JULY 2023
MATHEMATICS-II

Time Allowed: 2.5 Hours   *Full Marks: 60*

Answer to Question No. 1 is compulsory and Answer any Five (05) Questions from the rest.

1. Answer <u>any five</u> questions.   **3×5=15**

     a) Find the probability that getting at least two heads in a toss of unbiased coin 3 times.

     b) Evaluate: $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} x^2 \sin x^3 dx$.

     c) Check the exactness: $(xy - 2x^2) + xy\frac{dy}{dx} = 0$.

     d) Check whether the points $(-3, 4)$, $(0, -1)$ and $(1, 2)$ are the co-linear.

     e) Find the inverse of the matrix $\begin{bmatrix} 2 & 3 \\ 1 & 5 \end{bmatrix}$.

     f) State Euler's theorem on homogeneous function with two variables.

     g) Find the median of the data: 1, 2, 3, 4, 1, 2, 3, 4, 1, 2, 3, 4, 1, 2, 1.

2. (a) Solve by Cramer's rule:

$$x + 2y + 3z = 5$$
$$3x - 2y + z = -1$$
$$4x + 2y + z = 13$$

     5

     (b) Find the area bounded by the parabola $y^2 = 4x$ above x-axis and ordinate at x=2.   **4**

3. (a) Find the equation of the straight line making $60°$ angle with positive direction of x-axis and passing through the point of the points (7, 8).   **5**

     (b) If $U = sin^{-1}\frac{x-y}{\sqrt{x}+\sqrt{y}}$ show that $x\frac{\partial U}{\partial x} + y\frac{\partial U}{\partial x} = \frac{1}{2}\tan U$.   **4**

1

4. (a) Solve: $\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 6y = x^2 e^x$. 　　　　5

(b) Evaluate: $\int \frac{\cos x}{(2\sin x - 3)(\sin x + 2)} dx$. 　　　　4

5. (a) If $A = \frac{1}{3}\begin{pmatrix} -1 & 2 & -2 \\ -2 & 1 & 2 \\ 2 & 2 & 1 \end{pmatrix}$ show that $AA^T = A^T A = I_3$. Hence find

A^{-1}. 　　　　5

(b) An urn contains 2 white and 4 red balls. Two balls are drawn randomly one after another without replacement. What is the probability that the drawn balls are red? 　　　　4

6. (a) Solve: $(x + 2y)\frac{dy}{dx} = y$. 　　　　4

(b) Find the median of the distribution given below:

x:	15 – 19	20 – 24	25 – 29	30 – 34	35 – 39	40 – 44	45 – 49
f:	4	8	18	20	25	16	9

　　　　5

7. (a) The line segment $x + y = 1$, $0 \le y \le 1$ is revolved about y axis to generate a cone. Find the volume of the cone. 　　　　5

(b) Evaluate: $\int_0^{\frac{\pi}{2}} \frac{\sqrt{\tan x}}{\sqrt{\tan x} + \sqrt{\cot x}}$. 　　　　4

8. (a) A can hit a target 5 times out of 4, B 4 times out of 3 and C 2 times out of 3. Each is allowed to shoot once. What is the probability that the target is hit? 　　　　5

(b) Solve: $\cos y \, dx + (1 + 2e^{-x}) \sin y \, dy = 0$. 　　　　4

9. (a) Without expanding prove that $(a+b+c)$ is a factor of the determinant

$$\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$$

　　　　5

(b) Find the equation of the ellipse whose eccentricity is 4/5 and axes are along the coordinate axes and with foci at $(0, \pm 4)$. 　　　　4