NETWORK THEOREMS

Complete Study Notes

Electrical Engineering | Diploma / B.Tech

Topics Covered

1. Mesh Analysis and Node Analysis
2. Star/Delta and Delta/Star Transformation
3. Superposition Theorem
4. Thevenin's Theorem
5. Norton's Theorem
6. Maximum Power Transfer Theorem
7. Related Numerical Problems

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Introduction to Network Theorems

Network theorems are fundamental tools in electrical circuit analysis. They provide systematic methods to simplify complex electrical networks, making it easier to find unknown currents, voltages, and power in various branches of a circuit. Every theorem comes with specific conditions (applicability) and limitations that must be understood before applying them.

The major network theorems studied in this chapter are:

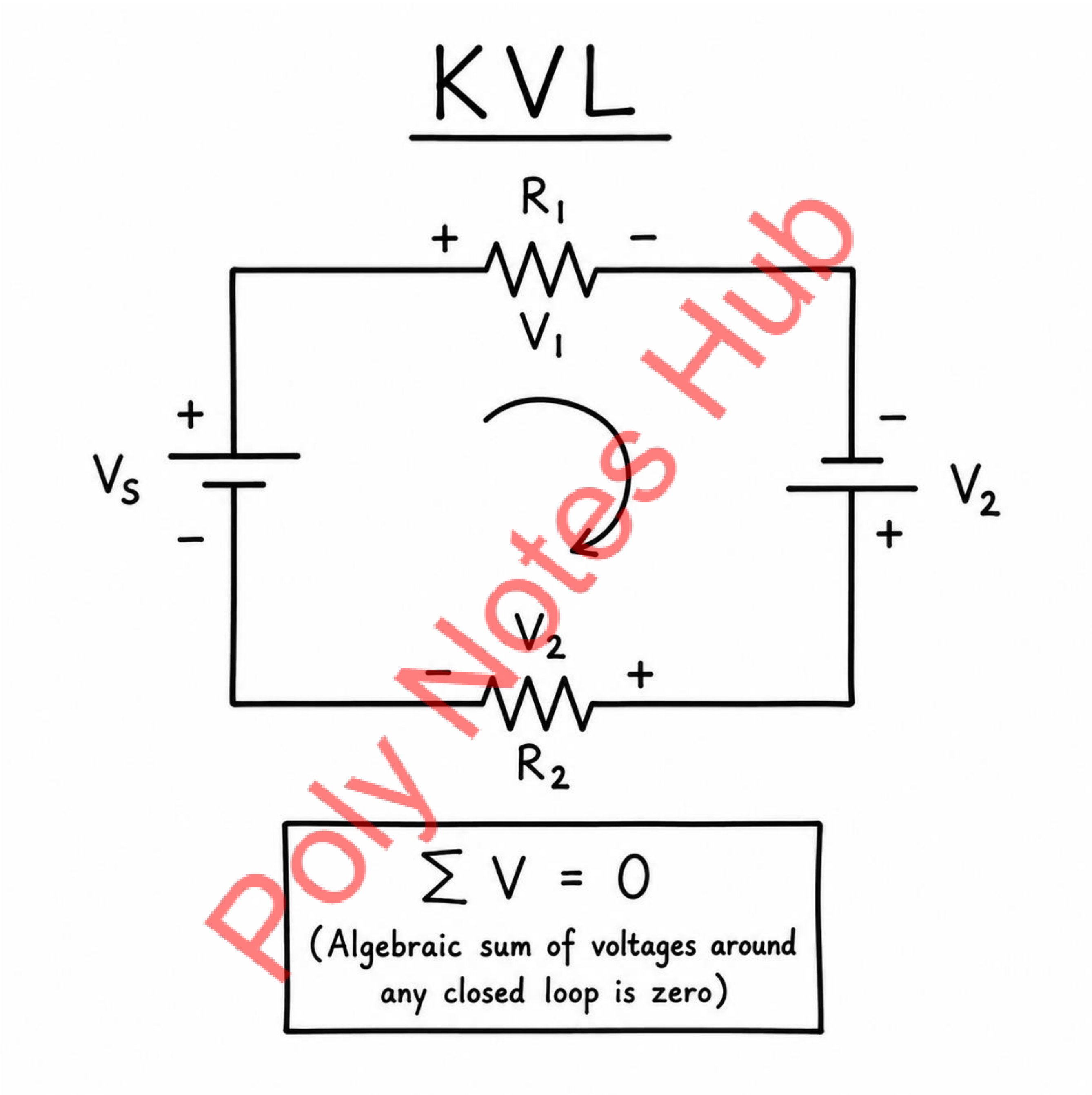
- Mesh Analysis and Node Analysis
- Star/Delta and Delta/Star Transformation
- Superposition Theorem
- Thevenin's Theorem
- Norton's Theorem
- Maximum Power Transfer Theorem

1. Mesh Analysis and Node Analysis

A. Mesh Analysis (Mesh Current Method)

Statement

Mesh Analysis is a method used to determine the current flowing in each mesh (loop) of a planar electrical network. It is based on Kirchhoff's Voltage Law (KVL), which states that the algebraic sum of all voltages around any closed loop in a circuit is zero.



Procedure

Follow these steps to apply Mesh Analysis:

1. Identify all the independent meshes (loops) in the circuit. A mesh is a loop that does not contain any other loop within it.
2. Assign mesh currents I1, I2, I3, ... to each mesh, typically in the clockwise direction.

3. Apply KVL to each mesh: write the voltage equation by summing up all voltage drops and rises. For a shared branch between two meshes, the net current is the algebraic difference of the two mesh currents.
4. Write the resulting simultaneous equations in matrix form (if needed) and solve for the unknown mesh currents using Cramer's rule, substitution, or matrix methods.
5. Once mesh currents are known, find branch currents, voltages, and power as required.

Areas of Application

- Analysis of planar circuits with multiple meshes
- Power system network analysis
- Electronic circuit analysis (transistor amplifiers, filters)
- Finding unknown voltages and currents in complex networks
- Communication circuit analysis

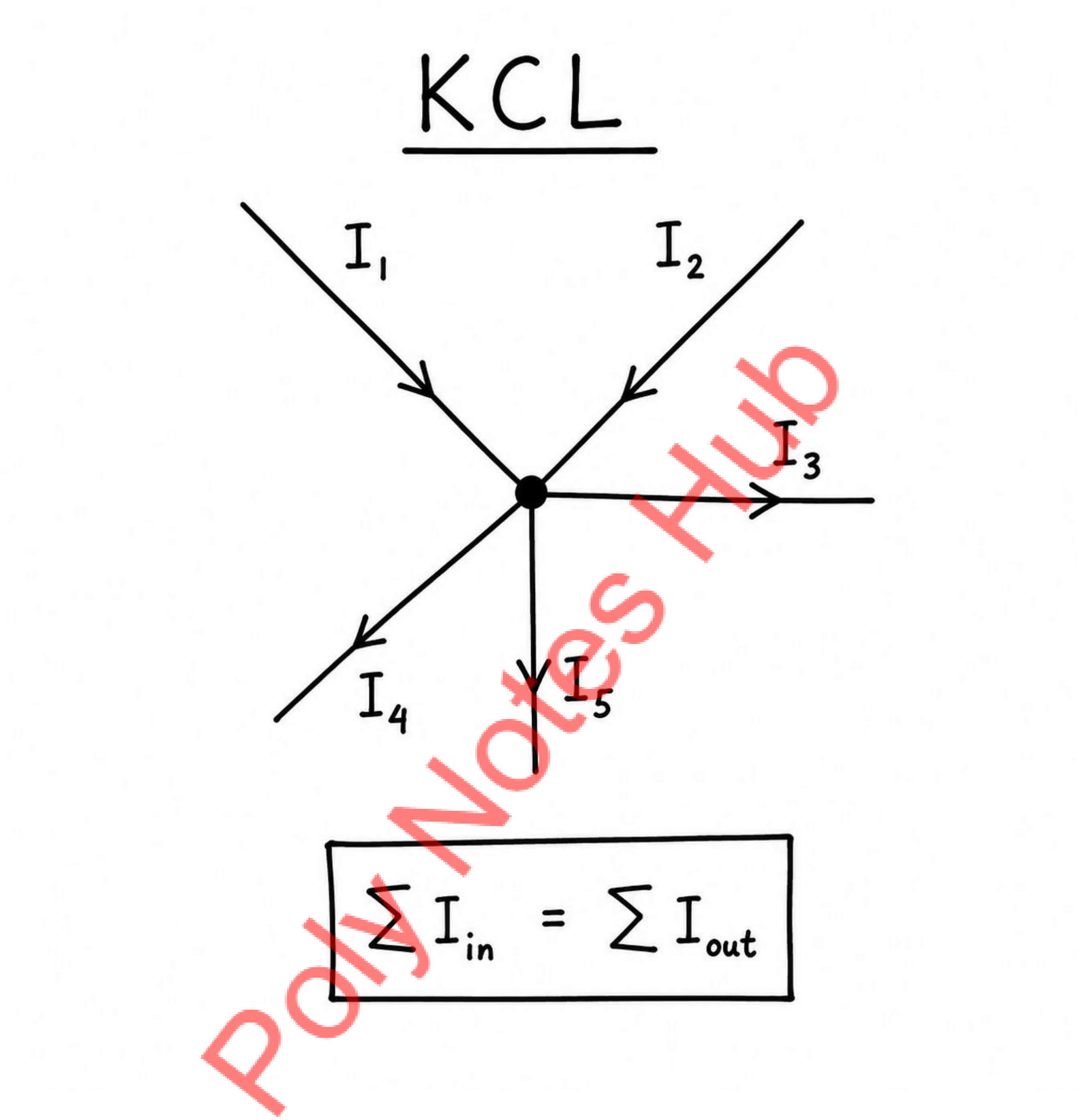
Limitations

- Applicable only to planar networks (circuits that can be drawn on a flat surface without crossings)
- Becomes complex when the number of meshes is very large
- Not directly applicable to circuits with dependent current sources without modification
- Requires the formation and solving of simultaneous linear equations

B. Node Analysis (Nodal Voltage Method)

Statement

Node Analysis is a method based on Kirchhoff's Current Law (KCL), which states that the algebraic sum of all currents entering and leaving a node is zero. In this method, node voltages with respect to a reference node (ground) are taken as the unknowns.



Procedure

6. Identify all nodes in the circuit. Select one node as the reference node (ground, V = 0). The remaining nodes are called independent nodes.
7. Assign voltage variables V1, V2, V3, ... to each independent node with respect to the reference node.
8. Apply KCL at each independent node: equate the sum of currents leaving the node to zero. Express all branch currents in terms of node voltages using Ohm's Law.
9. Solve the resulting simultaneous equations to find the node voltages.

10. Calculate any required branch currents, voltages, or power from the node voltages.

Areas of Application

- Analysis of circuits with many parallel branches
- Suitable for non-planar circuits as well
- Electronic circuit simulation (SPICE uses nodal analysis)
- Power distribution networks
- Analysis of circuits with current sources

Limitations

- Requires selection of a proper reference node for accurate results
- Can be complex for circuits with many nodes
- Voltage sources between two non-reference nodes require the supernode technique
- Less intuitive for circuits with voltage sources compared to mesh analysis

Key Comparison: Mesh vs. Node Analysis

• Mesh Analysis uses KVL; Node Analysis uses KCL
• Mesh is preferred when branch currents are required
• Node Analysis is preferred when node voltages are required
• Node Analysis works on non-planar circuits; Mesh Analysis does not

2. Star/Delta and Delta/Star Transformation

Statement

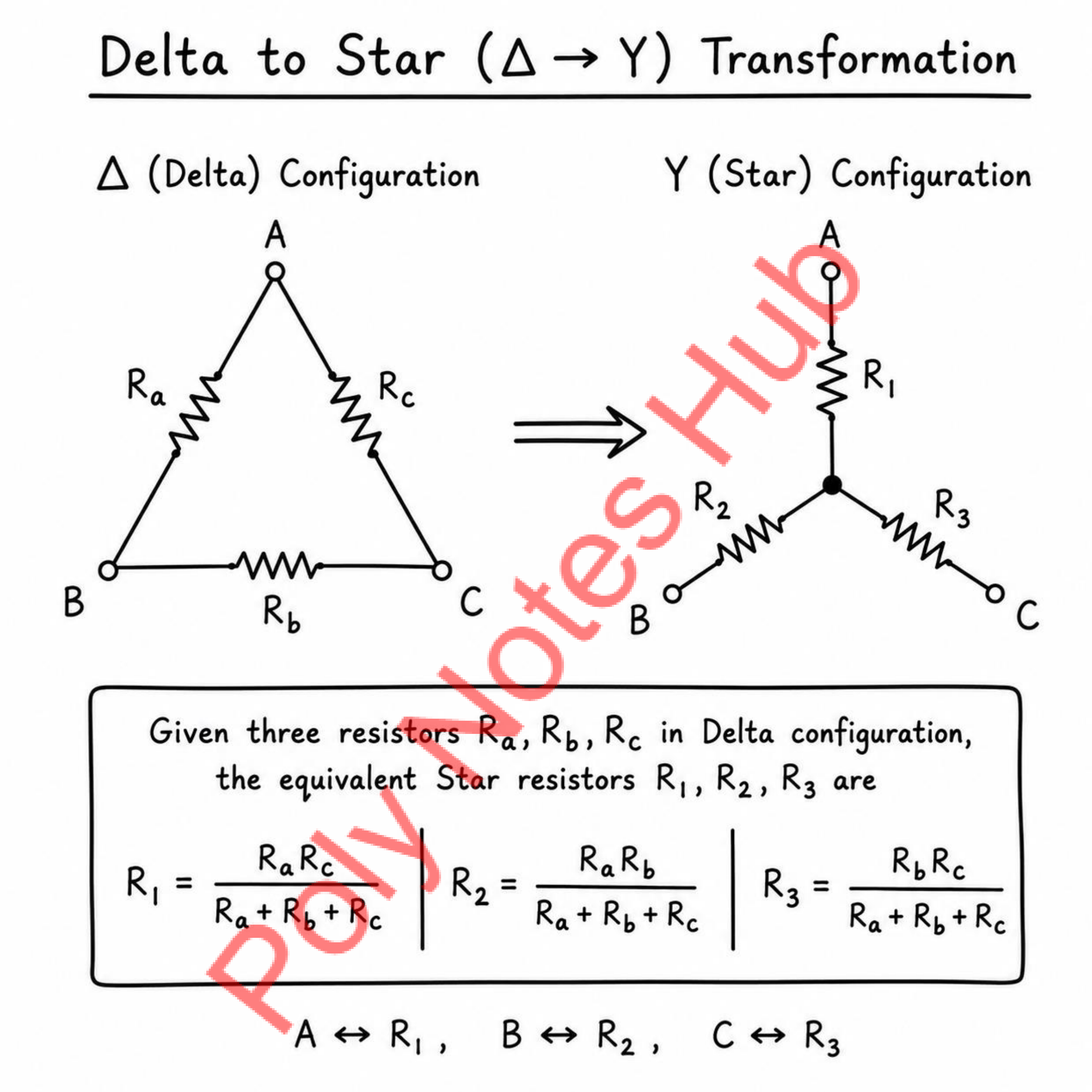
Star (Y) and Delta (Δ) are two common configurations for connecting three resistors (or impedances) in a three-terminal network. The Star-Delta and Delta-Star transformations allow conversion between these two equivalent configurations without changing the external behaviour of the network. These transformations simplify circuit analysis by reducing complex networks.

A. Delta to Star (Δ → Y) Transformation

Given three resistors Ra, Rb, Rc in Delta configuration, the equivalent Star resistors R1, R2, R3 are:

Delta Resistors	Star Equivalent Formula
Ra, Rb, Rc (Delta)	R1 = (Rb × Rc) / (Ra + Rb + Rc)
	R2 = (Ra × Rc) / (Ra + Rb + Rc)

	R3 = (Ra × Rb) / (Ra + Rb + Rc)

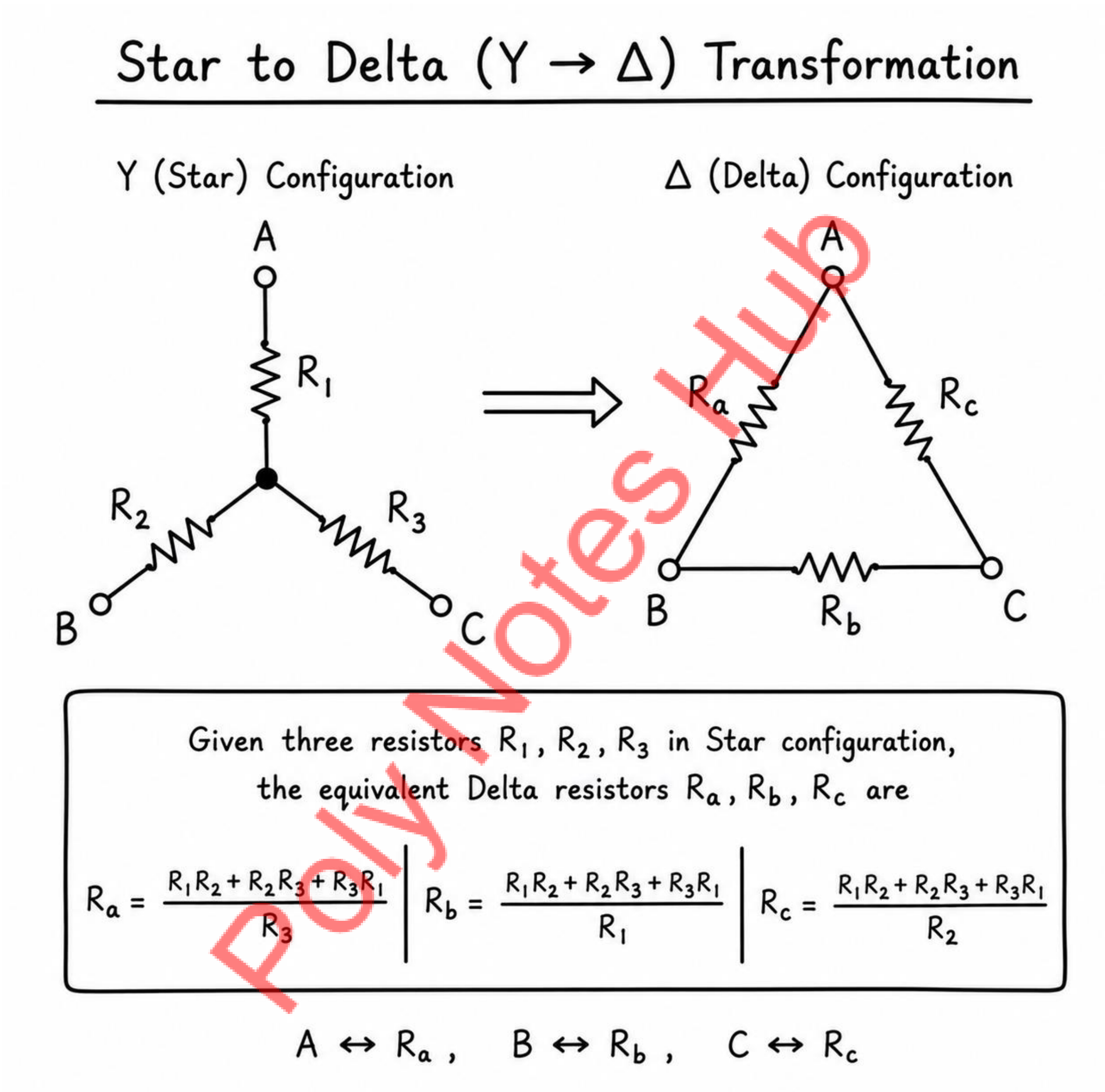


B. Star to Delta (Y → Δ) Transformation

Given three resistors R1, R2, R3 in Star configuration, the equivalent Delta resistors Ra, Rb, Rc are:

Star Resistors	Delta Equivalent Formula
R1, R2, R3 (Star)	Ra = (R1R2 + R2R3 + R3R1) / R1

	Rb = (R1R2 + R2R3 + R3R1) / R2
	Rc = (R1R2 + R2R3 + R3R1) / R3



Areas of Application

- Simplification of bridge circuits (Wheatstone bridge)
- Three-phase power system analysis
- Reducing complex resistor networks to simpler equivalent circuits
- Power electronics and transformer winding analysis

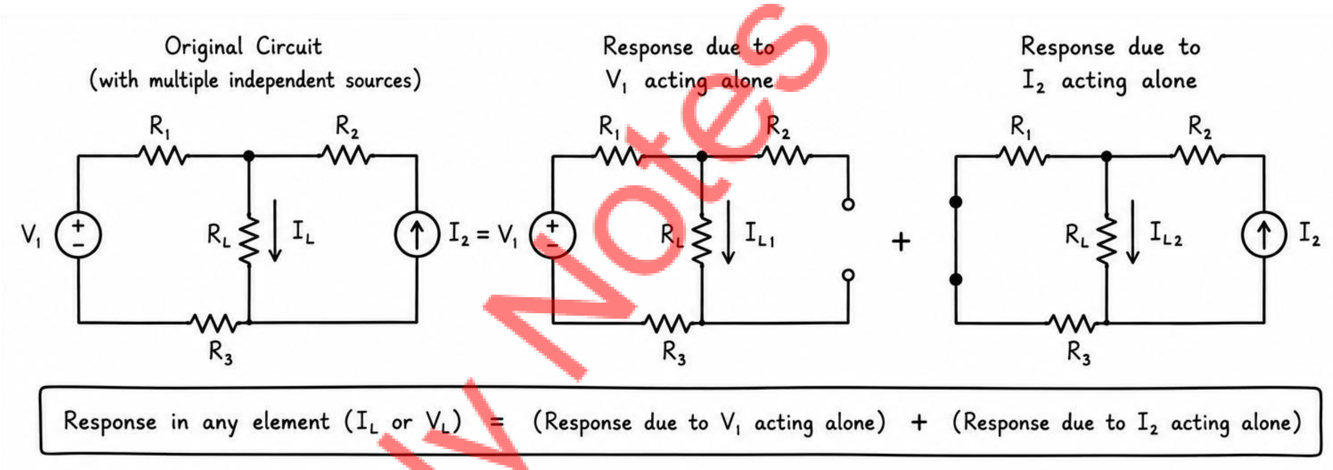
Limitations

- Applicable only to three-terminal networks
- Valid only when all elements are linear (resistors, impedances)
- Does not apply to non-linear or active elements directly
- Transformation changes internal structure — only external behaviour remains equivalent

3. Superposition Theorem

Statement

The Superposition Theorem states that in a linear bilateral network containing more than one independent source, the response (current or voltage) in any element is the algebraic sum of the responses caused by each independent source acting alone, while all other independent sources are replaced by their internal resistances (voltage sources are short-circuited and current sources are open-circuited).



Procedure

11. Identify the number of independent sources in the circuit.
12. Consider each source individually. Deactivate all other sources: replace voltage sources with short circuits (0 V) and current sources with open circuits (0 A).
13. Calculate the response (current or voltage) in the desired element due to the active source alone.
14. Repeat steps 2 and 3 for each source in the network.
15. Algebraically add all individual responses to get the total response. Pay careful attention to the direction/sign of each response.

Areas of Application

- Circuits containing multiple independent sources (AC and DC mixed sources)

- Used to find the contribution of each source individually
- Useful for sensitivity analysis — to check how each source affects circuit behaviour
- Commonly used in communications and signal processing circuits

Limitations

- Applicable only to linear bilateral networks — does not work for non-linear circuits (diodes, transistors in non-linear region)
- Cannot be used to calculate power directly — power is a non-linear quantity ($P = I^2R$); power must be calculated after finding total current/voltage
- Dependent sources are NOT deactivated — they remain active throughout
- More time-consuming for circuits with many sources

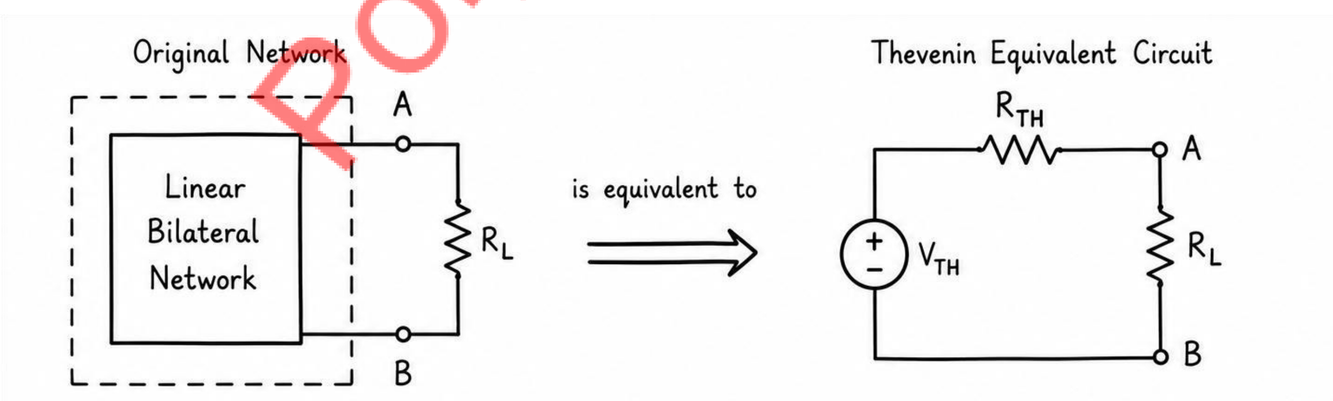
Important Note on Superposition

- Voltage sources → replaced by Short Circuit (wire) when deactivated
- Current sources → replaced by Open Circuit when deactivated
- Dependent sources → NEVER deactivated; always kept active
- Power CANNOT be superposed; only V and I can be superposed

4. Thevenin's Theorem

Statement

Thevenin's Theorem states that any linear, bilateral two-terminal network containing independent and/or dependent sources can be replaced by an equivalent circuit consisting of a single voltage source (Vth) in series with a single resistance (Rth). This equivalent circuit behaves identically to the original circuit as seen from the two terminals.

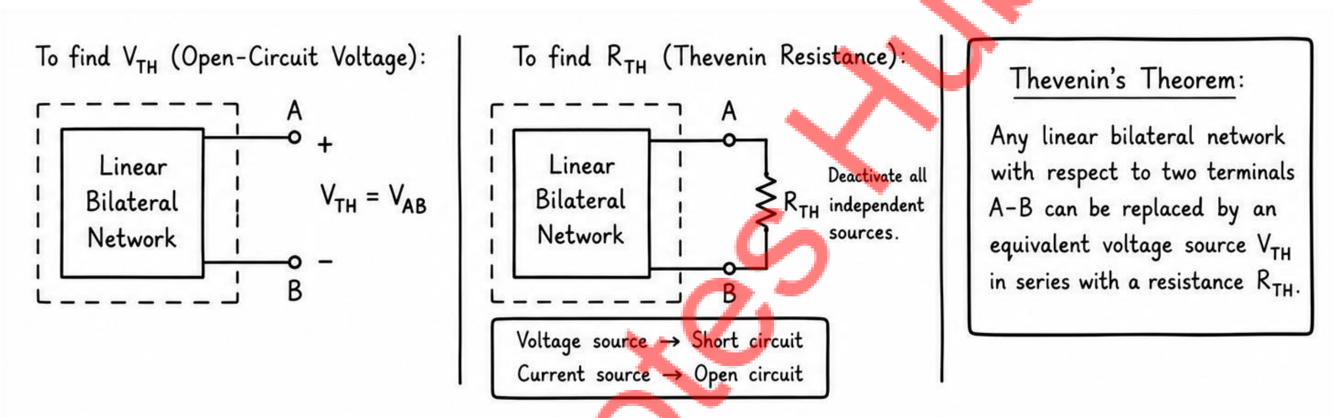


Vth (Thevenin Voltage): Open-circuit voltage across the two terminals

Rth (Thevenin Resistance): Equivalent resistance seen from the terminals with all independent sources deactivated

Procedure

16. Identify and remove the load resistor (RL) from the circuit. Mark the two open terminals as A and B.
17. Find Vth: Calculate the open-circuit voltage across terminals A and B. This is the Thevenin voltage.
18. Find Rth: Deactivate all independent sources (short-circuit voltage sources, open-circuit current sources). Then calculate the equivalent resistance seen from terminals A and B.
19. Draw the Thevenin equivalent circuit: Vth in series with Rth.
20. Reconnect the load resistor RL to the Thevenin equivalent circuit and calculate the desired response.



Areas of Application

- Simplification of complex networks to study the effect of varying load resistance
- Maximum power transfer calculations
- Analysis of electronic circuits: amplifiers, filters, and sensors
- Power system fault analysis
- Useful in circuit design when the same circuit supplies different loads

Limitations

- Applicable only to linear bilateral networks
- Not applicable to circuits with non-linear elements directly
- The theorem simplifies the circuit as seen from two terminals only — internal structure changes
- For circuits with dependent sources, Rth must be found using external test source method (Vx/Ix)
- Cannot be applied to the elements inside the network itself — only external load analysis

Thevenin Equivalent Summary

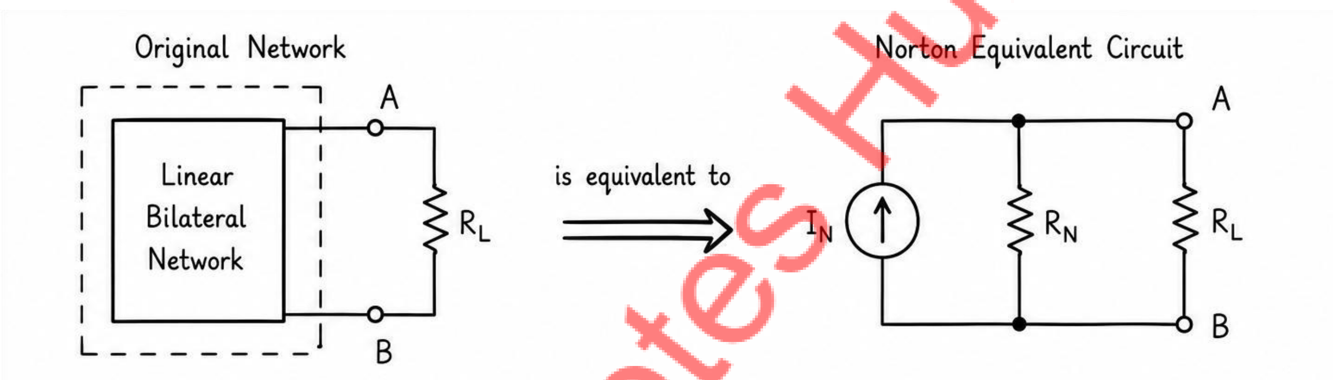
• Vth = Open circuit voltage at terminals A-B
• Rth = Resistance seen from A-B with all independent sources deactivated

• Final circuit: Vth (series) Rth (series) Load RL
• Load current IL = Vth / (Rth + RL)

5. Norton's Theorem

Statement

Norton's Theorem states that any linear, bilateral two-terminal network can be replaced by an equivalent circuit consisting of a single current source (IN) in parallel with a single resistance (RN). The Norton current IN is the short-circuit current flowing between the two terminals, and RN is the same as the Thevenin resistance Rth.



IN (Norton Current): Short-circuit current flowing between terminals A and B

RN (Norton Resistance): Same as Rth — equivalent resistance seen from the terminals with all independent sources deactivated

Procedure

21. Remove the load resistor RL from the circuit. Mark the terminals as A and B.
22. Short-circuit the terminals A and B (connect a wire between them).
23. Find IN: Calculate the current flowing through the short circuit wire — this is the Norton current.
24. Find RN: Deactivate all independent sources and calculate the equivalent resistance seen from terminals A and B. (RN = Rth)
25. Draw the Norton equivalent: IN (current source) in parallel with RN.
26. Reconnect the load RL in parallel and calculate required values.

Relationship between Thevenin and Norton

Thevenin and Norton equivalent circuits are interchangeable using source transformation:

- Vth = IN × RN
- IN = Vth / Rth

- RN = Rth

Areas of Application

- Simplification of circuits for current source-based analysis
- Useful when multiple loads are connected in parallel
- Transistor circuit analysis (Norton model of BJT)
- Power electronics and signal processing circuits

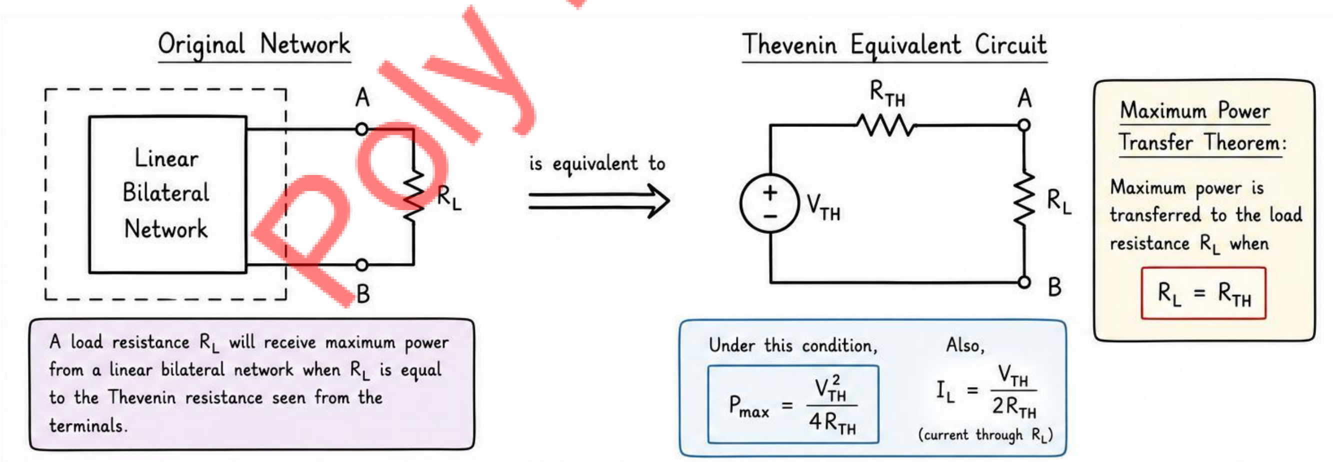
Limitations

- Applicable only to linear bilateral networks
- Non-linear elements (diodes, transistors in saturation) cannot be handled directly
- For dependent source circuits, IN is found by shorting terminals and computing current; RN via test-source method
- Internal structure is simplified — only the terminal behaviour is preserved

6. Maximum Power Transfer Theorem

Statement

The Maximum Power Transfer Theorem states that, in a linear network, maximum power is transferred from the source to the load when the load resistance (RL) is equal to the Thevenin's equivalent resistance (Rth) of the source network as seen from the load terminals.



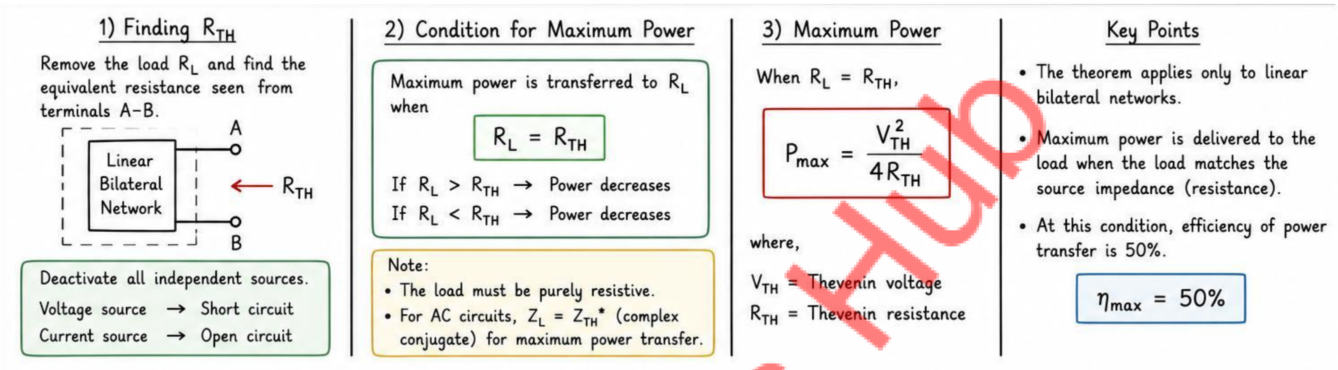
Condition for Maximum Power Transfer:

- RL = Rth (for DC circuits)
- ZL = Zth* (conjugate) for AC circuits — load impedance must equal the complex conjugate of source impedance

Derivation (Key Result)

For a Thevenin equivalent circuit with Vth and Rth:

- Load Current: IL = Vth / (Rth + RL)
- Power in load: PL = IL^2 × RL = Vth^2 × RL / $(Rth + RL)^2$
- Maximum power occurs when dPL/dRL = 0, which gives RL = Rth
- Maximum Power: Pmax = Vth^2 / (4 × Rth)
- Efficiency at maximum power transfer = 50% (half the source power is dissipated in Rth)



Procedure

27. Find the Thevenin equivalent (Vth and Rth) of the source network.
28. Set RL = Rth for maximum power transfer.
29. Calculate maximum power using: Pmax = Vth^2 / (4 × Rth)

Areas of Application

- Audio amplifier and speaker design — matching speaker impedance to amplifier output impedance
- Radio frequency (RF) and antenna design for maximum signal power
- Communication systems — matching transmission line impedance
- Power electronics and battery-powered circuit design
- Sensor circuits and instrumentation amplifiers

Limitations

- At maximum power transfer, efficiency is only 50% — not suitable for power transmission systems where high efficiency is required
- Applicable only to linear networks
- In AC circuits, requires complex conjugate matching which is more difficult to achieve
- The theorem gives the condition for maximum power, not maximum efficiency

Maximum Power Transfer Summary

- Condition: $R_L = R_{th}$ (DC) | $Z_L = Z_{th}^*$ (AC)
- Maximum Power: $P_{max} = V_{th}^2 / (4 \times R_{th})$
- Efficiency at Pmax = 50%
- Used in communication & audio systems, NOT in power transmission

7. Related Numerical Problems

The following solved examples illustrate the application of the theorems studied in this chapter. Understand each step carefully before attempting similar problems.

Problem 1 — Mesh Analysis

A circuit has two meshes. Mesh 1: 10V source, 2Ω and 4Ω resistors. Mesh 2: 5Ω and 3Ω resistors, no source. The 4Ω resistor is shared between both meshes.

Solution

Let mesh currents be I1 (Mesh 1) and I2 (Mesh 2).

30. Apply KVL to Mesh 1: $10 = 2I_1 + 4(I_1 - I_2) \rightarrow 10 = 6I_1 - 4I_2$ ...(i)
31. Apply KVL to Mesh 2: $0 = 5I_2 + 3I_2 + 4(I_2 - I_1) \rightarrow 0 = -4I_1 + 12I_2$ ...(ii)
32. From (ii): $I_1 = 3I_2$. Substituting in (i): $10 = 6(3I_2) - 4I_2 = 14I_2 \rightarrow I_2 = 5/7$ A
33. Therefore: $I_1 = 15/7$ A ≈ 2.14 A, $I_2 \approx 0.71$ A

Problem 2 — Star/Delta Transformation

Three resistors in Delta: Ra = 12Ω, Rb = 6Ω, Rc = 4Ω. Find the equivalent Star resistors.

Solution

34. $R_1 = (R_b \times R_c) / (R_a + R_b + R_c) = (6 \times 4) / (12 + 6 + 4) = 24/22 = 1.09\ \Omega$
35. $R_2 = (R_a \times R_c) / (R_a + R_b + R_c) = (12 \times 4) / 22 = 48/22 = 2.18\ \Omega$
36. $R_3 = (R_a \times R_b) / (R_a + R_b + R_c) = (12 \times 6) / 22 = 72/22 = 3.27\ \Omega$

Problem 3 — Superposition Theorem

A circuit has two voltage sources: V1 = 10V and V2 = 6V, with resistors R1 = 2Ω, R2 = 4Ω, R3 = 6Ω connected in a loop. Find the current through R3.

Solution (Using Superposition)

37. Consider V1 alone (short V2): I'3 = 10 / (2 + (4||6)) = 10 / (2 + 2.4) = 10/4.4 = 2.27 A. Current through R3 = 2.27 × 4/(4+6) = 0.91 A (↑)
38. Consider V2 alone (short V1): I''3 = 6 / (4 + (2||6)) = 6 / (4 + 1.5) = 6/5.5 = 1.09 A. Current through R3 = 1.09 × 2/(2+6) = 0.27 A (↓)
39. Total I3 = 0.91 − 0.27 = 0.64 A (in the direction of V1's contribution)

Problem 4 — Thevenin's Theorem

Find the Thevenin equivalent of a circuit across load terminals A-B. The circuit has a 12V source, R1 = 4Ω in series, and R2 = 8Ω in parallel with the terminal.

Solution

40. Find Vth (open circuit voltage): With RL removed, Vth = 12 × 8/(4+8) = 12 × 0.667 = 8 V
41. Find Rth (deactivate 12V source — short circuit): Rth = R1 || R2 = (4 × 8)/(4+8) = 32/12 = 2.67 Ω
42. Thevenin Equivalent: Vth = 8V in series with Rth = 2.67 Ω

Problem 5 — Maximum Power Transfer

Using the Thevenin equivalent from Problem 4 (Vth = 8V, Rth = 2.67 Ω), find the value of RL for maximum power transfer and the maximum power transferred.

Solution

43. For maximum power transfer: RL = Rth = 2.67 Ω
44. Maximum power: Pmax = Vth^2 / (4 × Rth) = $(8)^2$ / (4 × 2.67) = 64 / 10.68 = 5.99 W ≈ 6 W
45. Efficiency at this condition = 50%

Quick Revision Summary Table

Theorem	Basis/Law	Key Condition	Limitation
Mesh Analysis	KVL	Planar circuits only	Large mesh count = complex
Node Analysis	KCL	Reference node required	Voltage sources → supernode
Star/Delta	Equivalence	3-terminal network only	Linear elements only
Superposition	Linearity	One source at a time	Not for power directly
Thevenin's	Equivalent circuit	Linear bilateral network	Dependent src: test method
Norton's	Equivalent circuit	RL = Rth (max power)	Same as Thevenin
Max Power Xfer	Calculus (dP/dRL=0)	RL = Rth	Efficiency only 50%

— End of Chapter: Network Theorems —

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