January 2022 204N

MATHEMATICS-II

Time Allowed: 1.5 Hours Full Marks: 60

Answer to question number 1 is compulsory and Answer <u>any two</u> questions from the rest.

1. Answer any twenty questions from the following with proper justification of each: 20x2=40

i) Solve: $\frac{ydy+xdx}{x^2+y^2} = 0$.

ii) In the matrix $\begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & 0 \\ -1 & 2 & 1 \end{bmatrix}$ minor of "--1" is _____.

iii) If $u(x,y) = \sqrt{\frac{x^2}{x+y}}$ .Then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = ?$

iv) The area of the region bounded by $y = 2x - x^2$ and $y = x$ is_____.

v) Is the matrix $\begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 3 \\ -2 & -3 & 1 \end{bmatrix}$ non-singular? (Yes or No)

vi) Is the differential equation homogeneous: $\frac{dy}{dx} = \frac{3xy^2+2x^2y}{5y^3+x^2}$? Yes/No

vii) Check the co-linearity of the points: (2, 5), (0, 0) and (-1, -2.5).

viii) If two coins are tossed, the probability of getting at most one head is _____.

ix) If $P(A) = 0.4, P(B) = 0.35$, Find $P(\overline{A}) + P(\overline{B}) - 1 = ?$.

x) $\int_{-2}^{+2} x^3\,dx = ?$

xi) Find particular Integral (P.I.) of the differential equation $\frac{d^2y}{dx^2} - y = 1$.

xii) Find the probability of obtaining at least one '6' in a game of throwing of two dice.

xiii) Is the matrix $\begin{pmatrix} \cos\theta & \sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$ orthogonal? (Yes/No)

xiv) The mode of the data: 10, 12, 8, 10,12,11,10, 12,7,11,12 is_____.

xv) Find the angle between the lines $x + 2y = 5$ and $y = 2x$.

xvi) Write the degree of the differential equation $\left(1-9\frac{d^2y}{dx^2}\right)^{5/7}+7xy\frac{dy}{dx}-y^2=0.$

xvii) Compute the median of the data: 10,5,9, 6,4,8,7,6, 5.

xviii) The value of the determinant $\begin{vmatrix} 0 & \omega & -\omega^2 \\ -\omega & 0 & \omega^3 \\ \omega^2 & -\omega^3 & 0 \end{vmatrix}$ is _____.

xix) $2\int_0^{\pi/2}\frac{\sqrt{\cos x}}{\sqrt{\sin x}+\sqrt{\cos x}}=?$

xx) Test the differential equation $(ye^x-5x)\,dx-(e^x+5y)\,dy=0$ is exact or not.

xxi) The cofactor of 6 in the determinant $\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix}$ is _____.

xxii) Find the adjoin of the matrix $\begin{pmatrix} 5 & 0 \\ 1 & -2 \end{pmatrix}$.

xxiii) Evaluate: $\int_1^e \log_e x\ dx$.

xxiv) Find the integrating factor (I.F.) of $\frac{dy}{dx}+2xy=9x.$

xxv) Find the length of the semi- latus rectum of the hyperbola $\frac{x^2}{25}-\frac{y^2}{16}=1.$

2. a) Evaluate by Chio's method $\begin{vmatrix} 1 & 2 & -2 & 0 \\ 0 & 1 & 0 & 0 \\ -2 & 0 & 1 & 1 \\ 1 & 0 & -1 & 0 \end{vmatrix}$. 5

b) Evaluate : $\int\frac{x\,dx}{(x-1)\ (2x+1)}$. 5

3. a) Solve: $2xy+3x^2=\frac{dy}{dx}(2y-x^2+1)$ 5

b) If $p_1 and p_2$ be the length of perpendicular from the origin upon the lines $-4y-5\sin\alpha=0\ and\ 3y+4x=5\cos\alpha$, prove that $p_1^2+p_2^2=1.$ 5

4. a) Solve the equations by crammer's rule $x+y+z=3;\ 2x+z=5;\ x+2y=-1.$ 5

b) If $z=\tan^{-1}\left(\frac{x-y}{\sqrt{x}+\sqrt{y}}\right)$, show that $x\frac{\partial z}{\partial x}+y\frac{\partial z}{\partial y}=\frac{1}{4}Sin\ 2z$ 5

5. a) Evaluate : $\int e^x Cos\ x\ dx.$ 5

b) Solve: $\frac{d^2y}{dx^2}+2\frac{dy}{dx}+y=x^3.$ 5

6. a) Solve: $y\,dy = (y^2 - 1)(x + 1)\,dx$. 5

b) A straight line forms a right angle triangle with the coordinate axes. If the area of triangle is 6 and the hypotenuse is 5, find the equation of the straight line. 5

7. a) Find A^{-1}; if $A = \begin{pmatrix} 0 & -1 & 0 \\ 1 & 1 & 4 \\ 0 & -1 & 1 \end{pmatrix}$ and show that $AA^{-1} = I_3$. 3+2

b) Solve: $\frac{dy}{dx} + xy = xy^3$. 5

8. a) Find the volume of the solid generated by revolving; the area bounded by the curve $y = \sin x,\ x = 0, x = \frac{\pi}{2}$ about the x –axis. 5

b) What is the probability that the sum of outcomes is 13, when three dice are rolled? 5

9. a) A circle has radius 4 unit and its center lies on x+y=0 and it passes through (1, 1). Find its equation. 5

b) Find the median of the distribution given below: 5

x :	01– 10	11 – 20	21 – 30	31 – 40	41 – 50	51 – 60
f_i:	3	10	9	13	7	15