ORDINARY DIFFENTIAL EQUATION

(Finding order & degree)

Ex. of Ordinary differential eqn

① $\frac{dy}{dx} + 3 = 0$ or, $f'(x) + 3 = 0$ or, $y' + 3 = 0$

Here's

$f'(x) = \frac{dy}{dx} = y' = y_1$ (also)

② $\frac{d^2y}{dx^2} + 6 \cdot \frac{dy}{dx} + 5y = 0$

We can write this eqn if these forms also

ⓐ $f''(x) + 6f'(x) + 5y = 0$

ⓑ $y'' + 6y' + 5y = 0$

ⓒ $y_2 + 6y_1 + 5y = 0$

③ $\underbrace{y'''}_{\text{3rd order derivative}} + 3\underbrace{(y'')^2}_{\text{2nd orde derivate}} + \underbrace{y'}_{\text{1st order derivative}} = \cos x$

How to find the order & degree of the differential eqn_

The max. order of derivative in that diff. eqn is the "order" of the eqn.

The power of the max. order derrivative is the "degree" of the eqn.

① Find the order & degree of the given equations –

ⓐ $\left(\frac{d^3y}{dx^3}\right)^2 = x^2 \frac{dy}{dx}$

∴ Here's the max. order of derivative is 3

∴ Order = 3

And, the power of the max. derivative is 2

So, Degree = 2.

∴ Order = 3 & Degree = 2 Ans

ⓑ $\left(\frac{d^2y}{dx^2}\right)^2 = x^2\left(\frac{dy}{dx}\right)^4$

Here's Order = 2, Degree = 2 } Ans

ⓒ $\frac{d^2y}{dx^2} + 5\cdot\left(\frac{dy}{dx}\right)^2 + 2y = 0$

Here's Order = 2, Degree = 1 } Ans

ⓓ $\frac{d^2y}{dx^2} + 2\cdot\left(\frac{dy}{dx}\right)^2 + y = 2x$

Here's Order = 2, Degree = 1 } Ans

ⓔ $\frac{d^2y}{dx^2} - 3\left(\frac{dy}{dx}\right)^4 + 4y = 0$

Here's Order = 2, Degree = 1 } Ans

(f) $x\left[y\cdot\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2\right]=y\cdot\frac{dy}{dx}$

$\Rightarrow xy\cdot\frac{d^2y}{dx^2}+x\cdot\left(\frac{dy}{dx}\right)^2-y\cdot\frac{dy}{dx}=0$

$\left.\begin{array}{l}\therefore \text{Order} = 2\\ \therefore \text{Degree} = 1\end{array}\right\}$ An

(g) $D^2y-7Dy-44y=0$

$\Rightarrow \frac{d^2y}{dx^2}-7\cdot\frac{dy}{dx}-44\cdot y=0$

$\because D^2=\frac{d^2}{dx^2}$ & $D=\frac{d}{dx}$

$\therefore$ Here; $\left.\begin{array}{l}\text{order} = 2\\ \text{Degree} = 1\end{array}\right\}$ An

(h) $(D^3+2D^2+D)y=0$

$\therefore D^3y+2D^2y+Dy=0$

$\therefore \frac{d^3}{dx^3}y+2\cdot\frac{d^2}{dx^2}\cdot y+\frac{d}{dx}y=0$

$\therefore \frac{d^3y}{dx^3}+2\cdot\frac{d^2y}{dx^2}+\frac{dy}{dx}=0$

$\left.\begin{array}{l}\therefore \text{Order} = 3\\ \therefore \text{Degree} = 1\end{array}\right\}$ An

(i) $x^2y'' - xy' - 3y = 0$

$\Rightarrow x^2 \cdot \frac{d^2y}{dx^2} - x \cdot \frac{dy}{dx} - 3y = 0$

$$\left[\because y'' = \frac{d^2y}{dx^2} \ \& \ y' = \frac{dy}{dx}\right]$$

$\therefore$ Order $= 2$ & degree $= 1$ Ans

(j) $\sqrt{1 + \left(\frac{dy}{dx}\right)^2} = 1 + x$

$\Rightarrow 1 + \left(\frac{dy}{dx}\right)^2 = (1+x)^2$

$\Rightarrow \cancel{1} + \left(\frac{dy}{dx}\right)^2 = 1^2 + x^2 + 2x = x^2 + \cancel{1} + 2x$

$\Rightarrow \left(\frac{dy}{dx}\right)^2 - x^2 - 2x = 0$

$\therefore$ Here; $\left.\begin{array}{l}\text{Order} = 1 \\ \text{Degree} = 2\end{array}\right\}$ Ans

(2) Find the differential equation whose general solutions are $y = A\sin x + B\cos x$

$\Rightarrow$ Let; $y = A\sin x + B\cos x$ —— (i)

Now; differential (i) w.r.t. x

$\frac{dy}{dx} = A\cos x - B\sin x$ —— (ii)

Again; differential (ii) w.r.t. x

$\therefore \frac{d^2y}{dx^2} = A(-\sin x) - B\cos x$

$= -A\sin x - B\cos x$

$= -(A\sin x + B\cos x) = -y$

$\therefore \frac{d^2y}{dx^2} + y$ is the eqn or soln Ans

(3) Find the differential eqn whose general soln are $y = Ae^{x} + Be^{-x}$

$\Rightarrow \quad y = Ae^{x} + Be^{-x}$

$\therefore \frac{dy}{dx} = A \cdot e^{x} - Be^{-x}$

$\therefore \frac{d^2y}{dx^2} = A \cdot e^{x} + Be^{-x}$

$\therefore \frac{d^2y}{dx^2} = y$

$\therefore \frac{d^2y}{dx^2} - y = 0$ Ans