UNIT 2

Electric and Magnetic Circuits

Fundamentals of Electrical & Electronics Engineering

Polytechnic | 2nd Semester

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2.1 EMF, Current, Potential Difference, Power and Energy

Electromotive Force (EMF)

Electromotive force (EMF) is the energy provided by a source (battery, generator, etc.) per unit charge to drive current through a circuit. It is not a force in the mechanical sense, but rather a potential difference maintained by an energy source.

- **Symbol:** E | Unit: Volt (V)
- **Formula:** $E = W / Q$ (where W = work done in joules, Q = charge in coulombs)
- EMF is the cause; current is the effect.
- Sources of EMF: Chemical (batteries), Mechanical (generators), Thermal (thermocouples), Solar (photovoltaic cells).

Electric Current

Electric current is defined as the rate of flow of electric charge through a conductor. It is the movement of free electrons (in metallic conductors) or ions (in electrolytes) in a definite direction under the influence of an applied EMF.

- **Symbol:** I | Unit: Ampere (A)
- **Formula:** $I = Q / t$ (charge per unit time)
- **Types:** (i) Direct Current (DC) — unidirectional and constant in magnitude.
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- Conventional current flows from + to −; electron flow is from − to +.

Potential Difference (PD)

Potential difference between two points in a circuit is the work done in moving a unit positive charge from one point to the other. Unlike EMF, PD is measured across a load (resistor) and represents energy consumed.

- **Symbol:** V | Unit: Volt (V)
- **Formula:** $V = W / Q = I \times R$ (Ohm's Law)
- EMF is the open-circuit terminal voltage; PD is the voltage drop across a component.
- When current flows, PD = EMF − Voltage drop due to internal resistance.

Power

Electric power is the rate at which electrical energy is consumed or generated in a circuit. It represents how fast energy is transferred.

- **Symbol:** P | Unit: Watt (W)
- **Formulae:** $P = V \times I = I^2 \times R = V^2 / R$
- 1 Watt = 1 Joule per second.
- In a DC circuit, power is simply the product of voltage and current.

Energy

Electrical energy is the total work done by the electric current over a period of time. It is the product of power and time.

- **Symbol:** W | Unit: Joule (J) or Watt-hour (Wh) or kWh
- **Formula:** $W = P \times t = V \times I \times t = I^2 R t$
- Commercial unit of energy: 1 kWh = 3.6×10^6 J (1 Unit of electricity)
- Energy meters (electricity meters) measure energy in kWh.

2.2 Magnetic Circuit — Key Concepts

Magnetomotive Force (MMF)

Magnetomotive force (MMF) is the driving force that establishes magnetic flux in a magnetic circuit. It is analogous to EMF in an electric circuit.

- **Symbol:** F (or $\mathcal{F}$) | Unit: Ampere-turns (AT)
- **Formula:** MMF = N × I (N = number of turns, I = current in amperes)
- The greater the MMF, the stronger the magnetic field produced.

Magnetic Force

A magnetic force acts on a current-carrying conductor placed in a magnetic field. According to Ampere's law, a magnetic field exerts a force on moving charges.

- **Formula:** $F = B\,I\,L \sin\theta$ (B = flux density, I = current, L = length, θ = angle)
- Maximum force when the conductor is perpendicular to the field (θ = 90°).

Permeability

Permeability is the measure of how easily a magnetic field can be established in a material. It indicates the ability of a material to support the formation of magnetic flux.

- **Absolute permeability:** $\mu = \mu_0 \times \mu_r$
- **μ_0 (permeability of free space):** $4\pi \times 10^{-7}$ H/m
- **μ_r (relative permeability):** dimensionless; 1 for air, thousands for iron/silicon steel
- Higher permeability = lower reluctance = better magnetic conductor.

Reluctance

Reluctance is the opposition offered by a magnetic material to the establishment of magnetic flux. It is analogous to resistance in an electric circuit.

- **Symbol:** S (or $\mathcal{R}$) | Unit: Ampere-turns per Weber (AT/Wb)
- **Formula:** $S = l / (\mu_0 \mu_r A)$ (l = length, A = cross-sectional area)
- Relationship: MMF = Flux × Reluctance → $\mathcal{F} = \Phi \times S$

Leakage Factor

In a magnetic circuit, not all the flux produced by the MMF passes through the intended path (core). Some flux leaks through the surrounding air or non-magnetic materials. This is called leakage flux.

- **Leakage Factor (λ):** λ = Total flux / Useful flux = $(\Phi_{total}) / (\Phi_{useful})$
- λ is always greater than 1; a value close to 1 indicates good magnetic design.
- Leakage factor is considered in transformer and machine design.

B-H Curve (Magnetization Curve)

The B-H curve is a graph that shows the relationship between magnetic flux density (B) and the magnetic field intensity (H) for a given magnetic material. It is also called the magnetization curve.

- **B:** Magnetic flux density (Tesla, T)
- **H:** Magnetic field intensity (A/m); $H = N I / l$
- Initially, B increases almost linearly with H (unsaturated region).
- Beyond a point, the curve flattens — this is the saturation region.
- The slope of the B-H curve = permeability (μ) at that operating point.
- Soft magnetic materials (silicon steel) have steep B-H curves; hard materials (permanent magnets) have wide hysteresis loops.

Hysteresis Loop

When a magnetic material is subjected to a cycle of magnetization and demagnetization, the B-H curve forms a closed loop called the hysteresis loop. This loop demonstrates that the magnetization of the material lags behind the applied magnetic field.

- **Retentivity (Br):** Residual flux density when H = 0 (material retains magnetism).
- **Coercivity (Hc):** Reverse field needed to reduce B to zero (demagnetization force).
- Hard magnetic materials: large loop area → used for permanent magnets.
- Soft magnetic materials: small loop area → used for transformers and motors.

Hysteresis Loss

Every time the magnetic material goes through one complete cycle of magnetization, energy is dissipated as heat. This energy loss per cycle is proportional to the area of the hysteresis loop.

- **Steinmetz Formula:** $W_h = \eta \times B_m{}^{ax1.6} \times f \times V$ (η = Steinmetz constant, f = frequency)
- Hysteresis loss increases with frequency and maximum flux density.
- To minimize hysteresis loss, silicon steel (low Steinmetz constant) is used.

Eddy Current Loss

When a magnetic core is subjected to a changing magnetic flux, an EMF is induced in the core material itself. This causes circulating currents (eddy currents) to flow within the core, producing heat — this is the eddy current loss.

- **Formula:** $W_e = K_e \times B_m{}^2 \times f^2 \times t^2$ (K_e = eddy current constant, t = thickness of lamination)
- Eddy currents are minimized by laminating the core (thin sheets insulated from each other).
- Core laminations reduce the path for eddy currents, decreasing their magnitude and losses.

Analogy Between Electric and Magnetic Circuits

There is a strong mathematical analogy between electric circuits and magnetic circuits, which helps in analysis:

- **EMF (V) ↔ MMF (NI)**
- **Current (I, Ampere) ↔ Magnetic Flux (Φ, Weber)**
- **Resistance (R, Ohm) ↔ Reluctance (S, AT/Wb)**
- **Resistivity (ρ) ↔ Reluctivity (1/μ)**
- **Conductance ↔ Permeance**
- **Ohm's Law: V = IR ↔ MMF = Φ × S**

- **Kirchhoff's Voltage Law ↔ Sum of MMFs = Sum of ($\Phi \times S$)**

Note: Unlike electric circuits, magnetic flux does not actually 'flow'. Leakage is also more prevalent in magnetic circuits.

2.3 Electromagnetic Induction

Introduction

Electromagnetic induction is the phenomenon of producing an EMF (and hence a current, if the circuit is closed) in a conductor whenever the magnetic flux linked with it changes. This principle was discovered by Michael Faraday in 1831 and is the foundation for generators, transformers, and inductors.

Faraday's Laws of Electromagnetic Induction

Faraday proposed two fundamental laws:

- **First Law:** Whenever the magnetic flux linking with a conductor (or coil) changes, an EMF is induced in it. The induced EMF lasts only as long as the change in flux continues.
- **Second Law:** The magnitude of the induced EMF is directly proportional to the rate of change of flux linkage. Mathematically:

$$e = -N \times (d\Phi/dt)$$

- N = number of turns; dΦ/dt = rate of change of flux (Wb/s)
- The negative sign is explained by Lenz's Law.

Lenz's Law

Lenz's Law states that the direction of the induced EMF (and hence the induced current) is always such that it opposes the cause that produces it. This is a consequence of the law of conservation of energy.

- If a magnet approaches a coil, the induced current creates a magnetic field that repels the magnet.
- If a magnet moves away from a coil, the induced current creates a field that attracts the magnet.
- Lenz's Law gives the direction of induced EMF; Faraday's Second Law gives the magnitude.
- The negative sign in Faraday's equation ($e = -N\, d\Phi/dt$) represents Lenz's Law.

2.4 Types of Induced EMF & Inductance

Dynamically Induced EMF

Dynamically induced EMF is produced when a conductor physically moves through a stationary magnetic field (or the magnetic field moves past a stationary conductor). The flux linkage changes due to the motion of the conductor.

- **Principle:** A conductor of length L moving with velocity v in a field of flux density B experiences:

$$e = B\ L\ v\ \sin\theta$$

- θ is the angle between the direction of motion and the field.
- Maximum EMF when $\theta = 90°$ (conductor moves perpendicular to field).
- **Application:** Electrical generators (AC and DC) — mechanical energy → electrical energy.
- In a DC generator, the armature conductors move through the magnetic field of the stator poles.

Statically Induced EMF

Statically induced EMF is produced without any physical movement of the conductor. Instead, the magnetic flux linking with a stationary coil changes due to a changing current in the same or a neighboring coil. It is further divided into two types:

- **1. Self-Induced EMF:** When the current in a coil itself changes, the changing flux induces an EMF in the same coil. This opposes the change in current (Lenz's Law).

$$e = -L \times (dI/dt)$$

- **2. Mutually Induced EMF:** When the current in one coil changes, it induces an EMF in a neighboring coil due to the changing mutual flux linkage.

$$e_2 = -M \times (dI_1/dt)$$

- **Application:** Transformers operate on the principle of mutually induced EMF.

Self Inductance

Self inductance is the property of a coil by which it opposes any change in the current flowing through it, by inducing an EMF in itself. It is the electromagnetic inertia of the coil.

- **Symbol:** L | Unit: Henry (H)
- **Formula:** $L = N\Phi / I = N^2 / S$ (S = reluctance of the magnetic circuit)
- A coil is said to have 1 Henry inductance if a current change of 1 A/s induces 1 V.
- Factors: Number of turns (N^2), core permeability, cross-sectional area, length.

Mutual Inductance

Mutual inductance is the property by which a change of current in one coil induces an EMF in a neighboring coil. It is the basis of transformer action.

- **Symbol:** M | Unit: Henry (H)
- **Formula:** $M = k\sqrt{L_1 L_2}$ where k = coefficient of coupling ($0 \le k \le 1$)
- k = 1 means perfect coupling (all flux of coil 1 links with coil 2) — ideal transformer.
- k = 0 means no coupling (coils are magnetically isolated).
- Mutual inductance depends on the geometry, proximity, and orientation of the two coils.

2.5 Energy Stored in a Magnetic Circuit

When current flows through an inductor, energy is stored in the magnetic field of the coil. This energy is not dissipated but stored and can be released when the current decreases.

Formula for Energy Stored

$$W = \frac{1}{2} L I^2$$

- **W:** Energy stored in joules
- **L:** Self inductance in Henry
- **I:** Current through the coil in Ampere

Energy in Terms of Field Quantities

The energy stored can also be expressed in terms of magnetic field quantities (flux density B and field intensity H):

$$W = \frac{1}{2} \times (B^2/\mu) \times \text{Volume} = \frac{1}{2} \times B \times H \times \text{Volume}$$

- Energy density (energy per unit volume) = $\frac{1}{2} \times B \times H$ (J/m^3)
- This is analogous to $\frac{1}{2} \times \varepsilon \times E^2$ for energy stored in an electric field.

Physical Interpretation

When an inductor is connected to a DC source, current builds up gradually. During this build-up, energy is taken from the source and stored in the magnetic field. When the source is disconnected, the inductor tries to maintain the current — the stored energy is released. This is why an inductor in a circuit can generate high-voltage spikes when the current is suddenly interrupted.

- Applications: Energy storage in choke coils, smoothing inductors, magnetic energy storage systems.
- In a transformer, mutual energy is exchanged between primary and secondary coils.

2.6 Fleming's Left Hand and Right Hand Rules

Fleming's Left Hand Rule (FLHR) — Motor Action

Fleming's Left Hand Rule is used to determine the direction of force (motion) experienced by a current-carrying conductor placed in a magnetic field. It is applied in the study of electric motors.

Hold the left hand with the thumb, index finger, and middle finger mutually perpendicular to each other:

- **Index Finger (Forefinger):** Points in the direction of the magnetic Field (B) — North to South.
- **Middle Finger:** Points in the direction of the Current (I) in the conductor.
- **Thumb:** Points in the direction of the Thrust/Motion (Force, F) on the conductor.

Memory Aid: FBI — Forefinger = Field, Middle = current (I), Thumb = motion

- **Application:** DC motors, moving coil instruments (galvanometers).
- The force is given by: $F = B I L \sin\theta$

Fleming's Right Hand Rule (FRHR) — Generator Action

Fleming's Right Hand Rule is used to determine the direction of induced EMF (and hence current) in a conductor moving through a magnetic field. It is applied in the study of electric generators.

Hold the right hand with the thumb, index finger, and middle finger mutually perpendicular to each other:

- **Index Finger (Forefinger):** Points in the direction of the magnetic Field (B).
- **Thumb:** Points in the direction of Motion of the conductor.
- **Middle Finger:** Points in the direction of induced EMF (Current direction).

Memory Aid: TheMF — Thumb = Motion, Forefinger = Field, Middle = EMF

- **Application:** AC generators (alternators), DC generators.

Comparison: FLHR vs FRHR

- FLHR → LEFT hand → Motor (current-in → force-out)
- FRHR → RIGHT hand → Generator (motion-in → EMF-out)
- Both rules follow from the interaction of magnetic fields described by Maxwell's equations.
- The rules are consistent with the Lorentz force law: $F = q (v \times B)$.

2.7 Solved Problems

Problem 1: Calculate the current

A charge of 60 Coulombs passes through a conductor in 5 seconds. Find the current.

- Given: $Q = 60$ C, $t = 5$ s

$$I = Q / t = 60 / 5 = 12\ \text{A}$$

- Answer: Current $I = 12$ Ampere

Problem 2: Power in a circuit

A resistor of 10 Ω is connected across a 50 V supply. Calculate the power consumed.

- Given: $R = 10\ \Omega$, $V = 50$ V

$$P = V^2 / R = (50)^2 / 10 = 2500 / 10 = 250\ \text{W}$$

- Answer: Power $P = 250$ Watts

Problem 3: Induced EMF

A coil of 200 turns has a magnetic flux changing from 5 mWb to 25 mWb in 0.02 seconds. Find the induced EMF.

- Given: $N = 200$ turns, $d\Phi = (25 - 5) \times 10^{-3} = 20 \times 10^{-3}$ Wb, $dt = 0.02$ s

$$e = -N \times (d\Phi/dt) = -200 \times (20 \times 10^{-3} / 0.02) = -200 \times 1 = -200\ \text{V}$$

- Answer: Induced EMF = 200 V (magnitude)

Problem 4: Energy stored in an inductor

An inductor of 5 H carries a current of 4 A. Calculate the energy stored in its magnetic field.

- Given: $L = 5$ H, $I = 4$ A

$$W = \tfrac{1}{2} L I^2 = \tfrac{1}{2} \times 5 \times (4)^2 = \tfrac{1}{2} \times 5 \times 16 = 40\ \text{J}$$

- Answer: Energy stored $W = 40$ Joules

Problem 5: Reluctance of a magnetic circuit

A toroid has mean length 0.5 m and cross-sectional area 10 cm². The material has relative permeability $\mu_r = 1000$. Find the reluctance.

- Given: $l = 0.5$ m, $A = 10 \times 10^{-4}$ m², $\mu_0 = 4\pi \times 10^{-7}$ H/m, $\mu_r = 1000$

$$S = l / (\mu_0 \times \mu_r \times A) = 0.5 / (4\pi \times 10^{-7} \times 1000 \times 10 \times 10^{-4})$$

$$S \approx 0.5 / (1.2566 \times 10^{-6}) \approx 3.98 \times 10^5\ \text{AT/Wb}$$

- Answer: Reluctance $S \approx 3.98 \times 10^5$ AT/Wb

Problem 6: Mutual Inductance

Two coils have self inductances $L_1 = 9$ H and $L_2 = 4$ H. The coefficient of coupling is 0.6. Find the mutual inductance.

- Given: $L_1 = 9$ H, $L_2 = 4$ H, $k = 0.6$

$$M = k\sqrt{(L_1 \times L_2)} = 0.6 \times \sqrt{(9 \times 4)} = 0.6 \times \sqrt{36} = 0.6 \times 6 = 3.6\ \text{H}$$

- Answer: Mutual Inductance M = 3.6 Henry