 of $\begin{vmatrix} 2 & -2 & 3 \\ 1 & 5 & 6 \\ 2 & 7 & 3 \end{vmatrix}$

$\Rightarrow$ ∴ The co-factor of element 6 is

$= (-1)^{i+j} \begin{vmatrix} 2 & -2 \\ 2 & 7 \end{vmatrix}$    $i$ = Row, $j$ = column

$= (-1)^{2+3} [14+4]$

$= (-1)^5 (18)$

$= (-1) \times 18$

$= -18$ Ans

⑧ Find $\begin{vmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & \omega \\ \omega^2 & 1 & 1 \end{vmatrix}$ where, $\omega$ is a complex root of unity.

∴ $\omega^3 = 1$.

∴ $1 + \omega + \omega^2 = 0$

$\Rightarrow \begin{vmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & \omega \\ \omega^2 & 1 & 1 \end{vmatrix}$

$= 1 \begin{vmatrix} \omega^2 & \omega \\ 1 & 1 \end{vmatrix} - \omega \begin{vmatrix} \omega & \omega \\ \omega^2 & 1 \end{vmatrix} + \omega^2 \begin{vmatrix} \omega & \omega^2 \\ \omega^2 & 1 \end{vmatrix}$

$= 1(\omega^2 - \omega) - \omega(\omega - \omega^3) + \omega^2(\omega - \omega^4)$

$= 1(\omega^2 - \omega) - \omega(\omega - 1) + \omega^2(\cancel{\omega} - \cancel{\omega})$    $[\because \omega^3 = 1$, $\omega^3 . \omega = \omega^4$, $\therefore \omega^4 = \omega]$

$= \cancel{\omega^2} - \cancel{\omega} - \cancel{\omega^2} + \cancel{\omega} + \omega^2 \times 0$

$= 0$ Ans

(9) Find $\begin{vmatrix} \omega^3 & \omega^4 & \omega^5 \\ \omega^2 & 1 & \omega \\ \omega^7 & \omega^5 & \omega^6 \end{vmatrix}$

$\Rightarrow \omega^3 = 1$

$\omega^4 = \omega^3 \cdot \omega = \omega$

$\omega^5 = \omega^3 \cdot \omega^2 = \omega^2$

$\omega^7 = \omega^3 \cdot \omega^3 \cdot \omega = \omega$

$\omega^6 = \omega^3 \cdot \omega^3 = 1$

$\therefore \begin{vmatrix} 1 & \omega & \omega^2 \\ \omega^2 & 1 & \omega \\ \omega & \omega^2 & 1 \end{vmatrix}$

$= 1\begin{vmatrix} 1 & \omega \\ \omega^2 & 1 \end{vmatrix} - \omega\begin{vmatrix} \omega^2 & \omega \\ \omega & 1 \end{vmatrix} + \omega^2\begin{vmatrix} \omega^2 & 1 \\ \omega & \omega^2 \end{vmatrix}$

$= 1\,(1-\omega^3) - \omega(\omega^2-\omega^2) + \omega^2(\omega^4-\omega)$

$= 1\,(\cancel{1}-\cancel{1}) - \omega(\cancel{\omega^2}-\cancel{\omega^2}) + \omega^2(\cancel{\omega}-\cancel{\omega})$

$= 1\times 0 - \omega\times 0 + \omega^2\times 0$

$= 0$ Ans

(10) If $\begin{vmatrix} 1 & 3 & 9 \\ 1 & x & x^2 \\ 4 & 6 & 9 \end{vmatrix} = 0$, then the value of $x = ?$

$\Rightarrow \because \begin{vmatrix} 1 & 3 & 9 \\ 1 & x & x^2 \\ 4 & 6 & 9 \end{vmatrix} = 0$

$\Rightarrow 1\begin{vmatrix} x & x^2 \\ 6 & 9 \end{vmatrix} - 3\begin{vmatrix} 1 & x^2 \\ 4 & 9 \end{vmatrix} + 9\begin{vmatrix} 1 & x \\ 4 & 6 \end{vmatrix} = 0$

$\Rightarrow 1\,(9x-6x^2) - 3(9-4x^2) + 9(6-4x) = 0$

$\Rightarrow 9x - 6x^2 - 27 + 12x^2 + 54 - 36x = 0$

$\Rightarrow 6x^2 - 27x + 27 = 0$

$\Rightarrow 3(2x^2 - 9x + 9) = 0$

$\Rightarrow 2x^2 - 9x + 9 = 0$

$\Rightarrow 2x^2 - 6x - 3x + 9 = 0$

$\Rightarrow 2x(x-3) - 3(x-3) = 0$

$\Rightarrow (x-3)(2x-3) = 0$

| | |
|---|---|
| $\therefore x - 3 = 0$ | $\therefore 2x - 3 = 0$ |
| $\therefore x = 3$ | $\therefore x = \frac{3}{2}$ |

$\therefore x = 3, \frac{3}{2}$ Ans

(ii) Show that $\begin{vmatrix} 1 & a & a^3 \\ 1 & b & b^3 \\ 1 & c & c^3 \end{vmatrix} = (b-c)(c-a)(a-b)(a+b+c)$

$$\Rightarrow \text{L.H.S.} = \begin{vmatrix} 1 & a & a^3 \\ 1 & b & b^3 \\ 1 & c & c^3 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & a & a^3 \\ 0 & b-a & b^3-a^3 \\ 0 & c-a & c^3-a^3 \end{vmatrix} \quad \begin{bmatrix} R_2' = R_2 - R_1 \\ R_3' = R_3 - R_1 \end{bmatrix}$$

$$= \begin{vmatrix} 1 & a & a^3 \\ 0 & (b-a) & (b-a)(b^2+ab+a^2) \\ 0 & (c-a) & (c-a)(c^2+ac+a^2) \end{vmatrix}$$

$$= (b-a)(c-a) \begin{vmatrix} 1 & a & a^3 \\ 0 & 1 & b^2+ab+a^2 \\ 0 & 1 & c^2+ac+a^2 \end{vmatrix}$$

$$= (b-a)(c-a)\left\{1\begin{vmatrix} 1 & b^2+ab+a^2 \\ 1 & c^2+ac+a^2 \end{vmatrix} - 0 + 0\right\}$$

$$= (b-a)(c-a)(c^2+ac+a^2-b^2-ab-a^2)$$

$$= (b-a)(c-a)(c^2+ac-b^2-ab)$$

$$= (b-a)(c-a)\{c^2-b^2+ac-ab\}$$

$$= (b-a)(c-a)\{(c+b)(c-b)+a(c-b)\}$$

$$= (b-a)(c-a)(c-b)(c+b+a)$$

$$= (b-c)(c-a)(a-b)(a+b+c) = \text{R.H.S. (proved)}$$

(12) Show that $\begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} = (x-y)(y-z)(z-x)$

$$\Rightarrow \text{L.H.S.} = \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & x & x^2 \\ 0 & x-y & x^2-y^2 \\ 0 & x-z & x^2-z^2 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & x & x^2 \\ 0 & x-y & (x+y)(x-y) \\ 0 & x-z & (x+z)(x-z) \end{vmatrix}$$

$$= (x-y)(x-z)\begin{vmatrix} 1 & x & x^2 \\ 0 & 1 & x+y \\ 0 & 1 & x+z \end{vmatrix}$$

$$= (x-y)(x-z)\left\{1\begin{vmatrix} 1 & x+y \\ 1 & x+z \end{vmatrix} - 0 + 0\right\}$$

$$= (x-y)(x-z)(\cancel{x}+z-\cancel{x}-y)$$

$$= (x-y)(x-z)(z-y)$$

$$= (x-y)(z-x)(y-z) = \text{R.H.S. (proved)}$$

(13) Evaluate by chio's method $A = \begin{vmatrix} 0 & -1 & 2 & 3 \\ 1 & 2 & -1 & 1 \\ 1 & 1 & 2 & 3 \\ 3 & 2 & 1 & 0 \end{vmatrix}$

$\Rightarrow$ Since $a_{11} = 0$, so we need to interchange $c_1$ & $c_2$

$$\therefore A = -\begin{vmatrix} -1 & 0 & 2 & 3 \\ 2 & 1 & -1 & 1 \\ 1 & 1 & 2 & 3 \\ 2 & 3 & 1 & 0 \end{vmatrix}$$

$\therefore$ Applying Chio's method we get;

$$= -\frac{1}{(-1)^{4-2}} \begin{vmatrix} \begin{vmatrix} -1 & 0 \\ 2 & 1 \end{vmatrix} & \begin{vmatrix} -1 & 2 \\ 2 & -1 \end{vmatrix} & \begin{vmatrix} -1 & 3 \\ 2 & 1 \end{vmatrix} \\ \begin{vmatrix} -1 & 0 \\ 1 & 1 \end{vmatrix} & \begin{vmatrix} -1 & 2 \\ 1 & 2 \end{vmatrix} & \begin{vmatrix} -1 & 3 \\ 1 & 3 \end{vmatrix} \\ \begin{vmatrix} -1 & 0 \\ 2 & 3 \end{vmatrix} & \begin{vmatrix} -1 & 2 \\ 2 & 1 \end{vmatrix} & \begin{vmatrix} -1 & 3 \\ 2 & 0 \end{vmatrix} \end{vmatrix}$$

$$= (-1) \begin{vmatrix} -1 & -3 & -7 \\ -1 & -4 & -6 \\ -3 & -5 & -6 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 3 & 7 \\ 1 & 4 & 6 \\ 3 & 5 & 6 \end{vmatrix} = \frac{1}{(1)^{3-2}} \begin{vmatrix} \begin{vmatrix} 1 & 3 \\ 1 & 4 \end{vmatrix} & \begin{vmatrix} 1 & 7 \\ 1 & 6 \end{vmatrix} \\ \begin{vmatrix} 1 & 3 \\ 3 & 5 \end{vmatrix} & \begin{vmatrix} 1 & 7 \\ 3 & 6 \end{vmatrix} \end{vmatrix}$$

$$\left[ \begin{aligned} & -15-4 \\ & = -19 \end{aligned} \right.$$

$$= \begin{vmatrix} (4-3) & (6-7) \\ (5-9) & (6-21) \end{vmatrix}$$

$$= \begin{vmatrix} 1 & -1 \\ -4 & -15 \end{vmatrix} = (-19) = -19 \text{ Ans}$$

(14) $\begin{vmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 1 & 1 \\ 1 & 1 & 4 & 6 \\ 1 & 1 & 5 & 8 \end{vmatrix}$

$$\Rightarrow \Delta = \begin{vmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 1 & 1 \\ 1 & 1 & 4 & 6 \\ 1 & 1 & 5 & 8 \end{vmatrix}$$

$$= \frac{1}{(1)^{4-2}} \begin{vmatrix} \begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix} & \begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} & \begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} \\ \begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} & \begin{vmatrix} 1 & 1 \\ 1 & 4 \end{vmatrix} & \begin{vmatrix} 1 & 1 \\ 1 & 6 \end{vmatrix} \\ \begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} & \begin{vmatrix} 1 & 1 \\ 1 & 5 \end{vmatrix} & \begin{vmatrix} 1 & 1 \\ 1 & 8 \end{vmatrix} \end{vmatrix}$$

$$= \begin{vmatrix} (2-1) & (1-1) & (1-1) \\ (1-1) & (4-1) & (6-1) \\ (1-1) & (5-1) & (8-1) \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 0 & 0 \\ 0 & 3 & 5 \\ 0 & 4 & 7 \end{vmatrix}$$

Again applying Chio's method –

$$= \frac{1}{(1)^{3-2}} \begin{vmatrix} \begin{vmatrix} 1 & 0 \\ 0 & 3 \end{vmatrix} & \begin{vmatrix} 1 & 0 \\ 0 & 5 \end{vmatrix} \\ \begin{vmatrix} 1 & 0 \\ 0 & 4 \end{vmatrix} & \begin{vmatrix} 1 & 0 \\ 0 & 7 \end{vmatrix} \end{vmatrix}$$

$$= \begin{vmatrix} 3 & 5 \\ 4 & 7 \end{vmatrix}$$

$= 21 - 20$

$= 1$ Ans