ENGINEERING MECHANICS

Polytechnic Diploma | Study Notes

Chapter: Condition of Equilibrium

1. Introduction to Equilibrium

When a body is subjected to a number of forces and remains in a state of rest or uniform motion, it is said to be in equilibrium. In Engineering Mechanics, the study of equilibrium helps us analyse structures, machines, and members under various loading conditions. A body is in equilibrium when the resultant of all forces acting on it is zero and the resultant moment of all forces about any point is also zero.

Definition: Equilibrium is the state of a body in which the net force and net moment acting on it are both equal to zero, resulting in no acceleration (static equilibrium) or uniform motion (dynamic equilibrium).

Key Concept: For a body to be in static equilibrium, it must satisfy both force equilibrium (resultant force = 0) and moment equilibrium (resultant moment = 0) simultaneously.

2. Equilibrium of Co-planar Concurrent Force System

A co-planar concurrent force system consists of forces that lie in the same plane and whose lines of action all pass through a single common point. When such a system of forces is in equilibrium, two conditions must be satisfied. The algebraic sum of all horizontal components (X-direction) must equal zero, and the algebraic sum of all vertical components (Y-direction) must equal zero.

$$\Sigma F_x = 0 \quad \text{and} \quad \Sigma F_y = 0$$

2.1 Lami's Theorem

Lami's Theorem is a very useful result applicable to a body acted upon by exactly three concurrent coplanar forces in equilibrium. It states that when three concurrent forces are in equilibrium, each force is proportional to the sine of the angle between the other two forces.

Statement: If three concurrent forces are in equilibrium, then each force is proportional to the sine of the angle opposite to it (i.e., the angle between the other two forces).

$$P / \sin\alpha = Q / \sin\beta = R / \sin\gamma$$

Where P, Q, and R are the three concurrent forces in equilibrium, and α, β, and γ are the angles opposite to forces P, Q, and R respectively (i.e., the angles between the other two forces).

Important Note: The angles α, β, and γ are the angles between pairs of forces (not the angles each force makes with a reference axis), and their sum is always equal to 360°.

Conditions for Applying Lami's Theorem:

- There must be exactly three forces acting on the body.
- All three forces must be concurrent (passing through a single point).
- All three forces must be coplanar (lying in the same plane).
- The system must be in equilibrium (resultant = 0).

Application Steps:

- Draw a clear Free Body Diagram (FBD) of the body.
- Identify the three concurrent forces and label them P, Q, R.
- Measure or calculate the included angles α, β, γ between force pairs.
- Apply Lami's Theorem: $P/\sin\alpha = Q/\sin\beta = R/\sin\gamma$ and solve for unknowns.

2.2 Triangle Law of Equilibrium

The Triangle Law of Equilibrium states that if three concurrent forces acting on a body are in equilibrium, they can be represented in magnitude and direction by the three sides of a triangle taken in order. Conversely, if three forces can be represented as the sides of a closed triangle (head to tail), the system is in equilibrium.

> **Statement:** If three coplanar concurrent forces are in equilibrium, they can be represented as the three sides of a triangle taken in order (head to tail). The triangle so formed will be a closed figure.

Graphical Method: Draw the forces to scale, placing them head to tail. If the resulting figure is a closed triangle (the last vector ends at the starting point of the first), the system is in equilibrium. The unknown force(s) can be found by completing the triangle.

Key Points:

- Applicable only to three concurrent coplanar forces.
- Forces are drawn as vectors placed head-to-tail in sequence.
- A closed triangle confirms equilibrium of the force system.
- The direction of travel around the triangle must be consistent (all clockwise or all counter-clockwise).
- This method is a graphical verification tool and is complementary to analytical methods.

2.3 Polygon Law of Equilibrium

The Polygon Law of Equilibrium extends the Triangle Law to systems of more than three concurrent coplanar forces. It states that if a number of concurrent coplanar forces acting on a body are in equilibrium, they can be represented in magnitude and direction by the sides of a closed polygon taken in order.

Statement: If several coplanar concurrent forces acting on a body are in equilibrium, then these forces can be represented as the sides of a closed polygon when drawn head to tail in sequence.

Graphical Procedure: Select a convenient scale. Draw each force vector sequentially (head of one to tail of the next). If the system is in equilibrium, the polygon closes (the last vector's tip meets the starting point of the first vector). An open polygon indicates the system is not in equilibrium, and the closing vector represents the resultant.

Comparison: Triangle Law vs Polygon Law

Aspect	Triangle Law	Polygon Law
No. of Forces	Exactly 3	3 or more
Shape Formed	Triangle	Polygon
Applicability	Limited	General
Equilibrium Check	Closed triangle	Closed polygon

3. Concept of Free Body Diagram (FBD)

A Free Body Diagram (FBD) is a simplified, schematic representation of a body (or part of a body) isolated from its surroundings, showing all external forces and reactions acting on it. It is one of the most fundamental and powerful tools in Engineering Mechanics and is an essential first step in solving any equilibrium problem.

Definition: A Free Body Diagram is a diagram of a selected body or portion of a body in which the body is isolated from all supports and connections, and all forces (applied loads, support reactions, and self-weight) acting on it are drawn and labelled clearly.

3.1 Purpose and Importance of FBD

Drawing a correct FBD is critical because it allows the engineer to clearly visualise all forces acting on the body, systematically apply equilibrium equations, and avoid errors caused by missing forces or incorrect directions. Without an FBD, solving equilibrium problems becomes unreliable.

Purposes:

- Identifies all forces and moments acting on the body clearly.
- Provides a systematic basis for writing equilibrium equations.
- Helps distinguish between internal and external forces.
- Prevents common errors such as missing reaction forces.
- Serves as a communication tool between engineers.

3.2 Steps to Draw a Free Body Diagram

A proper FBD is constructed by following a systematic procedure to ensure no force is missed. The steps are as follows:

- **Identify the body:** Select the body or part of the body to be analysed and decide the system boundary.
- **Isolate the body:** Detach the body from all supports, surfaces, cables, and connected members.
- **Show all applied loads:** Draw all known external forces (loads, weights) with their correct magnitudes and directions.
- **Replace supports with reactions:** Substitute each support, joint, or connection with its corresponding reaction force(s) or moment.
- **Include self-weight:** If not negligible, include the weight of the body acting downward at its centre of gravity.
- **Label all forces:** Assign symbols or values to each force/reaction for use in equations.

3.3 Common Types of Supports and Their Reactions

Support Type	No. of Reactions	Reactions Provided
Simple / Roller Support	1	One reaction perpendicular to surface
Hinged / Pin Support	2	Horizontal (H) and Vertical (V) reactions
Fixed / Built-in Support	3	H, V reactions + Moment (M)
String / Cable	1	Tension along the string (away from body)
Smooth Surface	1	Normal reaction perpendicular to surface

4. Equilibrium of Co-planar Non-Concurrent Force System

A co-planar non-concurrent force system consists of forces that lie in the same plane but whose lines of action do not all pass through a single common point. Unlike concurrent systems (where only two equilibrium conditions are needed), non-concurrent force systems require three conditions to be fully in equilibrium.

$$\Sigma F_x = 0 \qquad \Sigma F_y = 0 \qquad \Sigma M = 0$$

Here, $\Sigma M = 0$ represents the algebraic sum of all moments of forces about any arbitrary point in the plane being equal to zero. This third condition is essential for non-concurrent systems because the forces, even if balanced in magnitude, could still cause rotation of the body.

4.1 Conditions of Equilibrium for Non-Concurrent Forces

The Three Conditions of Equilibrium:

- **Condition 1 – Horizontal Force Balance:** ΣFx = 0 (The algebraic sum of all horizontal force components must be zero, so there is no net translation in the x-direction.)
- **Condition 2 – Vertical Force Balance:** ΣFy = 0 (The algebraic sum of all vertical force components must be zero, so there is no net translation in the y-direction.)
- **Condition 3 – Moment Balance:** ΣM = 0 (The algebraic sum of moments of all forces about any point in the plane must be zero, so there is no net rotation of the body.)

Sign Convention: Typically, rightward forces and upward forces are taken as positive. For moments, counter-clockwise (CCW) moments are taken as positive and clockwise (CW) moments are negative. Always state and follow a consistent sign convention.

4.2 Parallel Force Systems (Like and Unlike Parallel Forces)

Parallel force systems are a special type of non-concurrent force system in which all forces act parallel to each other. There are two types of parallel force systems based on the direction of the forces.

Like Parallel Forces:

Like parallel forces are those which act parallel to each other and in the same direction. For a system of like parallel forces to be in equilibrium, the sum of all forces in one direction must be balanced by reactions, and the net moment about any point must be zero.

- All forces act in the same direction (e.g., all downward).
- The resultant force equals the algebraic sum of all forces.
- The resultant acts between the outermost forces.
- Example: Multiple loads acting downward on a simply supported beam.

Unlike Parallel Forces:

Unlike parallel forces are those which act parallel to each other but in opposite directions. These forces can form a couple if they are equal in magnitude, or create a net force if they are unequal.

- Forces act in opposite directions (some upward, some downward).
- The resultant is the algebraic difference of the forces.
- If two equal unlike parallel forces act at different points, they form a couple.
- Example: Beam reactions (upward) and applied loads (downward).

4.3 Conditions of Equilibrium for Parallel Force Systems

For a coplanar system of parallel forces (like or unlike) to be in equilibrium, two specific conditions must be satisfied. Since all forces are already parallel (say, vertical), the horizontal equilibrium (ΣFx = 0) is automatically satisfied. The two governing conditions are:

Condition 1: ΣF = 0 | Condition 2: ΣM = 0

- **Condition 1 (ΣF = 0):** The algebraic sum of all parallel forces must be zero. For vertical parallel forces: sum of upward forces = sum of downward forces.

- **Condition 2 (ΣM = 0):** The algebraic sum of moments of all forces about any point in the plane must be zero. This ensures no rotation of the body.

Tip for Solving Problems: Choose the moment centre wisely — take moments about a point where one or more unknown forces act. This eliminates those unknowns from the moment equation, making calculation simpler.

5. General Approach to Solving Equilibrium Problems

Solving equilibrium problems in Engineering Mechanics follows a structured, step-by-step procedure. A disciplined approach not only ensures correct answers but also develops engineering intuition and the ability to handle complex real-world problems.

Step-by-Step Procedure:

- **Step 1 – Read and Understand the Problem:** Carefully read the problem statement and identify what is given (known forces, angles, distances) and what is to be found (unknown forces, reactions).
- **Step 2 – Draw the Free Body Diagram:** Isolate the body and draw all forces acting on it including applied loads, self-weight, and support reactions. Label all forces and dimensions clearly.
- **Step 3 – Choose a Coordinate System and Sign Convention:** Establish the positive x and y directions and the sign convention for moments (CCW positive is standard).
- **Step 4 – Resolve Inclined Forces:** Resolve any inclined forces into their horizontal and vertical components using trigonometry ($F_x = F\cos\theta$, $F_y = F\sin\theta$).
- **Step 5 – Apply Equilibrium Equations:** Write and solve the applicable equilibrium equations ($\Sigma F_x = 0$, $\Sigma F_y = 0$, and $\Sigma M = 0$ as needed) to find unknown quantities.
- **Step 6 – Check the Results:** Verify the answers by substituting back into equilibrium equations or by taking moments about a different point. A correct solution must satisfy ALL equilibrium conditions.

6. Quick Summary and Key Formulae

Topic	Applicability	Key Condition / Formula
Lami's Theorem	3 concurrent forces only	**$P/\sin\alpha = Q/\sin\beta = R/\sin\gamma$**
Triangle Law	3 concurrent forces	**Closed triangle (graphical)**
Polygon Law	3 or more concurrent forces	**Closed polygon (graphical)**
Concurrent System	Forces meet at one point	**$\Sigma F_x=0$, $\Sigma F_y=0$**
Non-Concurrent System	Forces don't meet at one point	**$\Sigma F_x=0$, $\Sigma F_y=0$, $\Sigma M=0$**
Parallel Force System	Parallel forces (like/unlike)	**$\Sigma F=0$, $\Sigma M=0$**

Important Reminders

- Always draw the FBD before writing any equations — it is non-negotiable in Engineering Mechanics.
- For Lami's Theorem, angles are those between force pairs, and their sum must be 360°.
- For moment calculations, choose the moment centre strategically to simplify equations.
- Statically indeterminate problems (with more unknowns than equations) are excluded from this syllabus.
- Always check units and sign conventions consistently throughout a problem.
- Reaction forces at supports are always equal and opposite to the effect the support would exert on the body.

End of Chapter: Condition of Equilibrium

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