Engineering Mechanics

Engineering Mechanics is the branch of science that deals with the study of forces acting on bodies and the resulting effects of these forces.

It is mainly divided into two parts:

- Statics
- Dynamics

Statics

Statics is the branch of engineering mechanics that deals with bodies at rest or moving with constant velocity (i.e., acceleration is zero).

In statics, the net force and net moment acting on the body are zero.
Example: A book lying on a table, a stationary bridge.

Dynamics

Dynamics is the branch of engineering mechanics that deals with bodies in motion with acceleration.
Dynamics is further divided into:
A. **Kinematics** – Study of motion without considering forces.
B. **Kinetics** – Study of motion considering the forces causing it.

Example: Moving car, falling object.

Space

Space refers to the three-dimensional region in which a body exists and moves. It is defined using three mutually perpendicular axes (X, Y, Z).

Time

Time is the measure of the duration between two events. It is a fundamental quantity used to describe motion.
SI unit of time: second (s)

Mass

Mass is the quantity of matter contained in a body. It is a measure of the inertia of the body.

Mass remains constant everywhere.
SI unit: kilogram (kg)

Particle

A particle is an idealized body that has mass but negligible size.

- Shape and size are ignored.
- Used to simplify analysis in mechanics.

Example: Motion of a point on a path.

Flexible Body

A flexible body is a body that undergoes large deformation when forces are applied.
Shape changes significantly under load.
Example: Rope, cable, chain.

Rigid Body

A rigid body is an ideal body in which the distance between any two points remains constant even after applying forces.
Deformation is negligible.
Example: Beam, frame, machine parts.

Scalar Quantity

A scalar quantity is a physical quantity that has magnitude only and no direction.

Examples:

- Mass
- Time
- Temperature
- Speed
- Energy

Vector Quantity

A vector quantity is a physical quantity that has both magnitude and direction.

Examples:

- Force
- Velocity
- Acceleration
- Displacement
- Momentum

Addition of Vectors

Vectors can be added by:

1. Triangle Law of Vectors
2. Parallelogram Law of Vectors

Resultant vector represents the combined effect of given vectors.

Subtraction of Vectors

Vector subtraction is performed by:
Adding the negative of the vector to be subtracted.
$A - B = A + (-B)$

Basic (Fundamental) Units

Basic units are independent units that form the foundation of the measurement system.

Common basic units:

- Length → meter (m)
- Mass → kilogram (kg)
- Time → second (s)
- Electric current → ampere (A)
- Temperature → kelvin (K)

Derived Units

Derived units are obtained by combining basic units.

Examples:

- Velocity → m/s
- Acceleration → m/s^2
- Force → $kg \cdot m/s^2$
- Pressure → N/m^2

SI Units

SI (International System of Units) is the globally accepted system of measurement.

Advantages:

- Uniform and standard worldwide
- Easy conversion
- Based on decimal system

Example of SI unit:

- Force → newton (N)
- Work → joule (J)
- Power → watt (W)

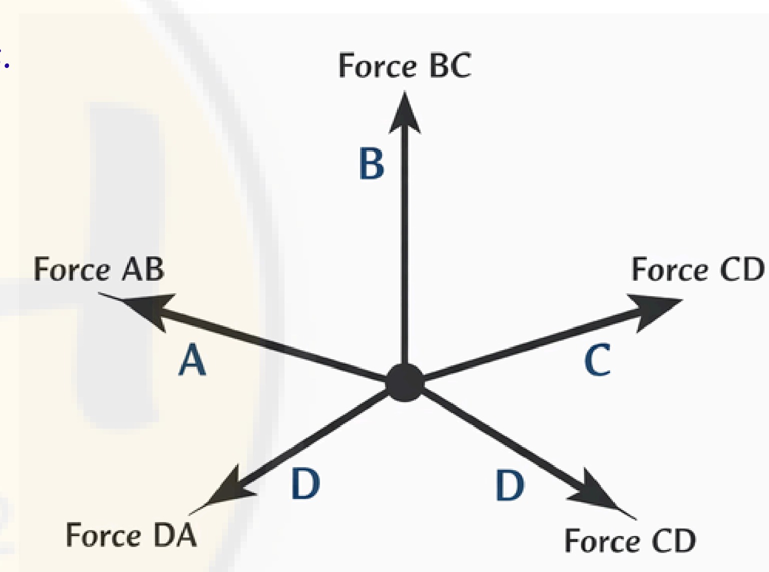
Definition of Force
Force is an external agency which changes or tends to change the state of rest or uniform motion of a body, or changes its shape or size.
Force can cause motion, stop motion, or deform a body

Unit of Force
SI unit of force: Newton (N)
Definition of 1 Newton: 1 Newton is the force required to accelerate a mass of 1 kg at a rate of 1 m/s^2.
1N=1 kg·m/s2

Force as a Vector Quantity
Force is a vector quantity because it has:
- Magnitude
- Direction
- Point of application

It is represented by a directed line segment (arrow):
- Length → Magnitude of force
- Arrow head → Direction of force

Representation of Force by Bow's Notation
Bow's notation is a graphical method used to represent forces, mainly in engineering drawing and force diagrams.
In this method:
- Capital letters (A, B, C, D) are written in the spaces between forces.
- Each force is named by the two letters on either side of it.



Example:
If force lies between space A and B, it is called Force AB.

Characteristics of a Force
A force is completely defined by the following characteristics:
- Magnitude – Size or strength of force
- Direction – Line of action of force
- Point of Application – Point where force acts
- Line of Action – Path along which force acts
- Nature of Force – Push or pull (tension or compression)

Effects of a Force
A force can produce the following effects:
- Change the state of rest or motion
- Change speed of a moving body
- Change direction of motion
- Produce rotation
- Cause deformation (change shape or size)

Principle of Transmissibility of Force
Statement:
The external effect of a force on a rigid body remains unchanged if the force is transmitted along its line of action to any other point on the body.
- Applicable only to rigid bodies
- Does not apply to deformable bodies

This principle simplifies force analysis.

Definition of Force System
A force system is a group of two or more forces acting simultaneously on a body.

Classification of Force Systems
Force systems are classified based on the relative position of their lines of action.
1. Coplanar Force System
All forces lie in the same plane.

Types of Coplanar Force System:
- Collinear Force System
- Concurrent Force System
- Parallel Force System
- General Coplanar Force System

(a) Coplanar Collinear Force System
A coplanar collinear force system is one in which all the forces act along the same straight line and lie in the same plane. Since the forces have the same line of action, their effect can be analyzed easily by algebraic addition.
Example: When a rope is pulled from both ends in opposite directions, the forces acting on the rope are coplanar and collinear.

(b) Coplanar Concurrent Force System
A coplanar concurrent force system is one in which all the forces lie in the same plane and their lines of action meet at a single point. Such force systems are common in jointed structures and frames.
Example: Forces acting at a pin joint of a truss, where several members meet at one point.

(c) Coplanar Parallel Force System
A coplanar parallel force system consists of forces that act in the same plane and are parallel to each other, but do not meet at a point. These forces may act in the same or opposite directions.
Example: Loads acting vertically downward on a horizontal beam.

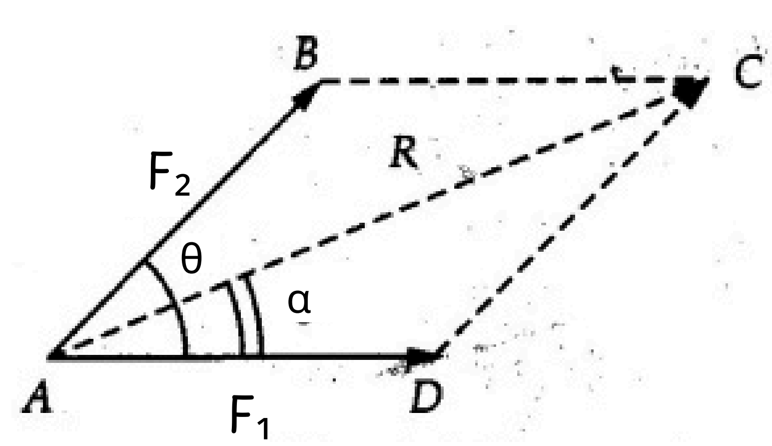
(d) General Coplanar Force System
A general coplanar force system is one in which all the forces act in the same plane, but their lines of action are neither parallel nor concurrent. This type of force system is the most common in practical engineering problems.
Example: Different forces acting on a machine component at various angles in the same plane.

(e) Co-planar Concurrent Force System
A coplanar concurrent force system is one in which all forces lie in the same plane and their lines of action intersect at a common point.
Example: Forces acting at a pin joint, hook, or ring.

Composition of Forces
Composition of forces means replacing two or more forces by a single force (resultant) that produces the same effect as the given forces.

Parallelogram Law of Forces
Statement:
If two forces acting at a point are represented in magnitude and direction by the two adjacent sides of a parallelogram, then the diagonal passing through that point represents the resultant in magnitude and direction.

Example: Two forces acting at a point making an angle with each other.



Formula of Parallelogram Law of Forces
If two forces F_1 and F_2 act at a point and the angle between them is θ, then:

Magnitude of Resultant Force (R)

$$\boxed{R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}}$$

Direction of Resultant (α)

(Angle made by resultant R with force F_1)

$$\boxed{\tan\alpha = \frac{F_2\sin\theta}{F_1 + F_2\cos\theta}}$$

Where:

- α = Angle between resultant R and force F_1
- $\sin\theta$ gives the perpendicular effect of F_2
- $F_1 + F_2\cos\theta$ gives the horizontal (parallel) effect

Special Cases (Very Important for Exams)

- If θ = 90° (Forces at right angles): $R = \sqrt{F_1^2 + F_2^2}$
- If θ = 0° (Same direction): $R = F_1 + F_2$
- If θ = 180° (Opposite direction): $R = |F_1 - F_2|$

Triangle Law of Forces

Statement:

If two forces acting at a point are represented in magnitude and direction by two sides of a triangle taken in order, then the third side of the triangle taken in opposite order represents the resultant.

Example: Pulling a body using two ropes one after another.

Polygon Law of Forces

Statement:

If a number of forces acting at a point are represented in magnitude and direction by the sides of a polygon taken in order, then the closing side of the polygon taken in opposite order represents the resultant.

Example: More than two forces acting at a joint.

Determination of Resultant Force (Analytical Method)

In this method, the resultant is calculated using mathematical equations.

Steps:

- Resolve each force into horizontal (x) and vertical (y) components.
- Add all horizontal components → ΣF_x
- Add all vertical components → ΣF_y

Resultant magnitude:

$$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$$

Direction:

$$\tan\theta = \frac{\Sigma F_y}{\Sigma F_x}$$

Simple problems

- Basics of Mechanics & Force systems

① Two forces of 100 N & 150 N are acting simultaneously at a point. What is the resultant of these two forces, if the angle between them is 45°?

⇒ Given: $F_1 = 100\,N$

$F_2 = 150\,N$

& $\theta = 45°$

∴ Resultant force (R)

$= \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$

$= \sqrt{100^2 + 150^2 + 2 \times 100 \times 150 \times \cos 45°}$

$= \sqrt{10000 + 22500 + (30000 \times 0.707)}$

$= 232\,N$ Ans

② Find the resultant of two forces equal to 50 N & 30 N acting at an angle 60°.

⇒ Given: $F_1 = 50\,N$ & $F_2 = 30\,N$ & $\theta = 60°$

∴ Resultant Force (R)

$= \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$

$= \sqrt{50^2 + 30^2 + 2 \times 50 \times 30 \times \cos 60°}$

$= \sqrt{2500 + 900 + 3000 \times 0.5}$

$= 70\,N$ Ans

& The direction

$$\therefore \tan\alpha = \frac{F_2 \sin\theta}{F_1 + F_2 \cos\theta}$$

$$= \frac{30 \times \sin 60^\circ}{50 + 30 \times \cos 60^\circ}$$

$$= \frac{30 \times 0.866}{50 + 30 \times 0.5}$$

$$= \frac{25.98}{65}$$

$$= 0.399$$

$$\therefore \alpha = \tan^{-1}(0.399)$$

$$= 21.75^\circ \text{ Ans}$$

③ Two forces of 80N & 70 N act at a point. Find the resultant force if the angle between them is 150°.

⇒ Given: $F_1 = 80N$, $F_2 = 70N$ & $\theta = 150^\circ$

$\therefore$ Resultant force (R)

$$= \sqrt{F_1^2 + F_2^2 + 2F_1 F_2 \cos\theta}$$

$$= \sqrt{80^2 + 70^2 + 2 \times 80 \times 70 \times \cos 150^\circ}$$

$$= \sqrt{6400 + 4900 + 11200 \times (-0.866)}$$

$$= \sqrt{6400 + 4900 - 9699.2}$$

$$= 40 \text{ N Ans}$$

∴ The direction

$$\therefore \alpha = \tan^{-1}\left(\frac{F_2 \sin\theta}{F_1 + F_2 \cos\theta}\right)$$

$$= \tan^{-1}\left(\frac{70 \times \sin 150^\circ}{80 + 70 \times \cos 150^\circ}\right)$$

$$= \tan^{-1}\left(\frac{35}{80 - 60.62}\right)$$

$$= \tan^{-1}\left(\frac{35}{19.38}\right)$$

$$= \tan^{-1}(1.805)$$

$$= 61.01^\circ \text{ Ans}$$

(4) Find the resultant of two forces 130N & 110N, acting at an angle whose tangent is 12/5.

⇒ Given; $F_1 = 130\,N$

$F_2 = 110\,N$

$$\therefore \text{Angle } (\theta) = \tan^{-1}(12/5)$$

$$= 67.3^\circ$$

$$\therefore \text{The Resultant force } (R) = \sqrt{F_1^2 + F_2^2 + 2F_1 F_2 \cos\theta}$$

$$= \sqrt{130^2 + 110^2 + 2 \times 130 \times 110 \times \cos 67.3}$$

$$= \sqrt{16900 + 12100 + 11036.9}$$

$$= \sqrt{40036.9}$$

$$= 200.09\ N \text{ Ans}$$

(5) Find the angle between two equal forces P, when their resultant is equal to (i) P & (ii) P/2

⇒ (i) When $R = P$

$\& F_1 = P = F_2$

∴ As per formula

$\therefore R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$

or, $P = \sqrt{P^2 + P^2 + 2P.P.\cos\theta}$

or, $P = \sqrt{2P^2 + 2P^2\cos\theta}$

or, $P^2 = 2P^2 + 2P^2\cos\theta$

or, $2P^2\cos\theta = P^2 - 2P^2 = -P^2$

or, $\cos\theta = -\frac{P^2}{2P^2} = -\frac{1}{2} = -0.5$

$\therefore \theta = \cos^{-1}(-0.5)$

$= 120°$ Ans

(ii) When $R = P/2$

∴ As per formula,

$\therefore \frac{P}{2} = \sqrt{P^2 + P^2 + 2.P.P.\cos\theta}$

or, $\frac{P}{2} = \sqrt{2P^2 + 2P^2\cos\theta}$

or, $\frac{P^2}{4} = 2P^2 + 2P^2\cos\theta$

or, $2P^2\cos\theta = \frac{P^2}{4} - 2P^2 = \frac{P^2 - 8P^2}{4} = -\frac{7P^2}{4}$

or, $\cos\theta = -\frac{7P^2}{4} \times \frac{1}{2P^2} = -\frac{7}{8}$

$\therefore \theta = \cos^{-1}(-7/8) = 151°$ Ans

(6) Find the magnitude of two forces, such that if they act at right angles, their resultant is $\sqrt{10}$ N. But if they act at 60°, their resultant is $\sqrt{13}$ N.

⇒ Let, those two forces are F_1 & F_2

∵ When they act at 90°, then

$$\sqrt{F_1^2+F_2^2}=\sqrt{10}$$

$$\therefore F_1^2+F_2^2=10 \quad \text{——— (i)}$$

But, when they act at 60°, then

$$\therefore \sqrt{F_1^2+F_2^2+2F_1F_2\cos\theta}=\sqrt{13}$$

$$\therefore F_1^2+F_2^2+2F_1F_2\cos 60^\circ=13$$

$$\therefore F_1^2+F_2^2+\cancel{2}F_1F_2\times\frac{1}{\cancel{2}}=13$$

$$\therefore F_1+F_2^2+F_1F_2=13$$

$$\therefore 10+F_1F_2=13 \quad [\because F_1^2+F_2^2=10]$$

$$\therefore F_1F_2=13-10=3$$

Now, we know that

$$(F_1+F_2)^2=F_1^2+F_2^2+2F_1F_2$$

$$\therefore (F_1+F_2)^2=10+2\times3=16.$$

$$\therefore F_1+F_2=\sqrt{16}=4 \quad \text{——— (ii)}$$

∴ Similarly, $(F_1-F_2)^2=F_1^2+F_2^2-2F_1F_2$

$$\therefore (F_1-F_2)^2=10-2\times3$$

$$\therefore (F_1-F_2)^2=10-6$$

$$\therefore (F_1-F_2)^2=4$$

$$\therefore F_1-F_2=2 \quad \text{——— (iii)}$$

$\therefore$ ① + ⑪, we get;

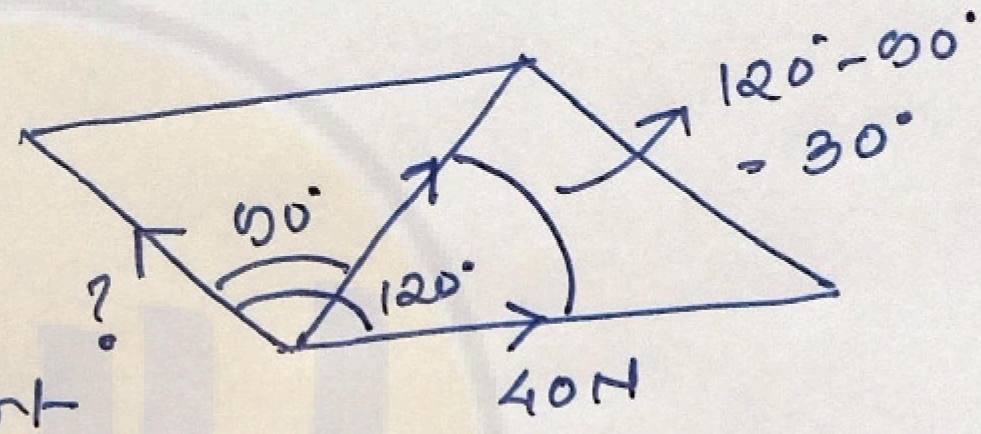
$\therefore F_1 + \cancel{F_2} + F_1 - \cancel{F_2} = 4 + 2$

$\therefore 2F_1 = 6$

$\therefore F_1 = 6/2 = 3\ N$ Ans

$\therefore F_2 = 4 - F_1 = 4 - 3 = 1\ N$ Ans

⑦ Two forces act at an angle of 120°. The bigger force is of 40N & the resultant is perpendicular to the smaller one. Find the smaller force.



$\Rightarrow \therefore$ Bigger Force

$F_1 = 40N$

$\because \theta = 120°$

$\therefore$ Angle between resultant & bigger one, $\alpha = 120° - 90°$

$= 30°$

$\therefore$ As per formula;

$$\because \tan\alpha = \frac{F_2 \sin\theta}{F_1 + F_2 \cos\theta}$$

$$\text{or, } \tan 30° = \frac{F_2 \sin 120°}{40 + F_2 \cos 120°} = \frac{F_2 \sin 60°}{40 + F_2(-\cos 60°)}$$

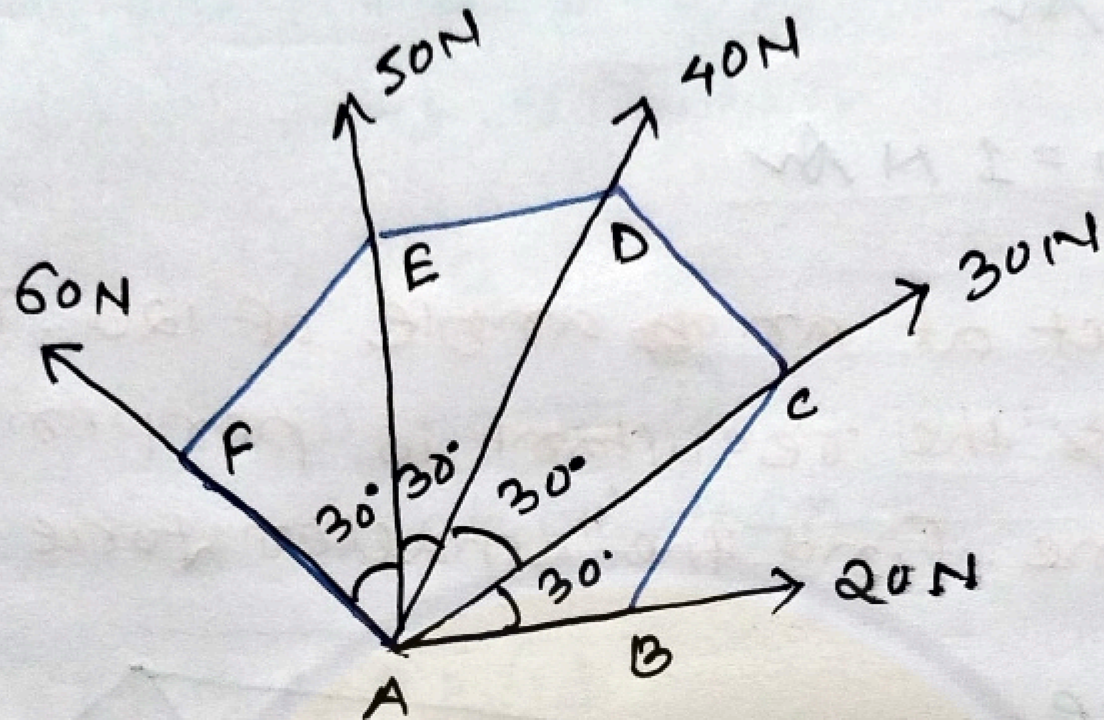
$$\text{or, } 0.577 = \frac{F_2 \times 0.866}{40 - F_2 \times 0.5}$$

$$\text{or, } 40 - F_2 \times 0.5 = \frac{F_2 \times 0.866}{0.577} = 1.5F_2$$

or, $2F_2 = 40$

$\therefore F_2 = 20N$ Ans

(8) The forces 20N, 30N, 40N, 50N & 60N are acting at a point of a regular hexagon, towards the other 5 angular points, taken in order. Find the magnitude & direction of the resultant force.



The magnitude of the resultant force

Resolving horizontally (ΣH)

$= 20\cos 0° + 30\cos 30° + 40\cos 60° + 50\cos 90° + 60\cos 120°$

$= 36.0\,N$

Now, vertically (ΣV)

$= 20\sin 0° + 30\sin 30° + 40\cos 60° + 50\sin 90° + 60\sin 120°$

$= 151.6\,N$

∴ The magnitude

$R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$

$= \sqrt{(36)^2 + (151.6)^2}$

$= 155.8\,N$ Ans

∴ The direction (θ)

$$\therefore \theta = \tan^{-1}\left(\frac{\Sigma V}{\Sigma H}\right)$$

$$= \tan^{-1}\left(\frac{151.6}{36}\right)$$

$$= \tan^{-1}(4.211)$$

$$= 76.64^\circ \text{ Ans}$$

(5) The following forces act at a point:

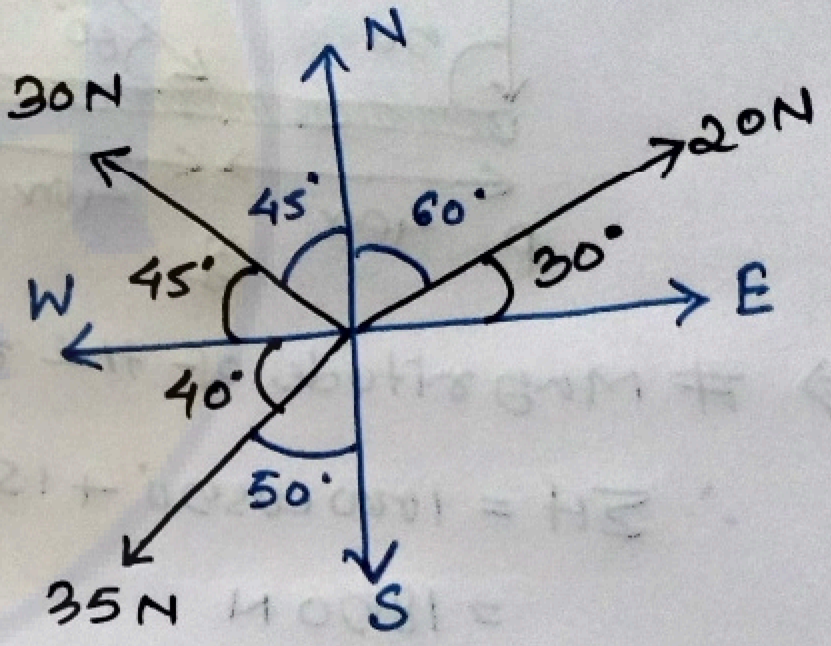
(i) 20 N inclined at 30° towards North of East

(ii) 25 N towards North

(iii) 30 N towards North West, &

(iv) 35 N inclined at 40° towards South of West

Find magnitude & direction of the resultant force.



⇒ # The magnitude of the resultant force;

∴ Resolving horizontally (ΣH)

$$= 20\cos 30^\circ + 25\cos 90^\circ + 30\cos 135^\circ + 35\cos 220^\circ$$

$$= -30.7 \text{ N}$$

∴ Now, vertically (ΣV)

$$= 20\sin 30^\circ + 25\sin 90^\circ + 30\sin 135^\circ + 35\sin 220^\circ$$

$$= 33.7 \text{ N}$$

∴ The magnitude $= \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$

$$= \sqrt{(33.7)^2 + (-30.7)^2}$$

$$= 45.6 \text{ N Ans}$$

∴ The direction (θ)

$$\therefore \theta = \tan^{-1}\left(\frac{\Sigma V}{\Sigma H}\right)$$

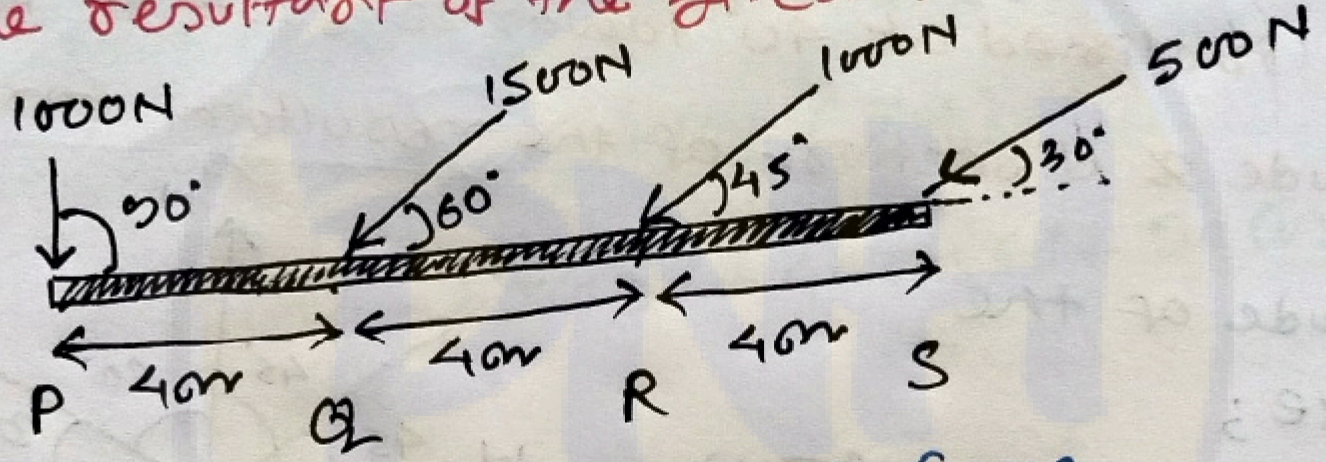
$$= \tan^{-1}\left(\frac{33.7}{-30.7}\right)$$

$$= \tan^{-1}(-1.098)$$

$$= 47.7^\circ$$

Since; ΣH is negative & ΣV is positive, therefore the resultant lies between 90° & 180°. Thus, the actual angle of the resultant $= 180^\circ - 47.7^\circ = 132.3^\circ$ Ans

(10) Find the resultant of the given structure –



⇒ # Magnitude of the resultant force

$$\therefore \Sigma H = 1000\cos 90^\circ + 1500\cos 60^\circ + 1000 \times \cos 45^\circ + 500 \times \cos 30^\circ$$

$$= 1890\text{ N}$$

$$\therefore \Sigma V = 1000\sin 90^\circ + 1500 \sin 90^\circ + 1000\sin 45^\circ + 500\sin 30^\circ$$

$$= 3256\text{ N}$$

∴ The magnitude of the resultant force

$$= \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$$

$$= \sqrt{(1890)^2 + (3256)^2}$$

$$= 3765\text{ N}$$ Ans

∴ The direction

$$\therefore \theta = \tan^{-1}\left(\frac{\Sigma V}{\Sigma H}\right)$$

$$= \tan^{-1}\left(\frac{3256}{1890}\right)$$

$$= \tan^{-1}(1.722)$$

$$= 59.85^\circ \text{ Ans}$$