1^{st} order Derivative formulas

1. $\frac{d}{dx}x^n = n.x^{n-1}$

2. $\frac{d}{dx}(x) = 1$

3. $\frac{d}{dx}(c) = 0$ where; c is a constant

4. $\frac{d}{dx}c.f'(x) = c.\frac{d}{dx}f(x)$

5. $\frac{d}{dx}e^x = e^x$

6. $\frac{d}{dx}a^{mx} = m.a^{mx}.\log a$

7. $\frac{d}{dx}a^x = \log a$

8. $\frac{d}{dx}\log x = \frac{1}{x}$

9. $\frac{d}{dx}\sin x = \cos x$

10. $\frac{d}{dx}\cos x = -\sin x$

11. $\frac{d}{dx}\tan x = \sec^2 x$

12. $\frac{d}{dx}\sec x = \sec x \tan x$

13. $\frac{d}{dx}\operatorname{cosec} x = -\operatorname{cosec} x \cot x$

14. $\frac{d}{dx}\cot x = -\operatorname{cosec}^2 x$

15. $\frac{d}{dx}\sin^{-1}x = \frac{1}{\sqrt{1-x^2}}$

16. $\frac{d}{dx}\cos^{-1}x = -\frac{1}{\sqrt{1-x^2}}$

17. $\frac{d}{dx}\tan^{-1}x = \frac{1}{1+x^2}$

18. $\frac{d}{dx}\cot^{-1}x = -\frac{1}{1+x^2}$

19. $\frac{d}{dx}\sec^{-1}x = \frac{1}{x\sqrt{x^2-1}}$

20. $\frac{d}{dx}\operatorname{cosec}^{-1}x = -\frac{1}{x\sqrt{x^2-1}}$

21. $\frac{d}{dx}(m \times n)$

$= m.\frac{d}{dx}(n) + n.\frac{d}{dx}(m)$

22. $\frac{d}{dx}\left(\frac{m}{n}\right)$

$= \frac{n.\frac{d}{dx}(m) - m.\frac{d}{dx}(n)}{n^2}$

where; $m = f(x)$
$n = f(x)$

23. $\frac{dy}{dx} = \frac{dy}{d\theta} \times \frac{d\theta}{dx} = \frac{dy/d\theta}{dx/d\theta}$

① If $y = \log \tan x$, find $\frac{dy}{dx} = ?$

$\Rightarrow \because y = \log \tan x$

$\therefore \frac{dy}{dx} = \frac{d}{dx} \log \tan x$

$= \frac{1}{\tan x} \frac{d}{dx}(\tan x) \quad \left[\because \frac{d}{dx} \log x = \frac{1}{x}\right]$

$= \frac{1}{\tan x} \cdot \sec^2 x \quad \left[\because \frac{d}{dx} \tan x = \sec^2 x\right]$

$= \frac{\cos x}{\sin x} \times \frac{1}{\cos^2 x}$

$= \frac{1}{\sin x} \times \frac{1}{\cos x}$

$= \operatorname{cosec} x \cdot \sec x$ Ans

② If $y = \log(\sec x + \tan x)$, find $\frac{dy}{dx} = ?$

$\Rightarrow \because y = \log(\sec x + \tan x)$

$\therefore \frac{dy}{dx} = \frac{d}{dx} \log(\sec x + \tan x)$

$= \frac{1}{\sec x + \tan x} \frac{d}{dx}(\sec x + \tan x)$

$= \frac{\frac{d}{dx}\sec x + \frac{d}{dx}\tan x}{\sec x + \tan x}$

$= \frac{\sec x \tan x + \sec^2 x}{\sec x + \tan x}$

$= \frac{\sec x \cancel{(\sec x + \tan x)}}{\cancel{(\sec x + \tan x)}} = \sec x$ Ans

(iii) If $y = \log(x^2 - \sin x)$, then find $\frac{dy}{dx}$

$\Rightarrow \because y = \log(x^2 - \sin x)$

$\therefore \frac{dy}{dx} = \frac{d}{dx}\log(x^2 - \sin x)$

$= \frac{1}{x^2 - \sin x} \cdot \frac{d}{dx}(x^2 - \sin x)$

$= \frac{1}{x^2 - \sin x} \cdot \left[\frac{d}{dx}x^2 - \frac{d}{dx}\sin x\right]$

$= \frac{1}{x^2 - \sin x}\left[2x - \cos x\right]$

$= \frac{2x - \cos x}{x^2 - \sin x}$ An

(iv) If $y = \log(\operatorname{cosec} x - \cot x)$, then find $\frac{dy}{dx}$

$\Rightarrow \because y = \log(\operatorname{cosec} x - \cot x)$

$\therefore \frac{dy}{dx} = \frac{d}{dx}\log(\operatorname{cosec} x - \cot x)$

$= \frac{1}{\operatorname{cosec} x - \cot x} \cdot \left[\frac{d}{dx}\operatorname{cosec} x - \frac{d}{dx}\cot x\right]$

$= \frac{-\operatorname{cosec} x \cot x + \operatorname{cosec}^2 x}{\operatorname{cosec} x - \cot x}$

$= \frac{\operatorname{cosec} x\,\cancel{(\operatorname{cosec} x - \cot x)}}{\cancel{(\operatorname{cosec} x - \cot x)}}$

$= \operatorname{cosec} x$ An

⑤ If $y = \frac{1-x}{1+x}$, then $\frac{dy}{dx} = ?$

$\Rightarrow \because y = \frac{1-x}{1+x}$

$\therefore \frac{dy}{dx} = \frac{d}{dx}\left(\frac{1-x}{1+x}\right)$

$= \frac{(1+x)\cdot\frac{d}{dx}(1-x) - (1-x)\cdot\frac{d}{dx}(1+x)}{(1+x)^2}$

$= \frac{(1+x)(0-1) - (1-x)(0+1)}{(1+x)^2}$

$= \frac{-1-\cancel{x}-1+\cancel{x}}{(1+x)^2}$

$= \frac{-2}{(1+x)^2}$ Ans

⑥ If $x^n + y^n = a^n$, then $\frac{dy}{dx} = ?$

$\Rightarrow \because x^n + y^n = a^n$

So, differentiat both sides, we get;

$\therefore \frac{d}{dx}x^n + \frac{d}{dx}y^n = \frac{d}{dx}a^n$

or, $n\cdot x^{n-1}\cdot 1 + n\cdot y^{n-1}\cdot\frac{dy}{dx} = 0$

or, $\cancel{n}\cdot y^{n-1}\cdot\frac{dy}{dx} = -\cancel{n}\cdot x^{n-1}$

$\therefore \frac{dy}{dx} = -\frac{x^{n-1}}{y^{n-1}} = -\left(\frac{x}{y}\right)^{n-1}$ Ans

⑦ If $f(x) = \log_e e^x + e^{\log_e x}$ then $f'(x) = ?$

$\Rightarrow \because f(x) = \log_e e^x + e^{\log_e x}$

$= x \log_e e + x$

$= x \times 1 + x$

$= 2x$

$\therefore f'(x) = \frac{d}{dx}(2x) = 2 \times \frac{dx}{dx} = 2 \times 1 = 2$ Ans

⑧ If $y = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$ then $\frac{dy}{dx} = ?$

$\Rightarrow$ Let's $x = \tan\theta$

$\therefore \frac{2x}{1+x^2} = \frac{2 \times \tan\theta}{1+\tan^2\theta} = \sin 2\theta$

$\because x = \tan\theta$

$\therefore \frac{dx}{d\theta} = \frac{d}{d\theta}\tan\theta = \sec^2\theta$

$\because y = \sin^{-1}\left(\frac{2x}{1+x^2}\right) = \sin^{-1}(\sin 2\theta)$

$\therefore y = \cancel{\sin}\, 2\theta$

$\therefore \frac{dy}{d\theta} = \frac{d}{d\theta} \cancel{\sin}\, 2\theta = 2$

$= \cancel{\cos 2\theta \cdot \frac{d}{d\theta} 2\theta}$

$= \cancel{2\cos 2\theta}$

$\therefore \frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{2}{\sec^2\theta} = \frac{2}{1+\tan^2\theta} = \frac{2}{1+x^2}$ Ans

⑨ If $y = \tan^{-1}\sqrt{\frac{1-x}{1+x}}$ then $\frac{dy}{dx} = ?$

$\Rightarrow$ Let's $x = \cos 2\theta$

$\therefore y = \tan^{-1}\sqrt{\frac{1-\cos 2\theta}{1+\cos 2\theta}}$

$= \tan^{-1}\sqrt{\frac{2\sin^2\theta}{2\cos^2\theta}}$

$= \tan^{-1}(\tan\theta)$

$= \theta$

Now's $\because x = \cos 2\theta$

$\therefore 2\theta = \cos^{-1}(x)$

$\therefore \theta = \frac{1}{2}\cos^{-1}(x)$

$\therefore y = \frac{1}{2}\cos^{-1}(x)$

$\therefore \frac{dy}{dx} = \frac{1}{2}\frac{dy}{dx}\cos^{-1}(x)$

$= \frac{1}{2}\left(-\frac{1}{\sqrt{1-x^2}}\right)$

$= -\frac{1}{2\sqrt{1-x^2}}$ Ans

⑩ If $y = \cos^{-1}\frac{1-x}{1+x}$ then's $\frac{dy}{dx} = ?$

$\Rightarrow$ Let's $x = \tan^2\theta$

$\therefore y = \cos^{-1}\frac{1-x}{1+x}$

$= \cos^{-1}\frac{1-\tan^2\theta}{1+\tan^2\theta}$

$= \cos^{-1}(\cos 2\theta)$

$= 2\theta$

$\because x = \tan^2\theta$

$\therefore \frac{dx}{d\theta} = \frac{d}{d\theta}\tan^2\theta$

$= 2\tan\theta \cdot \sec^2\theta$

$\therefore \frac{dy}{d\theta} = 2$

$\therefore \frac{dy}{dx} = \frac{dy}{d\theta} \Big/ \frac{dx}{d\theta} = \frac{2}{2\tan\theta\sec^2\theta} = \frac{1}{\tan\theta\sec^2\theta}$

Now's $\because x = \tan^2\theta$

$\therefore \tan\theta = \sqrt{x}$

$\therefore \sec^2\theta = 1+\tan^2\theta$

$= 1+x$

$= \frac{1}{\sqrt{x}(1+x)}$

$= \frac{1}{\sqrt{x}+x\sqrt{x}}$ Ans

(11) If $x^3+y^3=3axy$ then $\frac{dy}{dx}=?$

$\Rightarrow \because x^3+y^3=3axy$

$\therefore \frac{d}{dx}(x^3+y^3)=\frac{d}{dx}(3axy)$

or, $\frac{d}{dx}x^3+\frac{d}{dx}y^3=3a\cdot\frac{d}{dx}xy$

or, $3x^2+3y^2\cdot\frac{dy}{dx}=3a\left[x\cdot\frac{dy}{dx}+y\right]$

or, $3x^2+3y^2\cdot\frac{dy}{dx}=3ax\cdot\frac{dy}{dx}+3ay$

or, $3y^2\cdot\frac{dy}{dx}-3ax\cdot\frac{dy}{dx}=3ay-3x^2$

or, $\frac{dy}{dx}(3y^2-3ax)=3ay-3x^2$

$\therefore \frac{dy}{dx}=\frac{3ay-3x^2}{3y^2-3ax}$ Ans

(12) If $\log(xy)=x+y$, then $\frac{dy}{dx}=?$

$\Rightarrow \because \log(xy)=x+y$

$\therefore \cancel{\frac{d}{dx}}$

or, $\frac{d}{dx}\log(xy)=\frac{d}{dx}(x+y)$

or, $\frac{1}{xy}\cdot\frac{d}{dx}(xy)=\frac{d}{dx}(x)+\frac{d}{dx}(y)$

or, $\frac{1}{xy}\left[x\cdot\frac{dy}{dx}+y\cdot\frac{d}{dx}(x)\right]=1+\frac{dy}{dx}$

or, $\frac{1}{xy}\left[x\cdot\frac{dy}{dx}+y\right]=1+\frac{dy}{dx}$

or, $\frac{1}{y}\cdot\frac{dy}{dx}+\frac{1}{x}=1+\frac{dy}{dx}$

or, $\frac{1}{y}\cdot\frac{dy}{dx}-\frac{dy}{dx}=1-\frac{1}{x}$

or, $\frac{dy}{dx}\left(\frac{1}{y}-1\right) = \left(1-\frac{1}{x}\right)$

$\therefore \frac{dy}{dx} = \frac{1-1/x}{\frac{1}{y}-1} = \frac{\frac{x-1}{x}}{\frac{1-y}{y}} = \frac{y(x-1)}{x(1-y)}$ Ans

(13) If $xy = \tan(x+y)$ then $\frac{dy}{dx} = ?$

$\Rightarrow \because xy = \tan(x+y)$

$\therefore \frac{d}{dx}(xy) = \frac{d}{dx}\tan(x+y)$

or, $x\cdot\frac{dy}{dx}+y = \sec^2(x+y)\cdot\frac{d}{dx}(x+y)$

or, $x\cdot\frac{dy}{dx}+y = \sec^2(x+y)\left(1+\frac{dy}{dx}\right)$

or, $x\cdot\frac{dy}{dx}+y = \sec^2(x+y) + \sec^2(x+y)\cdot\frac{dy}{dx}$

or, $\frac{dy}{dx}\left[x-\sec^2(x+y)\right] = \sec^2(x+y)-y$

$\therefore \frac{dy}{dx} = \frac{\sec^2(x+y)-y}{x-\sec^2(x+y)}$ Ans

(14) Find $\frac{dy}{dx}$, if $y = (1+x)^x$

$\Rightarrow \because y = (1+x)^x$

$\therefore \log y = \log(1+x)^x$

or, $\log y = x\cdot\log(1+x)$

$\therefore \frac{d}{dx}\log y = \frac{d}{dx}\left[x\cdot\log(1+x)\right]$

or, $\frac{1}{y}\cdot\frac{dy}{dx} = x\cdot\frac{d}{dx}\log(1+x) + \log(1+x)\cdot\frac{d}{dx}(x)$

or, $\frac{1}{y}\cdot\frac{dy}{dx} = x\cdot\frac{1}{1+x}\cdot(0+1) + \log(1+x)\cdot 1$

or, $\frac{1}{y}\cdot\frac{dy}{dx} = \frac{x}{1+x} + \log(1+x)$

$\therefore \frac{dy}{dx} = y\left[\frac{x}{1+x} + \log(1+x)\right]$

$= (1+x)^x\left[\frac{x}{1+x} + \log(1+x)\right]$ Ans

(15) Find $\frac{dy}{dx}$, if $y = (\tan x)^{\sin x}$ at $x = \frac{\pi}{4}$

$\Rightarrow \because y = \tan x^{\sin x}$

$\therefore \log y = \log(\tan x)^{\sin x}$

$\therefore \log y = \sin x\cdot\log(\tan x)$

$\therefore \frac{d}{dx}\log y = \frac{d}{dx}[\sin x\cdot\log(\tan x)]$

or, $\frac{1}{y}\cdot\frac{dy}{dx} = \sin x\cdot\frac{d}{dx}\log(\tan x) + \log(\tan x)\cdot\frac{d}{dx}\sin x$

or, $\frac{1}{y}\cdot\frac{dy}{dx} = \sin x\cdot\frac{1}{\tan x}\cdot\sec^2 x + \log(\tan x)\cdot\cos x$

or, $\frac{1}{y}\cdot\frac{dy}{dx} = \cancel{\sin x}\cdot\frac{\cos x}{\cancel{\sin x}}\cdot\frac{1}{\cos^2 x} + \cos x\cdot\log(\tan x)$

or, $\frac{1}{y}\cdot\frac{dy}{dx} = \sec x + \cos x\cdot\log(\tan x)$

$\therefore \frac{dy}{dx} = y\left[\sec x + \cos x\cdot\log(\tan x)\right]$

$= (\tan x)^{\sin x}\left[\sec x + \cos x\log\tan x\right]$

$\left.\frac{dy}{dx}\right|_{x=\frac{\pi}{4}} = \left(\tan\frac{\pi}{4}\right)^{\sin\pi/4}\left[\sec\frac{\pi}{4} + \cos\frac{\pi}{4}\cdot\log\tan\frac{\pi}{4}\right]$

$$= (1)^{\frac{1}{\sqrt{2}}}\left[\sqrt{2} + \log(1)^{\frac{1}{\sqrt{2}}}\right]$$

$$= 1 \times [\sqrt{2} + 0]$$

$$= \sqrt{2} \text{ Ans}$$

(16) If $x^y = y^x$, then $\frac{dy}{dx} = ?$

$\Rightarrow \because x^y = y^x$

$\therefore \log x^y = \log y^x$

$\therefore \frac{d}{dx}(y \cdot \log x) = \frac{d}{dx}(x \cdot \log y)$

or, $y \cdot \frac{d}{dx}\log x + \log x \cdot \frac{dy}{dx} = x \cdot \frac{d}{dx}\log y + \log y \cdot \frac{d}{dx}(x)$

or, $\frac{y}{x} + \log x \cdot \frac{dy}{dx} = \frac{x}{y} \cdot \frac{dy}{dx} + \log y \cdot 1$

or, $\log x \cdot \frac{dy}{dx} - \frac{x}{y} \cdot \frac{dy}{dx} = \log y - \frac{y}{x}$

or, $\frac{dy}{dx}\left[\log x - \frac{x}{y}\right] = \left[\log y - \frac{y}{x}\right]$

$$\therefore \frac{dy}{dx} = \frac{\log y - y/x}{\log x - x/y}$$

$$= \frac{\frac{x\log y - y}{x}}{\frac{y\log x - x}{y}}$$

$$= \frac{y(x\log y - y)}{x(y\log x - x)} \text{ Ans}$$

(17) If $x^y = e^{x-y}$ then; $\frac{dy}{dx} = ?$

$\Rightarrow \because x^y = e^{x-y}$

$\therefore \log x^y = \log e^{x-y}$

or, $y \cdot \log x = (x-y)\log e$

or, $y \cdot \log x = (x-y)$

or, $\frac{d}{dx}(y \cdot \log x) = \frac{d}{dx}(x-y)$

or, $y \cdot \frac{d}{dx}\log x + \log x \cdot \frac{d}{dx}(y) = \frac{d}{dx}(x) - \frac{d}{dx}(y)$

or, $\frac{y}{x} + \log x \cdot \frac{dy}{dx} = 1 - \frac{dy}{dx}$

or, $\log x \cdot \frac{dy}{dx} + \frac{dy}{dx} = 1 - y/x$

or, $\frac{dy}{dx}(\log x + 1) = (1 - y/x)$

$\therefore \frac{dy}{dx} = \frac{1 - y/x}{\log x + 1} = \frac{x-y}{x(\log x + 1)} = \frac{x-y}{x\log x + x}$ Ans

(18) If $x = a\cos^3\theta$ & $y = b\sin^3\theta$, then $\frac{dy}{dx} = ?$

$\Rightarrow \because x = a\cos^3\theta$

$\therefore \frac{dx}{d\theta} = \frac{d}{d\theta} a\cos^3\theta$

$= a \cdot \frac{d}{d\theta}\cos^3\theta$

$= a \cdot 3 \cdot \cos^2\theta(-\sin\theta)$

$= -3a\sin\theta\cos^2\theta$

$\because y = b\sin^3\theta$

$\therefore \frac{dy}{d\theta} = \frac{d}{d\theta} b \cdot \sin^3\theta$

$= b \cdot \frac{d}{d\theta}\sin^3\theta$

$= b \cdot 3 \cdot \sin^2\theta \cdot \cos\theta$

$= 3b\sin^2\theta\cos\theta$

$$\therefore \frac{dy}{dx} = \frac{dy}{d\theta} \times \frac{d\theta}{dx}$$

$$= \frac{\cancel{3}b \sin^2\theta \cos\theta}{-\cancel{3}a \sin\theta \cos^2\theta}$$

$$= -\frac{b}{a} \cdot \frac{\sin\theta}{\cos\theta}$$

$$= -\frac{b}{a} \tan\theta \quad \text{Ans}$$

(15) If $x = a(\theta - \sin\theta)$ & $y = a(1 + \cos\theta)$, then $\frac{dy}{dx} = ?$

$\Rightarrow \because x = a(\theta - \sin\theta)$

$$\therefore \frac{dx}{d\theta} = \frac{d}{d\theta} a(\theta - \sin\theta)$$

$$= a \cdot (1 - \cos\theta)$$

$\because y = a(1 + \cos\theta)$

$$\therefore \frac{dy}{d\theta} = a \cdot \frac{d}{d\theta}(1 + \cos\theta)$$

$$= a \cdot (0 - \sin\theta)$$

$$= -a\sin\theta$$

$$\therefore \frac{dy}{dx} = \frac{dy}{d\theta} \times \frac{d\theta}{dx}$$

$$= \frac{-a\sin\theta}{a(1 - \cos\theta)}$$

$$= -\frac{\sin\theta}{1 - \cos\theta}$$

$$= \frac{\sin\theta}{\cos\theta - 1} \quad \text{Ans}$$

(20) If $y = a(1+\cos\theta)$ & $x = a(\theta+\sin\theta)$ at $\theta = \frac{\pi}{2}$ then $\frac{dy}{dx} = ?$

$\Rightarrow \because x = a(\theta+\sin\theta)$

$\therefore \frac{dx}{d\theta} = \frac{d}{d\theta}[a(\theta+\sin\theta)]$

$= a\cdot\frac{d}{d\theta}(\theta+\sin\theta)$

$= a\cdot\left[\frac{d}{d\theta}(\theta) + \frac{d}{d\theta}\sin\theta\right]$

$= a[1+\cos\theta]$

$\because \theta = \frac{\pi}{2}$

$\therefore \frac{dx}{d\theta} = a\left[1+\cos\frac{\pi}{2}\right]$

$= a[1+0]$ $\left[\because \cos\frac{\pi}{2} = 0\right]$

$= a\times 1$

$= a$

$\therefore$ Similarly

$\therefore y = a(1+\cos\theta)$

$\therefore \frac{dy}{d\theta} = \frac{d}{d\theta}[a(1+\cos\theta)]$

$= a\cdot\frac{d}{d\theta}(1+\cos\theta)$

$= a\cdot\left[\frac{d}{d\theta}(1) + \frac{d}{d\theta}\cos\theta\right]$

$= a[0-\sin\theta]$

$= -a\sin\theta$

$\therefore \frac{dy}{d\theta}$ at $\theta = \pi/2$ $= -a\times\sin\frac{\pi}{2} = -a\times 1$

$= -a$

$\therefore \frac{dy}{dx} = \frac{dy}{d\theta}\times\frac{d\theta}{dx} = \frac{-a}{a} = -1$ Ans

(21) Find the derivative of $\sin^{-1}\frac{2x}{1+x^2}$ with respect to $\tan^{-1}\frac{2x}{1-x^2}$

$\Rightarrow$ Let's $u = \sin^{-1}\frac{2x}{1+x^2}$

Now; Let's $x = \tan\theta$ $\therefore \theta = \tan^{-1}(x)$

$\therefore u = \sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right)$

$= \sin^{-1}(\sin 2\theta)$

$= 2\theta$

$= 2\tan^{-1}x$

$\therefore \frac{du}{dx} = \frac{d}{dx} 2\tan^{-1}x$

$= 2\frac{d}{dx}\tan^{-1}x$

$= 2\cdot\frac{1}{1+x^2}$

$= \frac{2}{1+x^2}$

Again; Let; $v = \tan^{-1}\frac{2x}{1-x^2}$

Let; $x = \tan\alpha \therefore \alpha = \tan^{-1}(x)$

$\therefore v = \tan^{-1}\left(\frac{2\tan\alpha}{1-\tan^2\alpha}\right)$

$= \tan^{-1}(\tan 2\alpha)$

$= 2\alpha$

$= 2\tan^{-1}x$

$\therefore \frac{dv}{dx} = \frac{d}{dx} 2\tan^{-1}x = \frac{2}{1+x^2}$

$\therefore \frac{du}{dv} = \frac{du}{dx}\times\frac{dx}{dv} = \frac{\frac{2}{1+x^2}}{\frac{2}{1+x^2}} = 1$ Ans

(22) Find derivative of $y = x^6$ w.r.t. x^3

$\Rightarrow \therefore y = x^6$ Let; $v = x^3$

$\therefore \frac{dy}{dx} = 6x^5$ $\therefore \frac{dv}{dx} = \frac{d}{dx}x^3 = 3x^2$

$\therefore \frac{dy}{dv} = \frac{dy}{dx}\times\frac{dx}{dv} = \frac{6x^5}{3x^2} = 2x^3$ Ans