

ENGINEERING MECHANICS

Unit IV: Friction (4L + 3T)

WBSCTE Diploma — 2nd Semester | Study Notes

1. Introduction to Friction

Friction is one of the most fundamental and practically important concepts in Engineering Mechanics. It is a force that resists the relative motion or tendency of motion between two surfaces in contact. Without friction, we could not walk, drive a vehicle, hold a pen, or apply brakes. Friction plays a dual role in engineering — it is both useful (in brakes, clutches, belts, and fasteners) and harmful (causing heat, wear, and energy loss in machines).

Engineering Relevance of Friction

Where Friction is USEFUL

- Brakes and clutches in automobiles — friction stops motion
- Belt drives and pulley systems — power transmission relies on friction
- Fasteners (bolts, screws, nuts) — friction prevents loosening
- Walking and running — foot-ground friction provides grip
- Writing with pen or chalk — friction transfers ink/chalk to surface
- Ignition of matchstick — frictional heat causes ignition

Where Friction is HARMFUL

- Causes wear and tear in machine parts (gears, bearings, shafts)
- Produces unwanted heat leading to energy loss
- Reduces efficiency of engines and mechanical systems
- Requires lubrication and maintenance, increasing cost

2. Types of Friction

Friction is classified based on the nature of motion and the type of surfaces in contact. The main classification is as follows:

Static Friction	The friction force acting on a body at rest when an external force tends to move it. It prevents the start of motion. Maximum static friction is called Limiting Friction.
Kinetic (Dynamic) Friction	The friction force acting on a body that is already in motion relative to another surface. It is less than static friction. Also called sliding friction.
Rolling Friction	Friction experienced by a body rolling over a surface (e.g., wheel on road). It is much less than sliding friction.
Fluid Friction	Friction between layers of fluid (viscosity) or between a body and a fluid medium. Not applicable for dry surfaces.
Dry (Coulomb) Friction	Friction between two dry solid surfaces in direct contact without any lubricant. Most commonly studied in Engineering Mechanics.

Note: In diploma-level Engineering Mechanics, we mainly study Dry Friction (also called Coulomb Friction), which deals with the frictional behavior of two solid surfaces in contact.

3. Laws of Friction

The behavior of friction between dry solid surfaces is governed by the following empirical laws, known as Coulomb's Laws of Friction (also called Laws of Dry Friction or Laws of Solid Friction):

Coulomb's Laws of Friction (Laws of Dry / Solid Friction)

1. The frictional force always acts in a direction opposite to the direction of relative motion (or tendency of motion) between the two surfaces.
2. The magnitude of the frictional force is directly proportional to the normal reaction force (N) between the surfaces. i.e., $F \propto N$, or $F = \mu N$, where μ is the coefficient of friction.
3. The frictional force is independent of the area of contact between the surfaces, as long as the normal reaction remains constant.
4. The frictional force is independent of the velocity of sliding (kinetic friction), though static friction may vary up to its limiting value.
5. The coefficient of static friction is slightly greater than the coefficient of kinetic friction for the same pair of surfaces.

These laws hold true for most engineering applications involving dry surfaces. Lubricated surfaces (fluid friction) do not follow these laws.

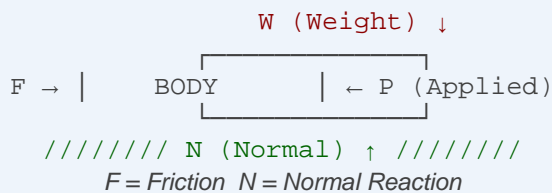
4. Limiting Friction

When an external force P is applied to a body at rest on a rough surface, the friction force F builds up to resist the motion. Initially, both the body and friction force increase together. The friction force increases until it reaches its maximum possible value — this maximum value of friction is called the Limiting Friction.

Definition & Key Points — Limiting Friction

- Limiting Friction (F_{lim}) is the maximum value of static friction that acts just before the body begins to slide.
- Once the applied force P exceeds limiting friction, the body starts moving (impending motion).
- After motion begins, friction reduces slightly to kinetic friction ($F_k < F_{lim}$).
- Limiting friction depends on the nature of the surfaces and the normal reaction force.
- Mathematically: $F_{lim} = \mu_s \times N$, where μ_s = coefficient of static friction.

FRICION FORCE DIAGRAM



KEY RELATIONSHIPS

$$\mu = F / N$$

$$\tan \phi = \mu$$

$$\phi = \tan^{-1}(\mu)$$

$$F_{lim} = \mu \times N$$

$$\text{Angle of Repose} = \phi$$

5. Coefficient of Friction (μ)

The coefficient of friction is a dimensionless constant that quantifies the ratio of the frictional force to the normal reaction force between two surfaces. It depends on the material and surface condition of the two contacting bodies, not on their size or the force applied.

$$\mu = F / N$$

$$\mu = \text{Coefficient of friction} \mid F = \text{Friction force (N)} \mid N = \text{Normal reaction (N)}$$

Types of Coefficient of Friction

- Coefficient of Static Friction (μ_s): **Ratio of limiting friction to normal reaction. Always higher.**
- Coefficient of Kinetic Friction (μ_k): **Ratio of kinetic friction to normal reaction. Slightly less than μ_s .**

Typical Values of μ for Common Material Pairs

Steel on Steel (dry)

$$\mu \approx 0.15 - 0.25$$

Steel on Steel (oiled)	$\mu \approx 0.05 - 0.10$
Wood on Wood	$\mu \approx 0.25 - 0.50$
Rubber on Concrete	$\mu \approx 0.60 - 0.80$
Cast Iron on Cast Iron	$\mu \approx 0.15 - 0.20$
Leather on Metal	$\mu \approx 0.30 - 0.60$

6. Angle of Friction (ϕ)

When a body is on the verge of sliding (i.e., limiting friction is acting), the normal reaction N and the limiting friction force F_{lim} combine to give a resultant reaction R . The angle made by this resultant R with the direction of the normal reaction N is called the Angle of Friction, denoted by ϕ (phi).

$$\tan \phi = F/N = \mu N/N = \mu \quad \therefore \phi = \tan^{-1}(\mu)$$

$\phi = \text{Angle of friction} \mid \mu = \text{Coefficient of friction}$

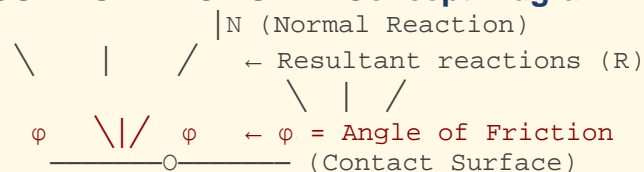
Key Facts about Angle of Friction

- The angle of friction ϕ is the angle between the resultant R and the normal reaction N at the point of contact.
- It represents the maximum angle of inclination of the resultant reaction from the normal.
- Relationship: **$\tan \phi = \mu$ or $\mu = \tan \phi$**
- The angle of friction is always equal to the angle of repose (proved later).
- If applied force is within the cone of friction, body remains at rest.

7. Cone of Friction

When a body is on the verge of motion, the resultant reaction R makes an angle ϕ (angle of friction) with the normal reaction N . If the direction of the applied force P is rotated through 360° around the normal, the resultant reaction R traces out the surface of a cone. This cone is known as the Cone of Friction.

CONE OF FRICTION — Concept Diagram



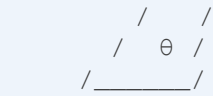
The surface traced by R as applied force P rotates 360° forms the CONE OF FRICTION

Properties of Cone of Friction

- The cone has its apex at the point of contact between the two surfaces.
- The axis of the cone is along the direction of the normal reaction N .
- The semi-vertical angle of the cone equals the angle of friction ϕ .
- If the resultant of all active forces on the body lies within the cone, the body remains in equilibrium (no sliding).
- If the resultant of active forces lies outside the cone, sliding occurs.
- **Semi-vertical angle of Cone of Friction = $\phi = \tan^{-1}(\mu)$**

8. Angle of Repose

Consider a body resting on a rough inclined plane. When the angle of inclination of the plane is gradually increased, a point is reached at which the body is just on the verge of sliding down. The maximum angle of inclination at which the body remains in equilibrium (just about to slide) is called the Angle of Repose.

BODY ON INCLINED PLANE	FORCE COMPONENTS
$W \downarrow$ $N \uparrow$ (perp to plane) $[\blacksquare] \leftarrow F$ (friction)  $\theta = \text{Angle of inclination}$	$N = W \cos \theta$ $F = W \sin \theta$ (limiting) $F = \mu N = \mu W \cos \theta$ $\tan \theta = \mu$ (at repose)

Derivation: Angle of Repose = Angle of Friction

Let θ be the angle of inclination at which a body of weight W just starts to slide down the plane.

- Forces acting: Weight W (downward), Normal reaction N (perpendicular to plane), Friction force F (up the plane, opposing tendency to slide)
- Resolving forces perpendicular to plane: $N = W \cos \theta$
- Resolving forces along the plane: $F = W \sin \theta$
- Since the body is about to slide: $F = \mu N$ (limiting friction)
- $\therefore W \sin \theta = \mu \times W \cos \theta$
- $\therefore \tan \theta = \mu$
- But we also know: $\tan \phi = \mu$ (where $\phi = \text{angle of friction}$)
- $\therefore \theta = \phi \rightarrow \text{Angle of Repose} = \text{Angle of Friction}$

$$\tan(\text{Angle of Repose}) = \mu = \tan \phi \quad \therefore \text{Angle of Repose} = \text{Angle of Friction}$$

This is a very important result — the angle of repose numerically equals the angle of friction

9. Relation Between Coefficient of Friction and Angle of Friction

The relationship between the coefficient of friction (μ) and the angle of friction (ϕ) is one of the most fundamental and frequently examined formulas in friction. This relation is derived from the geometry of forces when a body is on the verge of sliding.

Derivation of $\mu = \tan \phi$

When a body is in limiting equilibrium:

- Normal reaction N acts perpendicular to the surface
- Limiting friction F_{lim} acts along the surface, opposing motion
- Resultant R of N and F_{lim} makes angle ϕ with N

From the triangle of forces: $\tan \phi = F_{\text{lim}} / N = \mu N / N = \mu$

Therefore: $\mu = \tan \phi$ and $\phi = \tan^{-1}(\mu)$

10. Equilibrium of Bodies on Horizontal Surface

When a body rests on a horizontal surface, it is subjected to its own weight W acting vertically downward, the normal reaction N acting vertically upward, and any externally applied force. For equilibrium, the sum of all forces must be zero. If the applied force causes impending motion, friction equals its limiting value.

FORCE PARALLEL TO PLANE

$$P = F = \mu N, \quad N = W$$

FORCE INCLINED TO PLANE

$$N = W - P \sin \alpha$$

$$P \cos \alpha = F = \mu N$$

Case 1: Applied Force Parallel to the Horizontal Plane

When the force P is applied horizontally (parallel to the surface):

- Vertical equilibrium: $N = W$
- Horizontal equilibrium (impending motion): $P = F = \mu N = \mu W$

$$P = \mu W \quad (\text{for impending motion, force parallel to plane})$$

$P = \text{Applied force} \mid \mu = \text{Coefficient of friction} \mid W = \text{Weight of body}$

Case 2: Applied Force Inclined at Angle α to the Horizontal Plane

When force P is applied at angle α above the horizontal:

- Vertical equilibrium: $N + P \sin\alpha = W \rightarrow N = W - P \sin\alpha$
- Horizontal equilibrium: $P \cos\alpha = F = \mu N = \mu(W - P \sin\alpha)$

$$P \cos\alpha = \mu(W - P \sin\alpha) \rightarrow P = \mu W / (\cos\alpha + \mu \sin\alpha)$$

$\alpha = \text{angle of applied force with horizontal} \mid \text{Least force: } \alpha = \phi \text{ (angle of friction)}$

Least Force to Move a Body on Horizontal Surface

The minimum (least) force P required to move a body on a rough horizontal surface acts at an angle equal to the angle of friction ϕ with the horizontal:

$$P_{\min} = W \sin\phi \quad \text{where } \phi = \tan^{-1}(\mu)$$

Minimum force direction = angle of friction from horizontal

11. Equilibrium of Bodies on Inclined Plane

An inclined plane makes an angle θ (called the angle of inclination) with the horizontal. When a body rests on an inclined plane, the component of its weight along the plane tends to make it slide down. The friction force acts up the plane (when tendency is to slide down) or down the plane (when the body is being pushed up).

Case 1: Force P Applied Parallel to the Inclined Plane (Body Moving Up)

- Normal reaction: $N = W \cos\theta$
- Along the plane (equilibrium): $P = W \sin\theta + F = W \sin\theta + \mu W \cos\theta$

$$P \text{ (up)} = W(\sin\theta + \mu \cos\theta) = W(\sin\theta + \tan\phi \cos\theta)$$

Force parallel to inclined plane to push body UP

Case 2: Force P Applied Parallel to the Inclined Plane (Body Moving Down)

- Friction now acts UP the plane to prevent sliding:

$$P \text{ (down)} = W(\sin\theta - \mu \cos\theta) = W(\sin\theta - \tan\phi \cos\theta)$$

Force parallel to inclined plane to allow body to slide DOWN (P must not be zero if $\theta < \phi$)

Case 3: Force P Inclined at Angle α to the Inclined Plane (Moving Up)

When force P is applied at angle α above the inclined plane surface (body moving up):

- Normal: $N = W \cos\theta - P \sin\alpha$
- Along plane: $P \cos\alpha = W \sin\theta + \mu N = W \sin\theta + \mu(W \cos\theta - P \sin\alpha)$

$$P = W(\sin\theta + \mu \cos\theta) / (\cos\alpha + \mu \sin\alpha)$$

Force inclined at α to inclined plane to push body UP | Least force at $\alpha = \phi$

Case 4: Force P Inclined at Angle α to the Inclined Plane (Moving Down)

$$P = W(\sin\theta - \mu \cos\theta) / (\cos\alpha - \mu \sin\alpha)$$

Force inclined at α to inclined plane, body moving DOWN

Summary: Key Conditions for Inclined Plane

- If $\theta < \phi$: **Body is self-locking** — it will NOT slide down on its own. External force needed to move it down.
- If $\theta = \phi$: **Body is on the verge of sliding down.** This is the angle of repose.
- If $\theta > \phi$: **Body slides down on its own.** External force needed only to move it up.

12. Solved Example Problems

Example 1: Body on Horizontal Surface — Horizontal Force**Problem Statement**

A body of weight 500 N rests on a rough horizontal surface. The coefficient of friction is 0.35. Find the force required to move the body if applied (a) horizontally and (b) at 30° above horizontal.

Solution

Given: $W = 500 \text{ N}$, $\mu = 0.35$

(a) Force Applied Horizontally:

- $N = W = 500 \text{ N}$
- $F = \mu N = 0.35 \times 500 = 175 \text{ N}$
- $P = F = 175 \text{ N}$ ← Answer (a)

(b) Force Applied at $\alpha = 30^\circ$ above horizontal:

- $N = W - P \sin 30^\circ = 500 - 0.5P$
- $P \cos 30^\circ = \mu N = 0.35(500 - 0.5P)$
- $0.866P = 175 - 0.175P$
- $0.866P + 0.175P = 175$
- $1.041P = 175 \rightarrow P = 175 / 1.041 = 168.1 \text{ N} \leftarrow \text{Answer (b)}$

Example 2: Body on Inclined Plane — Force Parallel to Plane

Problem Statement

A block weighing 400 N rests on an inclined plane of inclination 30° . The coefficient of friction between the block and plane is 0.3. Find the minimum force required parallel to the plane to (a) pull the block UP the plane and (b) just prevent it from sliding DOWN.

Solution

Given: $W = 400 \text{ N}$, $\theta = 30^\circ$, $\mu = 0.3$

- $N = W \cos \theta = 400 \times \cos 30^\circ = 400 \times 0.866 = 346.4 \text{ N}$
- $F = \mu N = 0.3 \times 346.4 = 103.9 \text{ N}$
- $W \sin \theta = 400 \times \sin 30^\circ = 400 \times 0.5 = 200 \text{ N}$

(a) Force to pull UP:

- $P_{\text{up}} = W \sin \theta + \mu N = 200 + 103.9 = 303.9 \text{ N} \leftarrow \text{Answer (a)}$

(b) Force to just prevent sliding DOWN:

- $P_{\text{down}} = W \sin \theta - \mu N = 200 - 103.9 = 96.1 \text{ N} \leftarrow \text{Answer (b)}$
- (This force acts down the plane to hold the block, since $W \sin \theta > \mu N$, block would slide)

IMPORTANT EXAM QUESTIONS

Unit IV — Friction | Engineering Mechanics | WBSCTE Diploma 2nd Semester
(Based on WBSCTE Board Previous Year Final Exam Papers)

The following questions have been compiled based on WBSCTE Diploma 2nd Semester previous year final examination papers and the standard question bank for Engineering Mechanics (Unit IV – Friction). Students are strongly advised to practice all these questions thoroughly, as they represent frequently tested patterns.

Section A — Short Answer & Definitions (2–3 Marks)

These questions test your understanding of basic concepts, definitions, and short derivations. Most exam papers contain 4–6 questions of this type from Unit IV.

S.No.	Question	Type / Marks
1	Define friction. State its types with examples.	Short / 2M
2	State and explain the laws of friction (Coulomb's laws).	Short / 3M
3	Define limiting friction, coefficient of friction, and angle of friction.	Short / 3M
4	Explain the concept of Cone of Friction with a neat diagram.	Descriptive / 5M
5	Define angle of repose. Prove that angle of repose equals angle of friction.	Descriptive / 5M
6	Derive the relation: $\mu = \tan \phi$, where ϕ is the angle of friction.	Derivation / 5M

Section B — Descriptive & Derivation Questions (5 Marks)

These questions require complete explanations, neat diagrams, and step-by-step derivations. They appear in the long-answer section of WBSCTE exams.

S.No.	Question	Type / Marks
1	A body weighing 300 N rests on a rough horizontal surface with $\mu = 0.3$. Find the force required to move the body when applied horizontally.	Numerical / 5M
2	A body of weight 500 N lies on a rough horizontal surface. If the force applied at 30° to the horizontal is 200 N and the body just moves, find the coefficient of friction.	Numerical / 5M
3	Find the minimum force required to move a block of weight 400 N on a rough horizontal surface where $\mu = 0.4$. The force is applied at 25° to the horizontal.	Numerical / 5M

4	A block weighing 600 N rests on a rough inclined plane of inclination 30° . If $\mu = 0.3$, find the force required to (a) pull it up the plane and (b) just prevent it from sliding down. Force is parallel to the plane.	Numerical / 8M
5	A body of weight W is on a rough inclined plane inclined at angle α . A force P is applied at angle θ to the inclined plane. Find the expression for P to move the body up the plane.	Derivation / 5M
6	Distinguish between static friction and kinetic friction. Which is greater and why?	Short / 3M
7	A block of weight 200 N rests on an inclined plane at 20° to the horizontal. If $\mu = 0.25$, determine whether the block will remain in equilibrium or slide down.	Numerical / 5M

Section C — Numerical Problems (5–8 Marks)

Numerical problems carry the highest marks in Unit IV. Students must show complete free-body diagrams, equilibrium equations, and stepwise calculations. The following are the most frequently asked types:

S.No.	Question	Type / Marks
1	What is engineering significance of friction? Give examples where friction is (a) useful and (b) harmful.	Short / 3M
2	A force of 150 N at 20° above horizontal moves a 400 N block on a horizontal surface. Find μ .	Numerical / 5M
3	Prove that $\tan(\text{angle of repose}) = \text{coefficient of friction}$. Also state when the body on an inclined plane will slide down on its own.	Descriptive / 5M
4	A body resting on a rough inclined plane (angle 35° , $\mu = 0.4$) is subjected to a force of 100 N inclined at 15° to the plane. Check if the body is in equilibrium.	Numerical / 8M
5	Write short notes on: (a) Cone of Friction, (b) Angle of Repose, (c) Limiting Friction.	Short / 6M

Exam Tips for Unit IV — Friction

- Always draw a neat Free Body Diagram (FBD) before solving any friction problem. It fetches marks even if the final answer is wrong.
- Clearly state the direction of friction force (it opposes the direction of motion or tendency of motion).
- Remember: $\mu = \tan \phi$ and Angle of Repose = Angle of Friction — these are extremely common exam questions.
- For inclined plane problems: always resolve forces along and perpendicular to the plane, not vertically and horizontally.
- Check if $\theta < \phi$, $\theta = \phi$, or $\theta > \phi$ before concluding whether the body is self-locking or will slide.

- For the 'least force' concept: the minimum force to move a body acts at an angle equal to the angle of friction (ϕ) from the surface.
- Units: Force in Newtons (N), angle in degrees, μ is dimensionless.

— End of Unit IV Notes —

Prepared for WBSCTE Diploma 2nd Semester | Engineering Mechanics | Unit IV: Friction

