APPLICATION OF DEFINITE INTEGRAL

① Find out the area bounded by a straight line $y=2x$ above x axis from $x=1$ to $x=4$

⇒ Given; $y=2x$

$x=4\ (b) \rightarrow$ Upper limit

$x=1\ (a) \rightarrow$ Lower limit

∴ The Area $= \int_1^4 y\,dx$

$= \int_1^4 2x\,dx$

$= 2\int_1^4 x\,dx$

$= 2 \times \frac{1}{2}[x^2]_1^4$

$= [x^2]_1^4$

$= 4^2 - 1^2$

$= 16 - 1$

$= 15$ sq. unit.

② Find out the area bounded by the straight line $y=x$, from $y=1$ & $y=3$ (& $x=0$).

⇒ Given; $y=x$

$y=3 \rightarrow$ Upper Limit

$y=1 \rightarrow$ Lower Limit

∴ The area $= \int_1^3 x\,dy = \int_1^3 y\,dy = \frac{1}{2}[y^2]_1^3$

$= \frac{1}{2}[3^2 - 1^2]$

$= \frac{1}{2}[9 - 1]$

$= \frac{1}{2} \times 8$

$= 4$ sq. unit Ans

(3) Find out the area bounded by the curve $y = \tan x$, the x axis and $x = \frac{\pi}{4}$

$\Rightarrow$ Given: $y = \tan x$

$x = \frac{\pi}{4} \rightarrow$ upper limit

& $x = 0 \rightarrow$ Lower limit

$\therefore$ The area

$= \int_0^{\pi/4} y \, dx$

$= \int_0^{\pi/4} \tan x \, dx$

$= [\log|\sec x|]_0^{\pi/4}$

$= [\log|\sec \frac{\pi}{4}| - \log|\sec 0|]$

$= \log\sqrt{2} - \log 1$

$= \log\left(\frac{\sqrt{2}}{1}\right)$

$= \log\sqrt{2}$

$= \log 2^{1/2} = \frac{1}{2}\log 2$ sq. unit Ans

(4) Find out the area bounded by the x axis & one arc of the sine curve $y = \sin x$ is equal = ?

$\Rightarrow$ $\because$ Formed one arc, so x strats from 0 & ends to π

$\therefore x = \pi \rightarrow$ upper limit

$\therefore x = 0 \rightarrow$ Lower limit

& $y = \sin x$

$\therefore$ The area $= \int_0^\pi y \, dx$

$= \int_0^\pi \sin x \, dx$

$= [-\cos x]_0^\pi$

$= -[\cos x]_0^\pi$

$= -[\cos\pi - \cos 0]$

$= -[(-1) - 1]$

$= 2$ sq. unit Ans

(5) Find the area included between parabola $x^2 = 16y$; its focus & right hand side of y-axis.

$\Rightarrow$ Given; $x^2 = 16y = 4(4)y$

The area bounded by $x = 0$, $x^2 = 16y$ and $y = 4$

$\therefore$ The area is $= \int_0^4 x^2 \, dy$

$= \int_0^4 16y \, dy$

$= 16\int_0^4 y\,dy$

$= 16\int_0^4 y\,dy$

$= 16\left[\frac{1}{2}\cdot y^2\right]_0^4$

$= 16\times\frac{1}{2}\left[y^2\right]_0^4$

$= 8\left[4^2-0^2\right]$

$= 8\left[16-0\right]$

$= 8\times 16$

$= 128$ sq. unit Ans

⑥ Find the area above x axis which is bounded by the parabola $y^2=4x$ & the straight lines $x=0$ to $x=a$

$\Rightarrow$ Given, $y^2=4x$

$\therefore y=\sqrt{4x}=2\sqrt{x}=2\times x^{1/2}$

$\therefore$ The area $=\int_0^a y\,dx$

$=\int_0^a 2\times x^{1/2}\,dx$

$=2\int_0^a x^{1/2}\,dx$

$=2\times\left[\dfrac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1}\right]_0^a$

$$= 2\times\left[\frac{x^{3/2}}{3/2}\right]_0^a$$

$$= 2\times\frac{2}{3}\left[x^{3/2}\right]_0^a$$

$$= \frac{4}{3}\left[a^{\frac{3}{2}} - 0^{3/2}\right]$$

$$= \frac{4}{3}\times a^{\frac{3}{2}}$$

$$= \frac{4}{3}\times a^{3\times\frac{1}{2}}$$

$$= \frac{4}{3}\times a^1\times a^{1/2}$$

$= \frac{4}{3}a\sqrt{a}$ sq. unit Ans

(7) Find the area by the curves $y=\sqrt{x}$, $x=2y+3$ in the first quadrant and x-axis.

$\Rightarrow$ The area bounded by the curve $y=\sqrt{x}$, $x=2y+3$ & x-axis. The curve $y=\sqrt{x}$ i.e.; $y^2=x$ & straight line $x=2y+3$ intersects at the point;

$\therefore$ We have;

$\therefore\ y^2 = 2y+3$

$\therefore\ y^2-2y-3=0$

$\therefore\ y^2-3y-y-3=0$

$\therefore\ (y-3)y+1(y-3)=0$

$\therefore\ (y-3)(y+1)=0$

$\therefore\ y=3$ & $y=-1$

$\therefore$ The area is $=\int_0^3 \{(2y+3)-y^2\}\,dy$

$= \int_0^3 2y\,dy + \int_0^3 3\,dy - \int_0^3 y^2\,dy$

$= 2\left[\frac{1}{2}y^2\right]_0^3 + 3\left[y\right]_0^3 - \frac{1}{3}\left[y^3\right]_0^3$

$= \frac{1}{2}\times 2\left[3^2 - 0^2\right] + 3\left[3-0\right] - \frac{1}{3}\left[3^3 - 0^3\right]$

$= (9-0) + 3\times 3 - \frac{1}{3}(27)$

$= 9 + 9 - 9$

$= 9$ sq. unit Ans

(8) Find the area bounded by the curved line $y = \sin x$, x axis and at points $x = 0$ & $x = \pi$

$\Rightarrow$ Given; $y = \sin x$

& $x = \pi \rightarrow$ upper limit

$x = 0 \rightarrow$ Lower limit

$\therefore$ The area

$= \int_0^\pi y\,dx = \int_0^\pi \sin x\,dx$

$= \left[-\cos x\right]_0^\pi$

$= -\left[\cos x\right]_0^\pi$

$= -\left[\cos\pi - \cos 0\right]$

$= -\left[(-1) - 1\right]$

$= 1 + 1$

$= 2$ sq. unit Ans

(9) Find the area enclosed by the curve $y = \sin x$, $y=0$, $x=0$ & $x = \pi/2$

⇒ Given; $y = \sin x$

$x = \frac{\pi}{2} \rightarrow$ upper limit

$x = 0 \rightarrow$ Lower limit

∴ The area $= \int_0^{\pi/2} y \, dx$

$$= \int_0^{\pi/2} \sin x \, dx$$

$$= \left[-\cos x\right]_0^{\pi/2}$$

$$= -\left[\cos x\right]_0^{\pi/2}$$

$$= -\left[\cos \frac{\pi}{2} - \cos 0\right]$$

$$= -\left[0-1\right]$$

$= 1$ sq. unit Ans

(10) Find the area bounded by $y = x+4$, x axis $x=0$ & $x=6$.

⇒ ∵ The area $= \int_0^6 y \, dx$

$$= \int_0^6 x+4 \, dx = \int_0^6 x\,dx + \int_0^6 4\,dx$$

$$= \frac{1}{2}\left[x^2\right]_0^6 + 4\left[x\right]_0^6$$

$$= \frac{1}{2}\left[36-0\right] + 4\left[6-0\right]$$

$$= 18+24$$

$= 42$ sq. unit Ans