INDEFINITE INTEGRATION

(Formulas)

(1) $\int x^n dx = \frac{x^{n+1}}{n+1}$ $(n \neq -1)$

(2) $\int \frac{dx}{x^n} = -\frac{1}{(n-1)x^{n-1}}$, $(n \neq 1)$

(3) $\int dx = x$

(4) $\int \frac{dx}{\sqrt{x}} = 2\sqrt{x}$

(5) $\int \frac{dx}{x} = \log|x|$

(6) $\int e^x dx = e^x$

(7) $\int e^{mx} dx = \frac{e^{mx}}{m}$

(8) $\int a^x dx = \frac{a^x}{\log_e a}$ $(a>0)$

(9) $\int \sin x\, dx = -\cos x$

(10) $\int \sin mx\, dx = -\frac{\cos mx}{m}$

(11) $\int \cos x\, dx = \sin x$

(12) $\int \cos mx\, dx = \frac{\sin mx}{m}$

(13) $\int \sec^2 x\, dx = \tan x$

(14) $\int \operatorname{cosec}^2 x\, dx = -\cot x$

(15) $\int \sec x \tan x\, dx = \sec x$

(16) $\int \operatorname{cosec} x \cot x\, dx = -\operatorname{cosec} x$

(17) $\int p f(x)\, dx$

$= p\int f(x)\, dx$

where p is a constant

(18) $\int [f(x) \pm g(x)]\, dx$

$= \int f(x)\, dx \pm \int g(x)\, dx$

$f(x) \Rightarrow$ function of x

$g(x) \Rightarrow$ function of x

① Evaluate $\int \sin^2 x\, dx$

$= \int \sin^2 x\, dx$

$= \int \frac{1}{2} \times 2 \sin^2 x\, dx$

$= \frac{1}{2} \int 2\sin^2 x\, dx$

$= \frac{1}{2} \int (1 - \cos 2x)\, dx \quad [\because 2\sin^2\theta = 1 - \cos 2\theta]$

$= \frac{1}{2} \left[\int dx - \int \cos 2x\, dx \right] \quad \left[\because \int \cos mx\, dx = \frac{\sin mx}{m}\right]$

$= \frac{1}{2} \left[x - \frac{\sin 2x}{2} \right] + c$ where; c = integration constant

② Evaluate $\int \cos^2 x\, dx$

$= \int \cos^2 x\, dx$

$= \frac{1}{2} \int 2\cos^2 x\, dx$

$= \frac{1}{2} \int (1 + \cos 2x)\, dx \quad [\because 2\cos^2\theta = 1 + \cos 2\theta]$

$= \frac{1}{2} \left[\int dx + \int \cos 2x\, dx \right]$

$= \frac{1}{2} \left[x + \frac{\sin 2x}{2} \right] + c \quad \left[\int \cos mx\, dx = \frac{\sin mx}{m}\right]$

where; c is a integration constant

③ $\int \frac{dx}{\sin^2 x \cos^2 x}$

$= \int \frac{(\sin^2 x + \cos^2 x)}{\sin^2 x \cos^2 x}\, dx \quad [\because \sin^2\theta + \cos^2\theta = 1]$

$= \int \frac{\cancel{\sin^2 x}}{\cancel{\sin^2 x} \cos^2 x}\, dx + \int \frac{\cancel{\cos^2 x}}{\sin^2 x \cancel{\cos^2 x}}\, dx$

$= \int \sec^2 x\, dx + \int \operatorname{cosec}^2 x\, dx \quad \left[\because \frac{1}{\cos\theta} = \sec\theta \text{ and } \frac{1}{\sin\theta} = \operatorname{cosec}\theta\right]$

$= \tan x - \cot x + C$

where; C is a integration constant

④ $\int \tan^2 x \, dx$

$= \int (\sec^2 x - 1) dx$ [$\because \sec^2\theta - \tan^2\theta = 1$

$\therefore \sec^2\theta - 1 = \tan^2\theta$]

$= \int \sec^2 x \, dx - \int dx$

$= \tan x - x + C$ [$\int \sec^2 x \, dx = \tan x$]

where; C is a integration constant

⑤ $\int \cos x^\circ \, dx$

$\Rightarrow \because 180^\circ = \pi$

$\therefore x^\circ = \frac{\pi x}{180^\circ}$

$\therefore \int \cos x^\circ dx$

$= \int \cos\left(\frac{\pi x}{180}\right) dx$

$= \frac{\sin \frac{\pi x}{180^\circ}}{\frac{\pi}{180^\circ}}$ [$\because \int \cos mx \, dx = \frac{\sin mx}{m}$]

$= \frac{180^\circ}{\pi} \sin \frac{\pi x}{180^\circ}$

$= \frac{180^\circ}{\pi} \sin x^\circ + C$

where; C = integration constant

⑥ $\int \frac{x^2}{x-2} dx$

$= \int \frac{x^2 - 4 + 4}{x-2} dx$ [$\because 0 = -4 + 4$, we can write]

$= \int \frac{x^2 - 4}{x-2} dx + \int \frac{4}{x-2} dx$

$= \int \frac{(x+2)\cancel{(x-2)}}{\cancel{(x-2)}} dx + 4\int \frac{dx}{(x-2)}$

$= \int (x+2)\, dx + 4\int \frac{dx}{(x-2)}$

$= \int x\, dx + \int 2\, dx + 4\int \frac{dx}{(x-2)}$

$= \frac{1}{2}x^2 + 2x + 4\log|x-2| + c \quad \left[\because \int \frac{dx}{x \pm a} = \log|x \pm a|\right]$

where; c = integration constant

⑦ $\int \frac{e^{4x} + e^{8x}}{e^{-2x} + e^{2x}} dx$

$= \int \frac{e^{6x}(e^{-2x} + e^{2x})}{(e^{-2x} + e^{2x})} dx$

$= \int e^{6x} dx$

$= \frac{e^{6x}}{6} + c \quad \left[\because \int e^{mx} dx = \frac{e^{mx}}{m} + c\right]$

where; c = integration constant

⑧ $\int \frac{dx}{1 + \sin x}$

$= \int \frac{(1 - \sin x)}{(1 + \sin x)(1 - \sin x)} dx$

$= \int \frac{(1 - \sin x)}{1 - \sin^2 x} dx$

$= \int \frac{1 - \sin x}{\cos^2 x} dx \quad [\because 1 - \sin^2\theta = \cos^2\theta]$

$= \int \frac{dx}{\cos^2 x} - \int \frac{\sin x}{\cos^2 x} dx$

$= \int \sec^2 x \, dx - \int \tan x \sec x \, dx$

$= \tan x - \sec x + C$ $\left[\because \int \sec^2 x \, dx = \tan x \text{ \& } \int \sec x \tan x \, dx = \sec x\right]$

where; C = integration constant

(9) $\int \frac{dx}{1-\cos x}$

$= \int \frac{(1+\cos x)\, dx}{(1-\cos x)(1+\cos x)}$

$= \int \frac{(1+\cos x)\, dx}{1-\cos^2 x}$

$= \int \frac{(1+\cos x)\, dx}{\sin^2 x}$ $[\because 1-\cos^2\theta = \sin^2\theta]$

$= \int \frac{dx}{\sin^2 x} + \int \frac{\cos x \, dx}{\sin^2 x}$

$= \int \operatorname{cosec}^2 x + \int \operatorname{cosec} x \cot x \, dx$

$= -\cot x - \operatorname{cosec} x + C$ $\left[\because \int \operatorname{cosec}^2 x \, dx = -\cot x \text{ \& } \int \operatorname{cosec} x \cot x \, dx = -\operatorname{cosec} x\right]$

$= -\operatorname{cosec} x - \cot x + C$

$= -(\operatorname{cosec} x + \cot x) + C$

where; C = integration constant

(10) $\int \sqrt{1+\sin 2x} \, dx$

$\therefore 1+\sin 2x = \sin^2 x + \cos^2 x + 2\sin x \cos x$ $[\because \sin^2\theta + \cos^2\theta = 1$ & $\sin 2\theta = 2\sin\theta \cdot \cos\theta]$

$= (\sin x + \cos x)^2$ → $a^2+b^2+2ab = (a+b)^2$ is implemented.

$\therefore \sqrt{1+\sin 2x} = \sin x + \cos x$

$\therefore \int \sqrt{1+\sin x} \, dx = \int (\sin x + \cos x)\, dx$

$= \int \sin x \, dx + \int \cos x \, dx$

$= -\cos x + \sin x + C = \sin x - \cos x + C$ Ans

⑩ $\int \frac{e^{3x}+e^{5x}}{e^{x}+e^{-x}}\,dx$

$= \int \frac{e^{4x}\cdot e^{-x}+e^{4x}\cdot e^{x}}{e^{x}+e^{-x}}\,dx$

$= \int \frac{e^{4x}(e^{-x}+e^{x})}{e^{x}+e^{-x}}\,dx$

$= \int \frac{e^{4x}(e^{x}+e^{-x})}{(e^{x}+e^{-x})}\,dx$

$= \int e^{4x}\,dx$

$= \frac{1}{4}e^{4x}+c \quad \left[\because \int e^{mx}\,dx = \frac{e^{mx}}{m}\right]$

⑪ $\int \sin 3x \sin 5x\,dx$

$= \frac{1}{2}\int 2\sin 5x \sin 3x\,dx$

$= \frac{1}{2}\int [\cos(5x-3x)-\cos(5x+3x)]\,dx \quad [\because 2\sin A \sin B = \cos(A-B)-\cos(A+B)]$

$= \frac{1}{2}\int (\cos 2x - \cos 8x)\,dx$

$= \frac{1}{2}\left[\int \cos 2x\,dx - \int \cos 8x\,dx\right]$

$= \frac{1}{2}\left[\frac{\sin 2x}{2} - \frac{\sin 8x}{8}\right]+c$

$= \frac{\sin 2x}{4} - \frac{\sin 8x}{16}+c$

$= \frac{1}{4}\sin 2x - \frac{1}{16}\sin 8x + c$ Ans

⑫ $\int \sin 5x \cos 3x\,dx$

$= \frac{1}{2}\int 2\sin 5x \cos 3x\,dx$

$= \frac{1}{2}\int (\sin(5x+3x) + \sin(5x-3x))dx$ $[\because 2\sin A\cos B$
$= \sin(A+B) + \sin(A-B)]$

$= \frac{1}{2}\int(\sin 8x + \sin 2x)dx$

$= \frac{1}{2}\left[\int \sin 8x\, dx + \int \sin 2x\, dx\right]$

$= \frac{1}{2}\left[-\frac{\cos 8x}{8} - \frac{\cos 2x}{2}\right] + c$ $\left[\because \int \sin mx\, dx = -\frac{\cos mx}{m}\right]$

$= -\frac{1}{16}\cos 8x - \frac{1}{4}\cos 2x + c$ Ans

(13) $\int \sin^3 x\, dx$

$\because \sin 3\theta = 3\sin\theta - 4\sin^3\theta$

or, $4\sin^3\theta = 3\sin\theta - \sin 3\theta$

$\therefore \sin^3\theta = \frac{1}{4}(3\sin\theta - \sin 3\theta)$

$\therefore \int \sin^3 x\, dx$

$= \int \frac{1}{4}(3\sin x - \sin 3x)dx$ $\left[\because \sin^3\theta = \frac{1}{4}(3\sin\theta - \sin 3\theta)\right]$

$= \frac{1}{4}\left[\int 3\sin x\, dx - \int \sin 3x\, dx\right]$

$= \frac{1}{4}\left[3\cdot\int \sin x\, dx - \int \sin 3x\, dx\right]$

$= \frac{1}{4}\left[3(-\cos x) - \left(-\frac{\cos 3x}{3}\right)\right]$ $\left[\because \int \sin x\, dx = -\cos x\right]$

$= \frac{1}{4}\left[-3\cos x + \frac{\cos 3x}{3}\right] + c$

$= \frac{1}{4}\left[\frac{\cos 3x}{3} - 3\cos x\right] + c$

$= \frac{1}{12}\cos 3x - \frac{3}{4}\cos x + c$ Ans

(14) $\int \cos^3 x \, dx$

$\because \cos 3\theta = 4\cos^3\theta - 3\cos\theta$

or, $4\cos^3\theta = \cos 3\theta + 3\cos\theta$

$\therefore \cos^3\theta = \frac{1}{4}(\cos 3\theta + 3\cos\theta)$

$\therefore \int \cos^3 x \, dx$

$= \int \frac{1}{4}(\cos 3x + 3\cos x)\, dx \quad \left[\because \cos^3\theta = \frac{1}{4}(\cos 3\theta + 3\cos\theta)\right]$

$= \frac{1}{4}\left[\int \cos 3x \, dx + \int 3\cos x \, dx\right]$

$= \frac{1}{4}\left[\frac{\sin 3x}{3} + 3 \cdot \sin x\right] + c \quad \left[\because \int \cos mx \, dx = \frac{\sin mx}{m}\right]$

$= \frac{1}{12}\sin 3x + \frac{3}{4}\sin x + c$ Ans

(15) $\int \frac{p\sin^3 x + q\cos^3 x}{\sin^2 x \cos^2 x} dx$

$= p\int \frac{\sin^3 x}{\sin^2 x \cos^2 x} dx + q\int \frac{\cos^3 x}{\sin^2 x \cos^2 x} dx$

$= p\int \frac{\sin x}{\cos^2 x} dx + q\int \frac{\cos x}{\sin^2 x} dx$

$= p\int \tan x \sec x \, dx + q\int \cot x \operatorname{cosec} x \, dx$

$= p\sec x - q\operatorname{cosec} x + c$ Ans