CIRCUIT THEORY

Unit 1: Network Theorems, Mesh & Node Analysis, Star-Delta Transformation

Diploma / Polytechnic Engineering — Study Notes

Introduction

Network theorems are a set of principles used to simplify and analyse complex electrical circuits without having to redraw or solve the entire network again and again. They form the foundation of circuit theory and are widely used in electronics, power systems and communication engineering to reduce calculation effort and to design circuits efficiently. This chapter covers the statement, procedure, applications and limitations of the important network theorems, followed by Mesh and Node analysis techniques and Star-Delta / Delta-Star transformation, which are used to convert complex resistive networks into simpler equivalent forms.

Why Network Theorems Are Needed

- To simplify complex networks having multiple sources and branches.
- To find current, voltage or power in a particular branch without solving the whole circuit.
- To design and analyse circuits quickly in industries and laboratories.
- To study the behaviour of a circuit under varying load conditions.

1. Network Theorems

1.1 Superposition Theorem

Statement: In any linear, bilateral network containing more than one independent source, the current through (or voltage across) any element is equal to the algebraic sum of the currents (or voltages) produced by each independent source acting alone, with all other independent sources replaced by their internal resistances (voltage sources short-circuited and current sources open-circuited).

Procedure / Steps

1. Select one independent source and replace all other voltage sources by a short circuit and all other current sources by an open circuit.
2. Redraw the simplified circuit and find the current/voltage in the desired branch using Ohm's law and series-parallel reduction.
3. Repeat step 1 and 2 for every independent source present in the network, one at a time.
4. Algebraically add all the individual current/voltage values obtained to get the actual (resultant) current or voltage.

Areas of Application

- Analysis of circuits having more than one source (AC and DC combined circuits).
- Used in amplifier circuits to study the effect of AC and DC sources separately.
- Useful in communication and signal processing circuits.

Limitations

- Applicable only to linear, bilateral networks; cannot be applied to non-linear elements like diodes and transistors.

- Not applicable for power calculation directly, since power is a non-linear (squared) function of current/voltage.
- Time consuming for networks having a large number of sources.

1.2 Thevenin's Theorem

Statement: Any linear, bilateral two-terminal network containing sources and resistances can be replaced by an equivalent circuit consisting of a single voltage source (Vth) in series with a single resistance (Rth), where Vth is the open-circuit voltage across the terminals and Rth is the resistance measured across the terminals with all independent sources deactivated.

Procedure / Steps

1. Remove the load resistance (RL) from the circuit across which Thevenin's equivalent is required.
2. Find the open-circuit voltage (Vth) across the open terminals using any circuit analysis method.
3. Deactivate all independent sources (short voltage sources, open current sources) and find the equivalent resistance (Rth) as seen from the open terminals.
4. Draw the Thevenin's equivalent circuit with Vth in series with Rth, and reconnect the load resistance RL.
5. Calculate the required current or voltage across RL using this simplified equivalent circuit.

Areas of Application

- Used to simplify complex networks for repeated calculations with different load resistances.
- Widely used in power system analysis and in designing matching networks.
- Useful in analysing amplifier and transducer circuits.

Limitations

- Applicable only to linear, bilateral networks.
- Not suitable for circuits containing non-linear elements.
- Additional calculation is required each time internal source values change.

1.3 Norton's Theorem

Statement: Any linear, bilateral two-terminal network containing sources and resistances can be replaced by an equivalent circuit consisting of a single current source (IN) in parallel with a single resistance (RN), where IN is the short-circuit current through the terminals and RN is the resistance measured across the terminals with all independent sources deactivated (RN is equal to Rth).

Procedure / Steps

1. Remove the load resistance from the circuit across the terminals of interest.
2. Short the two open terminals and calculate the short-circuit current (IN) flowing through them.
3. Deactivate all independent sources and calculate the equivalent resistance (RN) as seen from the open terminals (same as Rth).
4. Draw the Norton's equivalent circuit with IN in parallel with RN, and reconnect the load resistance.
5. Use current division to find the current through the load resistance.

Areas of Application

- Used as an alternative to Thevenin's theorem, especially when a current-source equivalent is convenient.
- Applied in transistor and semiconductor device modelling.

- Used in network simplification for repeated load analysis.

Limitations

- Applicable only to linear, bilateral circuits.
- Cannot be used directly for non-linear or unilateral elements.
- Requires re-calculation whenever source values are changed.

1.4 Maximum Power Transfer Theorem

Statement: Maximum power is transferred from a source (network) to a load when the load resistance (RL) is equal to the internal (Thevenin's) resistance (Rth) of the source network as seen from the load terminals, i.e., RL = Rth.

Procedure / Steps

1. Remove the load resistance and find the Thevenin's equivalent (Vth and Rth) of the remaining network across the load terminals.
2. Set the load resistance equal to the Thevenin resistance, RL = Rth, for maximum power transfer.
3. Calculate the maximum power delivered using the formula Pmax = Vth^2 / (4Rth).

Areas of Application

- Used in communication systems and amplifier design for impedance matching.
- Applied in antenna design to maximise signal transfer.
- Used in audio systems for matching speaker impedance with amplifier output.

Limitations

- Efficiency at maximum power transfer is only 50%, so it is not used where high efficiency is required (e.g., power transmission systems).
- Applicable mainly to circuits where load resistance can be varied/matched.
- Not practical in high power systems where efficiency is more important than maximum power.

1.5 Reciprocity Theorem

Statement: In any linear, bilateral, passive network, if a source of voltage (V) placed in one branch produces a current (I) in another branch, then the same voltage source placed in the second branch will produce the same current (I) in the first branch. In short, the ratio of response to excitation remains the same even after interchanging the positions of source and response (ammeter).

Procedure / Steps

1. Select the branch where the source is applied and the branch where the response (current) is to be measured.
2. Calculate the current in the response branch due to the applied source using standard network analysis.
3. Interchange the positions of the source and the ammeter (response branch).
4. Recalculate the current; it should be equal to the current obtained in step 2, verifying the theorem.

Areas of Application

- Used in the design and analysis of passive filter networks and transmission lines.
- Useful in antenna theory to establish equivalence between transmitting and receiving properties.
- Applied in the study of two-port networks and attenuators.

Limitations

- Applicable only to single-source, linear, bilateral, passive networks.
- Not applicable to networks containing dependent sources or active/non-linear elements.
- Cannot be applied when more than one source is present simultaneously.

2. Mesh Analysis and Node Analysis

2.1 Mesh Analysis (Loop Analysis)

Mesh analysis is a method of circuit analysis based on Kirchhoff's Voltage Law (KVL), used for planar networks. In this method, mesh (loop) currents are assumed for each independent loop of the circuit, and KVL equations are written for each mesh in terms of these loop currents. Solving the simultaneous equations gives the value of each mesh current, from which branch currents and voltages can be determined.

Procedure for Mesh Analysis

1. Identify all the independent meshes (loops) in the network and assign a circulating mesh current (I1, I2, I3, ...) to each, usually in the clockwise direction.
2. Apply Kirchhoff's Voltage Law (KVL) to each mesh, taking the sum of voltage rises and drops around the loop equal to zero.
3. Express the voltage drop across shared (common) branches in terms of the difference of adjoining mesh currents.
4. Form a set of simultaneous linear equations, one for each mesh.
5. Solve the equations using elimination, substitution, Cramer's rule or matrix method to obtain the mesh currents.
6. Calculate the actual branch currents from the mesh currents obtained.

Applications and Limitations

Type	Details
Applications	Analysis of planar networks, filter and amplifier circuits, useful when the number of meshes is less than the number of nodes.
Limitations	Applicable only to planar networks (cannot be applied directly to non-planar circuits); becomes complex for networks with a large number of meshes; not convenient for circuits with many current sources.

2.2 Node Analysis (Nodal Analysis)

Node analysis is a method of circuit analysis based on Kirchhoff's Current Law (KCL). In this method, one node is selected as the reference (ground) node, and unknown node voltages are assumed for all other nodes. KCL equations are written at each non-reference node in terms of these node voltages, and the resulting simultaneous equations are solved to find all node voltages.

Procedure for Node Analysis

1. Identify all the nodes in the circuit and select one node (usually with maximum connections) as the reference or ground node (0V).
2. Assign unknown voltage variables (V1, V2, V3, ...) to all the remaining non-reference nodes.
3. Apply Kirchhoff's Current Law (KCL) at each non-reference node, i.e., the sum of currents leaving (or entering) the node is equal to zero.

4. Express each branch current in terms of node voltages using Ohm's law.
5. Form a set of simultaneous linear equations, one for each non-reference node.
6. Solve the equations to obtain the node voltages, and use them to calculate branch currents and voltages as required.

Applications and Limitations

Type	Details
Applications	Preferred for circuits with a large number of parallel branches and current sources; widely used in computer-aided circuit simulation (SPICE) software; convenient when the number of nodes is less than the number of meshes.
Limitations	Becomes complicated when the circuit has many voltage sources (super-node concept required); not convenient for circuits having a small number of nodes but many meshes; requires careful selection of the reference node.

> *Tip: Choose Mesh Analysis when the number of independent loops is less than the number of nodes; choose Node Analysis when the number of nodes is less than the number of loops, to minimise the number of simultaneous equations to be solved.*

3. Star/Delta and Delta/Star Transformation

In many resistive networks, three resistors may be connected either in a Star (Y) form or in a Delta (Δ) form between three terminals. Such networks can be converted from one form to another using Star-Delta and Delta-Star transformation formulas. This conversion simplifies complex bridge-type or ladder-type networks into simple series-parallel combinations, making circuit analysis much easier.

3.1 Delta (Δ) to Star (Y) Transformation

When a Delta-connected network (resistances RAB, RBC, RCA between terminals A, B, C) is converted into an equivalent Star network (resistances RA, RB, RC from each terminal to a common star point), the equivalent star resistances are obtained by the following relations:

- RA = (RAB × RCA) / (RAB + RBC + RCA)
- RB = (RAB × RBC) / (RAB + RBC + RCA)
- RC = (RBC × RCA) / (RAB + RBC + RCA)

> *Rule: Star resistance at a terminal = (Product of the two Delta resistances connected to that terminal) ÷ (Sum of all three Delta resistances).*

3.2 Star (Y) to Delta (Δ) Transformation

When a Star-connected network (resistances RA, RB, RC) is converted into an equivalent Delta network (resistances RAB, RBC, RCA), the equivalent delta resistances are obtained by the following relations:

- RAB = (RA·RB + RB·RC + RC·RA) / RC
- RBC = (RA·RB + RB·RC + RC·RA) / RA
- RCA = (RA·RB + RB·RC + RC·RA) / RB

> *Rule: Delta resistance between two terminals = (Sum of the products of Star resistances taken two at a time) ÷ (Star resistance of the opposite/third terminal).*

3.3 Procedure for Applying the Transformation

1. Identify the three-terminal network connected either in Star or Delta form within the larger circuit.

2. Select the appropriate set of formulas (Delta-to-Star or Star-to-Delta) depending on the required simplification.
3. Calculate the equivalent resistance values using the transformation formulas.
4. Replace the original Star/Delta network with the calculated equivalent network in the circuit diagram.
5. Reduce the resulting series-parallel network using standard resistance combination rules to solve for the required current, voltage or equivalent resistance.

Applications

- Simplification of unbalanced Wheatstone bridge networks that cannot be solved by simple series-parallel reduction.
- Used in three-phase power system analysis for transformer and load connections.
- Applied in the design of attenuator and filter networks.
- Useful in reducing complex ladder networks to find equivalent resistance between two terminals.

Limitations

- Applicable only to purely resistive (or purely reactive/impedance) three-terminal networks.
- Increases the number of resistive elements when converting Star to Delta, which may complicate simple circuits unnecessarily.
- Not helpful for networks that are already in simple series-parallel form.

Multiple Choice Questions (MCQs)

Choose the correct option for each question. The correct answer is shown in bold within the options, and a separate answer key is also given at the end.

Q1. Superposition theorem is applicable only to:

a) Linear, bilateral networks
b) Non-linear networks
c) Unilateral networks only
d) Networks with a single source only

Q2. In Thevenin's theorem, Rth is calculated by:

a) Short-circuiting all sources
b) Deactivating all independent sources and finding resistance across open terminals
c) Opening all resistances
d) Measuring resistance with sources active

Q3. Norton's equivalent circuit consists of:

a) A voltage source in series with resistance
b) A current source in parallel with resistance
c) Two voltage sources in series
d) A resistance only

Q4. For maximum power transfer, the condition is:

a) RL = 0
b) RL = infinity
c) RL = Rth
d) RL = 2Rth

Q5. The efficiency of a circuit at maximum power transfer condition is:

a) 100%

b) 75%
c) 50%
d) 25%

Q6. Reciprocity theorem is applicable to networks containing:
a) Only one independent source
b) Multiple independent sources
c) Only dependent sources
d) Non-linear elements

Q7. Mesh analysis is based on:
a) Kirchhoff's Current Law
b) Kirchhoff's Voltage Law
c) Ohm's Law only
d) Superposition theorem

Q8. Node analysis is based on:
a) Kirchhoff's Voltage Law
b) Kirchhoff's Current Law
c) Thevenin's theorem
d) Norton's theorem

Q9. In Delta to Star conversion, RA is given by:
a) (RAB + RCA) / RBC
b) (RAB × RCA) / (RAB + RBC + RCA)
c) (RAB × RBC × RCA)
d) RAB + RBC + RCA

Q10. Mesh analysis is preferred over node analysis when:
a) Number of nodes is less than number of meshes
b) Number of meshes is less than number of nodes
c) Circuit has only current sources
d) Circuit is non-planar

Answer Key

1-a | 2-b | 3-b | 4-c | 5-c | 6-a | 7-b | 8-b | 9-b | 10-b

Broad / Long Answer Questions

1. State and explain the Superposition theorem with its procedure, applications and limitations.
2. State Thevenin's theorem. Explain the step-by-step procedure to find the Thevenin's equivalent circuit of a given network.
3. State and explain Norton's theorem. Differentiate between Thevenin's and Norton's equivalent circuits.
4. State the Maximum Power Transfer theorem and derive the condition for maximum power transfer along with its practical applications.
5. State and explain the Reciprocity theorem with a suitable example and mention its limitations.
6. Explain the procedure for solving a network using Mesh Analysis. State its applications and limitations.
7. Explain the procedure for solving a network using Node (Nodal) Analysis. State its applications and limitations.

8. Differentiate between Mesh Analysis and Node Analysis, and explain when each method is preferred.
9. Derive the formulas for Delta to Star transformation with a neat circuit diagram.
10. Derive the formulas for Star to Delta transformation and explain its applications in circuit simplification.

— End of Notes —