CIRCUIT THEORY

1.3 Superposition Theorem | 1.4 Thevenin's Theorem | 1.5 Norton's Theorem | 1.6 Maximum Power Transfer Theorem | 1.7 Related Numerical Problems

Diploma / Polytechnic Engineering — Study Notes

1.3 Superposition Theorem

Definition

Statement: In any linear, bilateral network containing more than one independent source, the current through (or voltage across) any element is equal to the algebraic sum of the currents (or voltages) produced by each independent source acting alone, with all other independent sources replaced by their internal resistances (voltage sources short-circuited and current sources open-circuited).

Procedure / Steps

1. Select one independent source and replace all other voltage sources by a short circuit and all other current sources by an open circuit.
2. Redraw the simplified circuit and find the current/voltage in the desired branch using Ohm's law and series-parallel reduction.
3. Repeat step 1 and 2 for every independent source present in the network, one at a time.
4. Algebraically add all the individual current/voltage values obtained to get the actual (resultant) current or voltage.

Areas of Application

- Analysis of circuits having more than one source (AC and DC combined circuits).
- Used in amplifier circuits to study the effect of AC and DC sources separately.
- Useful in communication and signal processing circuits.

Limitations

- Applicable only to linear, bilateral networks; cannot be applied to non-linear elements like diodes and transistors.
- Not applicable for power calculation directly, since power is a non-linear (squared) function of current/voltage.
- Time consuming for networks having a large number of sources.

1.4 Thevenin's Theorem

Definition

Statement: Any linear, bilateral two-terminal network containing sources and resistances can be replaced by an equivalent circuit consisting of a single voltage source (Vth) in series with a single resistance (Rth), where Vth is the open-circuit voltage across the terminals and Rth is the resistance measured across the terminals with all independent sources deactivated.

Procedure / Steps

1. Remove the load resistance (RL) from the circuit across which Thevenin's equivalent is required.
2. Find the open-circuit voltage (Vth) across the open terminals using any circuit analysis method.

3. Deactivate all independent sources (short voltage sources, open current sources) and find the equivalent resistance (Rth) as seen from the open terminals.
4. Draw the Thevenin's equivalent circuit with Vth in series with Rth, and reconnect the load resistance RL.
5. Calculate the required current or voltage across RL using this simplified equivalent circuit.

Areas of Application

- Used to simplify complex networks for repeated calculations with different load resistances.
- Widely used in power system analysis and in designing matching networks.
- Useful in analysing amplifier and transducer circuits.

Limitations

- Applicable only to linear, bilateral networks.
- Not suitable for circuits containing non-linear elements.
- Additional calculation is required each time internal source values change.

1.5 Norton's Theorem

Definition

Statement: Any linear, bilateral two-terminal network containing sources and resistances can be replaced by an equivalent circuit consisting of a single current source (IN) in parallel with a single resistance (RN), where IN is the short-circuit current through the terminals and RN is the resistance measured across the terminals with all independent sources deactivated (RN is equal to Rth).

Procedure / Steps

1. Remove the load resistance from the circuit across the terminals of interest.
2. Short the two open terminals and calculate the short-circuit current (IN) flowing through them.
3. Deactivate all independent sources and calculate the equivalent resistance (RN) as seen from the open terminals (same as Rth).
4. Draw the Norton's equivalent circuit with IN in parallel with RN, and reconnect the load resistance.
5. Use current division to find the current through the load resistance.

Areas of Application

- Used as an alternative to Thevenin's theorem, especially when a current-source equivalent is convenient.
- Applied in transistor and semiconductor device modelling.
- Used in network simplification for repeated load analysis.

Limitations

- Applicable only to linear, bilateral circuits.
- Cannot be used directly for non-linear or unilateral elements.
- Requires re-calculation whenever source values are changed.

> *Relation: Vth = IN × RN, and Rth = RN. Once one equivalent (Thevenin or Norton) is found, the other can be obtained directly using this relation.*

1.6 Maximum Power Transfer Theorem

Definition

Statement: Maximum power is transferred from a source (network) to a load when the load resistance (RL) is equal to the internal (Thevenin's) resistance (Rth) of the source network as seen from the load terminals, i.e., RL = Rth.

Procedure / Steps

1. Remove the load resistance and find the Thevenin's equivalent (Vth and Rth) of the remaining network across the load terminals.
2. Set the load resistance equal to the Thevenin resistance, RL = Rth, for maximum power transfer.
3. Calculate the maximum power delivered using the formula Pmax = Vth^2 / (4Rth).

Areas of Application

- Used in communication systems and amplifier design for impedance matching.
- Applied in antenna design to maximise signal transfer.
- Used in audio systems for matching speaker impedance with amplifier output.

Limitations

- Efficiency at maximum power transfer is only 50%, so it is not used where high efficiency is required (e.g., power transmission systems).
- Applicable mainly to circuits where load resistance can be varied/matched.
- Not practical in high power systems where efficiency is more important than maximum power.

1.7 Related Numerical Problems

This section presents solved numerical examples based on Superposition, Thevenin's, Norton's and Maximum Power Transfer theorems, following the step-by-step procedures explained above.

Numericals on Superposition Theorem

Example 1: Two-source resistive circuit using Superposition theorem

Given: A circuit has two voltage sources, V1 = 20 V and V2 = 10 V, connected to a common resistor R = 5 Ω, with series resistances R1 = 2 Ω (with V1) and R2 = 3 Ω (with V2). Find the current through R using the Superposition theorem.

Solution

Step 1 (V1 acting alone, V2 shorted): The circuit reduces to V1 = 20 V in series with R1 = 2 Ω, and R = 5 Ω, R2 = 3 Ω in parallel/series combination as per the network. Solving this reduced circuit, the current through R due to V1 alone is calculated as I1' = 2 A (using series-parallel reduction and Ohm's law on the simplified network).

Step 2 (V2 acting alone, V1 shorted): Similarly, with V1 replaced by a short circuit, the current through R due to V2 alone is calculated as I2' = 1 A, following the same reduction procedure.

Step 3 (Superposition): Since both currents through R flow in the same direction (as per the assumed circuit convention), the total current is the algebraic sum: I = I1' + I2'.

Answer: I = I1' + I2' = 2 A + 1 A = 3 A (current through resistor R)

Example 2: Circuit with a voltage source and a current source

Given: A network has a 12 V voltage source in series with a 4 Ω resistor, and a 2 A current source in parallel with a 6 Ω resistor, both feeding a common load resistor RL = 6 Ω. Find the current through RL using the Superposition theorem.

Solution

Step 1 (Voltage source acting alone): Open-circuit the current source (2 A source removed/opened). The circuit now has the 12 V source with 4 Ω in series, feeding the parallel combination of 6 Ω and RL = 6 Ω. Applying the voltage divider and current divider rule, the current through RL due to the voltage source alone is found as I1' = 0.857 A (approx.).

Step 2 (Current source acting alone): Short-circuit the voltage source (12 V source replaced by a short). The 2 A current source now divides between the 6 Ω resistor and RL = 6 Ω (equal resistances, so the current divides equally). The current through RL due to the current source alone is I2' = 1 A.

Step 3 (Superposition): Add the two individual currents algebraically (assuming both currents are in the same reference direction through RL): I = I1' + I2'.

> **Answer: I = 0.857 A + 1 A ≈ 1.857 A (current through load resistor RL)**

Numericals on Thevenin's Theorem

Example 3: Finding Thevenin's equivalent and load current

Given: A network has a 24 V source in series with R1 = 4 Ω, feeding a parallel combination of R2 = 6 Ω and load resistor RL = 8 Ω, connected across terminals A-B. Find the current through RL using Thevenin's theorem.

Solution

Step 1 (Remove RL and find Vth): With RL removed, the open-circuit voltage across A-B equals the voltage across R2 (since no current flows into the open terminal through R2's branch other than the source loop). Using the voltage divider rule on the loop formed by the 24 V source, R1 and R2: Vth = 24 × R2 / (R1 + R2) = 24 × 6 / (4 + 6) = 14.4 V.

Step 2 (Find Rth): Deactivate the 24 V source (replace with a short circuit). Looking into terminals A-B, R1 and R2 appear in parallel: Rth = (R1 × R2) / (R1 + R2) = (4 × 6) / (4 + 6) = 2.4 Ω.

Step 3 (Draw Thevenin's equivalent and reconnect RL): The equivalent circuit is Vth = 14.4 V in series with Rth = 2.4 Ω and RL = 8 Ω. The current through RL is IL = Vth / (Rth + RL).

> **Answer: IL = 14.4 / (2.4 + 8) = 14.4 / 10.4 ≈ 1.385 A (current through load resistor RL)**

Numericals on Norton's Theorem

Example 4: Finding Norton's equivalent and load current

Given: For the same network as Example 3 (24 V source, R1 = 4 Ω, R2 = 6 Ω, RL = 8 Ω across A-B), find the current through RL using Norton's theorem.

Solution

Step 1 (Remove RL and short A-B to find IN): With A-B shorted, R2 gets short-circuited, so the entire source current flows through R1 alone: IN = 24 / R1 = 24 / 4 = 6 A.

Step 2 (Find RN): Deactivate the 24 V source (short it) and find resistance across A-B, which is R1 parallel with R2: RN = (R1 × R2) / (R1 + R2) = (4 × 6) / (4 + 6) = 2.4 Ω (same as Rth, as expected).

Step 3 (Draw Norton's equivalent and apply current division): The Norton equivalent is IN = 6 A in parallel with RN = 2.4 Ω, feeding RL = 8 Ω. Using the current divider rule: IL = IN × RN / (RN + RL).

> **Answer: IL = 6 × 2.4 / (2.4 + 8) = 14.4 / 10.4 ≈ 1.385 A (current through RL — matches the result obtained by Thevenin's theorem in Example 3)**

Numericals on Maximum Power Transfer Theorem

Example 5: Finding RL for maximum power transfer and the maximum power

Given: Using the Thevenin's equivalent obtained in Example 3 (Vth = 14.4 V, Rth = 2.4 Ω), find the value of load resistance RL for maximum power transfer and calculate the maximum power delivered to the load.

Solution

Step 1 (Condition for maximum power transfer): According to the theorem, maximum power is transferred when RL = Rth, so RL = 2.4 Ω.

Step 2 (Calculate maximum power): Using the formula Pmax = Vth^2 / (4 × Rth), substitute the known values: Pmax = (14.4)^2 / (4 × 2.4).

Step 3 (Simplify): (14.4)^2 = 207.36, and 4 × 2.4 = 9.6, so Pmax = 207.36 / 9.6.

> **Answer: RL = 2.4 Ω for maximum power transfer, and Pmax = 207.36 / 9.6 = 21.6 W**

Multiple Choice Questions (MCQs)

Choose the correct option for each question. The correct answer is shown in bold within the options, and a separate answer key is also given at the end.

Q1. Superposition theorem can be applied only to networks that are:

- **a) Linear and bilateral**
- b) Non-linear only
- c) Unilateral only
- d) Purely resistive AC circuits only

Q2. While applying Superposition theorem, an ideal current source is replaced by:

- a) A short circuit
- **b) An open circuit**
- c) A resistance equal to its internal resistance
- d) Another current source

Q3. Thevenin's voltage (Vth) is defined as the:

- a) Short-circuit current at the terminals
- **b) Open-circuit voltage at the terminals**
- c) Voltage across the source
- d) Voltage across Rth only

Q4. Norton's resistance (RN) is related to Thevenin's resistance (Rth) as:

- a) RN = 2Rth
- **b) RN = Rth**
- c) RN = Rth / 2
- d) RN = 0

Q5. The relationship between Vth, IN and Rth is given by:

- a) Vth = IN + Rth
- **b) Vth = IN × Rth**
- c) Vth = IN / Rth

d) Vth = IN − Rth

Q6. For maximum power transfer from a source to a load, the load resistance must be:

a) Equal to zero
b) Equal to Thevenin's resistance
c) Twice the Thevenin's resistance
d) As large as possible

Q7. The maximum power transferred to the load is given by the formula:

a) Pmax = Vth^2 / Rth
b) Pmax = Vth^2 / (2Rth)
c) Pmax = Vth^2 / (4Rth)
d) Pmax = Vth / (4Rth)

Q8. At maximum power transfer condition, the efficiency of the circuit is:

a) 100%
b) 75%
c) 50%
d) 25%

Q9. Norton's short-circuit current (IN) is measured by:

a) Opening the load terminals
b) Short-circuiting the load terminals
c) Removing all sources
d) Connecting a voltmeter across the load

Q10. Thevenin's and Norton's theorems are both applicable to:

a) Linear, bilateral two-terminal networks
b) Non-linear networks only
c) Single-source circuits only
d) Purely capacitive circuits only

Answer Key

1-a | 2-b | 3-b | 4-b | 5-b | 6-b | 7-c | 8-c | 9-b | 10-a

Broad / Long Answer Questions

1. State and explain the Superposition theorem. Explain its step-by-step procedure with the help of a suitable circuit.
2. Explain the applications and limitations of the Superposition theorem.
3. State Thevenin's theorem and explain the procedure to find the Thevenin's equivalent circuit of a given network.
4. State and explain Norton's theorem with its procedure, applications and limitations.
5. Derive the relationship between Thevenin's equivalent and Norton's equivalent circuits.
6. State the Maximum Power Transfer theorem and derive the expression for maximum power transferred to the load.
7. Explain why the efficiency of a circuit is only 50% at the maximum power transfer condition.
8. Using Thevenin's theorem, explain how to find the current through a load resistor connected to a complex network, with a numerical example.

9. Using Norton's theorem, explain how to find the current through a load resistor connected to a complex network, with a numerical example.

10. A network has a 24 V source in series with a 4 Ω resistor, feeding a parallel combination of a 6 Ω resistor and a variable load resistor RL. Find the value of RL for maximum power transfer and calculate the maximum power delivered.

— End of Notes —