MATRICES [2nd sem/Math-2]

① If $A = \begin{bmatrix} 2 & -1 & 0 \\ 0 & 2 & 4 \end{bmatrix}$ then $A^T = ?$

$\Rightarrow$ Given; $A = \begin{bmatrix} 2 & -1 & 0 \\ 0 & 2 & 4 \end{bmatrix}$

$\therefore A^T = \begin{bmatrix} 2 & 0 \\ -1 & 2 \\ 0 & 4 \end{bmatrix}$ Ans

② If $A = \begin{bmatrix} 3 & -1 & 4 \\ 2 & 5 & 6 \\ 1 & 2 & 3 \end{bmatrix}$ then $A^T = ?$

$\Rightarrow$ Given; $A = \begin{bmatrix} 3 & -1 & 4 \\ 2 & 5 & 6 \\ 1 & 2 & 3 \end{bmatrix}$

$\therefore A^T = \begin{bmatrix} 3 & 2 & 1 \\ -1 & 5 & 2 \\ 4 & 6 & 3 \end{bmatrix}$ Ans

③ Given $\begin{bmatrix} 3 & x+1 \\ y & 5 \end{bmatrix} = \begin{bmatrix} x-1 & 5 \\ x-3y & 5 \end{bmatrix}$, find x & y

$\Rightarrow \because \begin{bmatrix} 3 & x+1 \\ y & 5 \end{bmatrix} = \begin{bmatrix} x-1 & 5 \\ x-3y & 5 \end{bmatrix}$

$\therefore 3 = x-1$

$\therefore x = 3+1$

$= 4$ Ans

$\therefore y = x-3y$

$\therefore y+3y = x$

$\therefore 4y = x$

$\therefore y = \frac{x}{4} = \frac{4}{4} = 1$

$\therefore x = 4$ & $y = 1$ Ans

④ If $\begin{bmatrix} 2x-y & 5 \\ 3 & y \end{bmatrix} = \begin{bmatrix} 6 & 5 \\ 3 & -2 \end{bmatrix}$ then x & y = ?

$\Rightarrow \because \begin{bmatrix} 2x-y & 5 \\ 3 & y \end{bmatrix} = \begin{bmatrix} 6 & 5 \\ 3 & -2 \end{bmatrix}$

$\therefore 2x-y=6$ | $\because y=-2$

$\therefore 2x-(-2)=6$

$\therefore 2x+2=6$

$\therefore 2x=6-2$

$\therefore 2x=4$

$\therefore x=2$

$\therefore x=2$ & $y=-2$ Ans

⑤ If $3\begin{bmatrix} x & y \\ z & t \end{bmatrix} = \begin{bmatrix} 4 & 3 \\ 5 & 0 \end{bmatrix} + \begin{bmatrix} 2 & 6 \\ 4 & 12 \end{bmatrix}$ then $(x, y, z, t) = ?$

$\Rightarrow 3\begin{bmatrix} x & y \\ z & t \end{bmatrix} = \begin{bmatrix} 4 & 3 \\ 5 & 0 \end{bmatrix} + \begin{bmatrix} 2 & 6 \\ 4 & 12 \end{bmatrix}$

or, $\begin{bmatrix} 3x & 3y \\ 3z & 3t \end{bmatrix} = \begin{bmatrix} 6 & 9 \\ 9 & 12 \end{bmatrix}$

or, $3x=6$ | or, $3y=9$ | or, $3z=9$ | or, $3t=12$

$\therefore x=6/3$ | $\therefore y=9/3$ | $\therefore z=9/3$ | $\therefore t=12/3$

$=2$ | $=3$ | $=3$ | $=4$

$\therefore (x, y, z, t) = (2, 3, 3, 4)$ Ans

(6) Multiply $\begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$ & $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix} \times \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 2\times1+(-1)\times3 & 2\times2+(-1)\times4 \\ 3\times1+2\times3 & 3\times2+2\times4 \end{bmatrix}$$

$$= \begin{bmatrix} 2-3 & 4-4 \\ 3+6 & 6+8 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 0 \\ 9 & 14 \end{bmatrix} \text{ Ans}$$

(7) If $A = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$ & $B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ then $AB = ?$

$$\Rightarrow AB = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix} \times \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 1\times1+(-1)\times3 & 1\times2+(-1)\times4 \\ 2\times1+3\times3 & 2\times2+3\times4 \end{bmatrix}$$

$$= \begin{bmatrix} 1-3 & 2-4 \\ 2+9 & 4+12 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & -2 \\ 11 & 16 \end{bmatrix} \text{ Ans}$$

(8) Show that A is a singular matrix if A

$$= \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 2 \\ 1 & 3 & 4 \end{bmatrix}$$

⇒ If $|A| = 0$, then A will be a singular matrix

$$\therefore \begin{vmatrix} 1 & 2 & 3 \\ 1 & 1 & 2 \\ 1 & 3 & 4 \end{vmatrix} = 1\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} - 2\begin{vmatrix} 1 & 2 \\ 1 & 4 \end{vmatrix} + 3\begin{vmatrix} 1 & 1 \\ 1 & 3 \end{vmatrix}$$

$$= 1(4-6) - 2(4-2) + 3(3-1)$$

$$= 1(-2) - 2(2) + 3(2)$$

$$= -2 - 4 + 6$$

$$= \cancel{-6} + \cancel{6}$$

$$= 0$$

As, $|A| = 0$, so it is a singular matrix (proved)

(9) If $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ then $A^2 = ?$

⇒ $\because A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\therefore A^2 = A \times A$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1\times1+0\times0 & 1\times0+0\times1 \\ 0\times1+1\times0 & 0\times0+1\times1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+0 & 0+0 \\ 0+0 & 0+1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$ Ans

⑩ If $I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ then $I_3^2 + 2I_3 = ?$

$\Rightarrow I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\therefore I_3^2 = I_3 \times I_3$

$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\therefore 2I_3 = 2\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

$\therefore I_3^2 + 2I_3$

$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

$= \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ Ans

⑪ If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ & $B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, find or show that

$AB = BA$

$\Rightarrow \therefore AB = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \times \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1\times1 + 2\times0 & 1\times0 + 2\times1 \\ 3\times1 + 4\times0 & 3\times0 + 4\times1 \end{bmatrix}$

$= \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

$\therefore BA = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

$= \begin{bmatrix} 1\times1+0\times3 & 1\times2+0\times4 \\ 0\times1+1\times3 & 0\times2+1\times4 \end{bmatrix}$

$= \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

$\therefore AB = BA$ (proved)

(12) If $A = \begin{pmatrix} 1 & 4 & 5 \\ 2 & 0 & 3 \\ 3 & 2 & 1 \end{pmatrix}$ then $A+A^T$ is what type of matrix?

$\Rightarrow \because A = \begin{bmatrix} 1 & 4 & 5 \\ 2 & 0 & 3 \\ 3 & 2 & 1 \end{bmatrix}$ $\therefore A^T = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 0 & 2 \\ 5 & 3 & 1 \end{bmatrix}$

$\therefore A+A^T = \begin{bmatrix} 1 & 4 & 5 \\ 2 & 0 & 3 \\ 3 & 2 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 2 & 3 \\ 4 & 0 & 2 \\ 5 & 3 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1+1 & 4+2 & 5+3 \\ 2+4 & 0+0 & 3+2 \\ 3+5 & 2+3 & 1+1 \end{bmatrix}$

$= \begin{bmatrix} 2 & 6 & 8 \\ 6 & 0 & 5 \\ 8 & 5 & 2 \end{bmatrix}$

As, $A_{12} = A_{21}$, $A_{13} = A_{31}$ & $A_{23} = A_{32}$

$\therefore A+A^T$ is is symmetric matrix.

(13) If $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 4 & 5 \\ 5 & 6 & 7 \end{bmatrix}$ then show that $A+A^T$ is symmetric matrix & $A-A^T$ is a skew-symmetric matrix

$\Rightarrow \because A^T = \begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \\ 3 & 5 & 7 \end{bmatrix}$

$\therefore A+A^T = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 4 & 5 \\ 5 & 6 & 7 \end{bmatrix} + \begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \\ 3 & 5 & 7 \end{bmatrix}$

$= \begin{bmatrix} 2 & 5 & 8 \\ 5 & 8 & 11 \\ 8 & 11 & 14 \end{bmatrix}$

As, $A_{12} = A_{21}$, $A_{13} = A_{31}$, & $A_{23} = A_{32}$

$\therefore A+A^T$ is a symmetric matrix.

$\therefore A-A^T = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 4 & 5 \\ 5 & 6 & 7 \end{bmatrix} - \begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \\ 3 & 5 & 7 \end{bmatrix}$

$= \begin{bmatrix} 0 & -1 & -2 \\ 1 & 0 & -1 \\ 2 & 1 & 0 \end{bmatrix}$

As, $-A_{12} = A_{21}$, $-A_{13} = A_{31}$ & $-A_{23} = A_{32}$

$\therefore A-A^T$ is a skew-symmetric matrix.

(14) If $A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$ then $Adj(A) = ?$

$\Rightarrow \quad Adj(A) = \begin{bmatrix} A_{11} & -A_{12} & A_{13} \\ -A_{21} & A_{22} & -A_{23} \\ A_{31} & -A_{32} & A_{33} \end{bmatrix}$

$\therefore A_{11} = \begin{vmatrix} -3 & 4 \\ -1 & 1 \end{vmatrix} = 1$

$\therefore A_{12} = \begin{vmatrix} 2 & 4 \\ 0 & 1 \end{vmatrix} = 2$

$\therefore A_{13} = \begin{vmatrix} 2 & -3 \\ 0 & -1 \end{vmatrix} = -2$

$\therefore A_{21} = \begin{vmatrix} -3 & 4 \\ -1 & 1 \end{vmatrix} = -3+4 = 1$

$\therefore A_{22} = \begin{vmatrix} 3 & 4 \\ 0 & 1 \end{vmatrix} = 3$

$\therefore A_{23} = \begin{vmatrix} 3 & -3 \\ 0 & -1 \end{vmatrix} = -3$

$\therefore A_{31} = \begin{vmatrix} -3 & 4 \\ -3 & 4 \end{vmatrix} = -\cancel{12} + \cancel{12} = 0$

$\therefore A_{32} = \begin{vmatrix} 3 & 4 \\ 2 & 4 \end{vmatrix} = 12 - 8 = 4$

$\therefore A_{33} = \begin{vmatrix} 3 & -3 \\ 2 & -3 \end{vmatrix} = -9 + 6 = -3$

$\therefore Adj(A) = \begin{bmatrix} A_{11} & -A_{12} & A_{13} \\ -A_{21} & A_{22} & -A_{23} \\ A_{31} & -A_{32} & A_{33} \end{bmatrix} = \begin{bmatrix} 1 & -2 & -2 \\ -1 & 3 & 3 \\ 0 & -4 & -3 \end{bmatrix}$ Ans

(15) If $A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$ then $A^{-1} = ?$

$\Rightarrow \because A^{-1} = \dfrac{Adj(A)}{|A|}$

$\therefore |A| = \begin{vmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{vmatrix}$

$= 3\begin{vmatrix} -3 & 4 \\ -1 & 1 \end{vmatrix} + 3\begin{vmatrix} 2 & 4 \\ 0 & 1 \end{vmatrix} + 4\begin{vmatrix} 2 & -3 \\ 0 & -1 \end{vmatrix}$

$= 3(-3+4) + 3(2-0) + 4(-2+0)$

$= 3(1) + 3\times2 + 4(-2)$

$= 3+6-8$

$= 9-8$

$= 1$

$\therefore Adj(A) = \begin{bmatrix} 1 & -2 & -2 \\ -1 & 3 & 3 \\ 0 & -4 & -3 \end{bmatrix}$ {see 14 NO. math}

$\therefore A^{-1} = \dfrac{Adj(A)}{|A|} = \begin{bmatrix} 1 & -2 & -2 \\ -1 & 3 & 3 \\ 0 & -4 & -3 \end{bmatrix}$ Ans

(16) If $P = \begin{pmatrix} a & 0 \\ -1 & -2 \end{pmatrix}$ & $Q = \begin{pmatrix} 1 & 0 \\ 3 & 4 \end{pmatrix}$ & $P^2 = Q$, then

Show that $a = -1$

$\Rightarrow \therefore P^2 = P \times P$

$$= \begin{bmatrix} a & 0 \\ -1 & -2 \end{bmatrix} \times \begin{bmatrix} a & 0 \\ -1 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} a \times a + 0 \times (-1) & a \times 0 + 0 \times (-2) \\ (-1) \times a + (-2) \times (-1) & (-1) \times 0 + (-2) \times (-2) \end{bmatrix}$$

$$= \begin{bmatrix} a^2 & 0 \\ -a+2 & 4 \end{bmatrix}$$

$\because P^2 = Q$

$$\therefore \begin{bmatrix} a^2 & 0 \\ -a+2 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 3 & 4 \end{bmatrix}$$

$\therefore a^2 = 1$

$\therefore a = \pm\sqrt{1}$

$= +1, -1$

$\therefore -a + 2 = 3$

$\therefore -a = 3 - 2$

$\therefore -a = 1$

$\therefore a = -1$

$\therefore a = -1$ (proved)