Compound Angles, Multiple Angles & Sub-multiple Angles

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1. Introduction

A compound angle is an angle formed by the algebraic sum (or difference) of two or more angles, such as (A+B) or (A−B). A multiple angle is an angle that is an integer multiple of another angle, such as 2A or 3A, expressed in terms of the trigonometric ratios of A. A sub-multiple (or half) angle expresses the ratios of A in terms of A/2. As per the WBSCTE syllabus, the compound angle formulas are stated WITHOUT PROOF, and the multiple/sub-multiple angle formulas are derived directly from them. This topic is extremely important, since it forms the basis of nearly all further trigonometric simplification and equation-solving.

2. Compound Angle Formulas (Without Proof)

- **$\sin(A+B) = \sin A\cdot\cos B + \cos A\cdot\sin B$**
- **$\sin(A-B) = \sin A\cdot\cos B - \cos A\cdot\sin B$**
- **$\cos(A+B) = \cos A\cdot\cos B - \sin A\cdot\sin B$**
- **$\cos(A-B) = \cos A\cdot\cos B + \sin A\cdot\sin B$**
- **$\tan(A+B) = (\tan A+\tan B) / (1-\tan A\cdot\tan B)$**
- **$\tan(A-B) = (\tan A-\tan B) / (1+\tan A\cdot\tan B)$**

3. Multiple Angle Formulas (2A and 3A)

These are derived by putting B=A in the compound angle formulas:

- **$\sin 2A = 2\cdot\sin A\cdot\cos A$**
- **$\cos 2A = \cos^2 A-\sin^2 A = 2\cos^2 A-1 = 1-2\sin^2 A$**
- **$\tan 2A = 2\tan A / (1-\tan^2 A)$**
- **$\sin 3A = 3\sin A - 4\sin^3 A$**
- **$\cos 3A = 4\cos^3 A - 3\cos A$**
- **$\tan 3A = (3\tan A-\tan^3 A) / (1-3\tan^2 A)$**

4. Sub-multiple (Half) Angle Formulas

These are the same formulas as above, with A replaced by A/2 (and 2A replaced by A):

- **$\sin A = 2\cdot\sin(A/2)\cdot\cos(A/2)$**
- **$\cos A = \cos^2(A/2)-\sin^2(A/2) = 2\cos^2(A/2)-1 = 1-2\sin^2(A/2)$**
- **$\tan A = 2\tan(A/2) / (1-\tan^2(A/2))$**

Useful rearranged forms for solving directly for sin(A/2) or cos(A/2):

- **$\sin(A/2) = \sqrt{(1-\cos A)/2}$**
- **$\cos(A/2) = \sqrt{(1+\cos A)/2}$**

5. Important Points for Exams

- Standard angles like 15°, 75°, 105° are always evaluated by writing them as a sum/difference of 45°, 30°, or 60° (e.g. 75°=45°+30°, 15°=45°−30°).
- The three forms of cos2A are interchangeable — pick whichever form matches the given data (in terms of sinA only, cosA only, or both).
- For half-angle problems, always check the sign (+ or −) of sin(A/2), cos(A/2) based on which quadrant A/2 lies in before taking the square root.
- Compound and multiple angle formulas are the building blocks for proving more complex trigonometric identities — practice recognising when an expression like $4\sin A\cos^2 A$ can be rewritten using sin3A+sinA.
- Always double check triple-angle answers using a known standard angle (e.g. A=30° so 3A=90°) as a quick verification, as shown in the worked examples.

6. 30 Important Questions & Answers (With Step-by-Step Solutions)

The following questions are compiled based on the WBSCTE Mathematics I syllabus for this topic (Compound Angles, Multiple Angles, and Sub-multiple Angles) and repeatedly tested question patterns seen in external examination papers. Practising all of these thoroughly gives very strong exam coverage.

Q1. State the compound angle formula for sin(A+B).

[Core formula — direct recall — asked almost every year]

Solution:

This is a standard compound angle formula (stated without proof, as per syllabus):

$\sin(A+B) = \sin A\cdot\cos B + \cos A\cdot\sin B$.

Answer: $\sin(A+B) = \sin A\cdot\cos B + \cos A\cdot\sin B$

Q2. State the compound angle formula for cos(A−B).

[Core formula — direct recall]

Solution:

This is a standard compound angle formula (stated without proof, as per syllabus):

$\cos(A-B) = \cos A\cdot\cos B + \sin A\cdot\sin B$.

Answer: $\cos(A-B) = \cos A\cdot\cos B + \sin A\cdot\sin B$

Q3. Using the compound angle formula, find the value of sin75°.

[Standard application (45°+30°) — very important]

Solution:

Write $75° = 45°+30°$. $\sin 75° = \sin(45°+30°) = \sin 45°\cos 30° + \cos 45°\sin 30°$.

$= (1/\sqrt{2})(\sqrt{3}/2) + (1/\sqrt{2})(1/2) = (\sqrt{3}+1)/(2\sqrt{2})$.

Rationalizing: $= (\sqrt{3}+1)\sqrt{2}/4 = (\sqrt{6}+\sqrt{2})/4$.

Answer: $\sin 75° = (\sqrt{6}+\sqrt{2})/4$

Q4. Using the compound angle formula, find the value of cos15°.

[Standard application (45°−30°) — very important]

Solution:

Write $15° = 45°-30°$. $\cos 15° = \cos(45°-30°) = \cos 45°\cos 30° + \sin 45°\sin 30°$.

$= (1/\sqrt{2})(\sqrt{3}/2) + (1/\sqrt{2})(1/2) = (\sqrt{3}+1)/(2\sqrt{2})$.

Rationalizing: $= (\sqrt{6}+\sqrt{2})/4$.

Answer: $\cos 15° = (\sqrt{6}+\sqrt{2})/4$

Q5. Find the value of tan15°.

[Standard application — important]

Solution:

$\tan 15° = \tan(45°-30°) = (\tan 45°-\tan 30°)/(1+\tan 45°\cdot\tan 30°)$.

$= (1-1/\sqrt{3})/(1+1/\sqrt{3})$. Multiplying numerator and denominator by $\sqrt{3}$: $= (\sqrt{3}-1)/(\sqrt{3}+1)$.

Rationalizing by multiplying by $(\sqrt{3}-1)/(\sqrt{3}-1)$: $= (\sqrt{3}-1)^2/((\sqrt{3})^2-1^2) = (4-2\sqrt{3})/2 = 2-\sqrt{3}$.

Answer: $\tan 15° = 2 - \sqrt{3}$

Q6. Find the value of sin105°.

[Standard application (60°+45°) — common]

Solution:

Write $105° = 60°+45°$. $\sin 105° = \sin 60°\cos 45° + \cos 60°\sin 45°$.

$= (\sqrt{3}/2)(1/\sqrt{2}) + (1/2)(1/\sqrt{2}) = (\sqrt{3}+1)/(2\sqrt{2})$.

$= (\sqrt{6}+\sqrt{2})/4$.

Answer: $\sin 105° = (\sqrt{6}+\sqrt{2})/4$

Q7. Find the value of cos75°.

[Standard application (45°+30°) — common]

Solution:

$\cos 75° = \cos(45°+30°) = \cos 45°\cos 30° - \sin 45°\sin 30°$.

$= (1/\sqrt{2})(\sqrt{3}/2) - (1/\sqrt{2})(1/2) = (\sqrt{3}-1)/(2\sqrt{2})$.

$= (\sqrt{6}-\sqrt{2})/4$.

Answer: $\cos 75° = (\sqrt{6}-\sqrt{2})/4$

Q8. Prove that $\sin(A+B)\cdot\sin(A-B) = \sin^2 A - \sin^2 B$.

[Standard identity proof — very important]

Solution:

$\sin(A+B)\sin(A-B) = (\sin A\cos B+\cos A\sin B)(\sin A\cos B-\cos A\sin B) = \sin^2 A\cos^2 B - \cos^2 A\sin^2 B$.

$= \sin^2 A(1-\sin^2 B) - (1-\sin^2 A)\sin^2 B$ [using $\cos^2=1-\sin^2$]

$= \sin^2 A - \sin^2 A\sin^2 B - \sin^2 B + \sin^2 A\sin^2 B$.

$= \sin^2 A - \sin^2 B$. (Proved)

Answer: $\sin(A+B)\sin(A-B) = \sin^2 A - \sin^2 B$ (proved)

Q9. Prove that $\cos(A+B)\cdot\cos(A-B) = \cos^2 A - \sin^2 B$.

[Standard identity proof — very important]

Solution:

$\cos(A+B)\cos(A-B) = (\cos A\cos B-\sin A\sin B)(\cos A\cos B+\sin A\sin B) = \cos^2 A\cos^2 B - \sin^2 A\sin^2 B$.

$= \cos^2A(1-\sin^2B) - (1-\cos^2A)\sin^2B.$

$= \cos^2A - \cos^2A\sin^2B - \sin^2B + \cos^2A\sin^2B.$

$= \cos^2A - \sin^2B.$ (Proved)

Answer: $\cos(A+B)\cos(A-B) = \cos^2A - \sin^2B$ (proved)

Q10. If $\sin A=3/5$ and $\cos B=12/13$, where A and B are acute, find $\sin(A+B)$.

[Classic important application question]

Solution:

Since A is acute: $\cos A = 4/5$. Since B is acute: $\sin B = 5/13$.

$\sin(A+B) = \sin A\cdot\cos B + \cos A\cdot\sin B = (3/5)(12/13) + (4/5)(5/13).$

$= 36/65 + 20/65 = 56/65.$

Answer: $\sin(A+B) = 56/65$

Q11. If $\sin A=3/5$ and $\cos B=12/13$, where A and B are acute, find $\cos(A+B)$.

[Classic important application question]

Solution:

$\cos A = 4/5$, $\sin B = 5/13$ (as found from the acute-angle triangles).

$\cos(A+B) = \cos A\cdot\cos B - \sin A\cdot\sin B = (4/5)(12/13) - (3/5)(5/13).$

$= 48/65 - 15/65 = 33/65.$

Answer: $\cos(A+B) = 33/65$

Q12. If $\sin A=8/17$ and $\cos B=3/5$, where A and B are acute, find $\sin(A-B)$.

[Classic important application question]

Solution:

$\cos A = 15/17$, $\sin B = 4/5$ (from the acute-angle triangles).

$\sin(A-B) = \sin A\cdot\cos B - \cos A\cdot\sin B = (8/17)(3/5) - (15/17)(4/5).$

$= 24/85 - 60/85 = -36/85.$

Answer: $\sin(A-B) = -36/85$

Q13. State the double-angle formula for $\sin 2A$ in terms of $\sin A$ and $\cos A$.

[Core multiple-angle formula]

Solution:

$\sin 2A = 2\cdot\sin A\cdot\cos A.$

Answer: $\sin 2A = 2\sin A\cdot\cos A$

Q14. State the three equivalent forms of $\cos 2A$.

[Core multiple-angle formula — very important]

Solution:

$\cos 2A = \cos^2 A - \sin^2 A$.

$= 2\cos^2 A - 1$ [using $\sin^2 A = 1 - \cos^2 A$]

$= 1 - 2\sin^2 A$ [using $\cos^2 A = 1 - \sin^2 A$]

Answer: $\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$

Q15. State the double-angle formula for tan2A.

[Core multiple-angle formula]

Solution:

$\tan 2A = 2\tan A / (1 - \tan^2 A)$.

Answer: $\tan 2A = 2\tan A/(1 - \tan^2 A)$

Q16. If sinA=3/5 and A is acute, find sin2A and cos2A.

[Very common application question]

Solution:

$\cos A = 4/5$ (since A is acute).

$\sin 2A = 2\sin A\cos A = 2(3/5)(4/5) = 24/25$.

$\cos 2A = 1 - 2\sin^2 A = 1 - 2(9/25) = 1 - 18/25 = 7/25$.

Answer: $\sin 2A = 24/25$, $\cos 2A = 7/25$

Q17. If cosA=5/13 and A is acute, find sin2A and cos2A.

[Very common application question]

Solution:

$\sin A = 12/13$ (since A is acute).

$\sin 2A = 2\sin A\cos A = 2(12/13)(5/13) = 120/169$.

$\cos 2A = 2\cos^2 A - 1 = 2(25/169) - 1 = 50/169 - 1 = -119/169$.

Answer: $\sin 2A = 120/169$, $\cos 2A = -119/169$

Q18. If tanA=3/4, find the value of tan2A.

[Common application question]

Solution:

$\tan 2A = 2\tan A/(1 - \tan^2 A) = 2(3/4) / (1 - (3/4)^2)$.

$= (3/2) / (1 - 9/16) = (3/2) / (7/16)$.

$= (3/2) \times (16/7) = 24/7$.

Answer: $\tan 2A = 24/7$

Q19. State the triple-angle formula for sin3A.

[Core multiple-angle formula — very important]

Solution:

$\sin 3A = 3\sin A - 4\sin^3 A$.

Answer: $\sin 3A = 3\sin A - 4\sin^3 A$

Q20. State the triple-angle formula for cos3A.

[Core multiple-angle formula — very important]

Solution:

$\cos 3A = 4\cos^3 A - 3\cos A$.

Answer: $\cos 3A = 4\cos^3 A - 3\cos A$

Q21. If sinA=1/2, find sin3A using the triple-angle formula, and verify the result.

[Verification-style application question]

Solution:

$\sin 3A = 3\sin A - 4\sin^3 A = 3(1/2) - 4(1/2)^3 = 3/2 - 4(1/8) = 3/2 - 1/2 = 1$.

Verification: since $\sin A=1/2$, $A=30°$, so $3A=90°$, and $\sin 90°=1$. ✓(Matches)

Answer: $\sin 3A = 1$

Q22. If cosA=1/2, find cos3A using the triple-angle formula, and verify the result.

[Verification-style application question]

Solution:

$\cos 3A = 4\cos^3 A - 3\cos A = 4(1/2)^3 - 3(1/2) = 4(1/8) - 3/2 = 1/2 - 3/2 = -1$.

Verification: since $\cos A=1/2$, $A=60°$, so $3A=180°$, and $\cos 180°=-1$. ✓(Matches)

Answer: $\cos 3A = -1$

Q23. State the sub-multiple (half-angle) formula for sinA in terms of A/2.

[Core sub-multiple formula]

Solution:

This follows directly from $\sin 2\theta=2\sin\theta\cos\theta$ by replacing θ with $A/2$:

$\sin A = 2\sin(A/2)\cdot\cos(A/2)$.

Answer: $\sin A = 2\sin(A/2)\cos(A/2)$

Q24. State the three equivalent sub-multiple (half-angle) forms of cosA in terms of A/2.

[Core sub-multiple formula — important]

Solution:

$\cos A = \cos^2(A/2) - \sin^2(A/2)$.

$= 2\cos^2(A/2) - 1$.

$= 1 - 2\sin^2(A/2)$.

Answer: $\cos A = \cos^2(A/2)-\sin^2(A/2) = 2\cos^2(A/2)-1 = 1-2\sin^2(A/2)$

Q25. If cosA=7/25 and A is acute, find sin(A/2) and cos(A/2).

[Classic important application of half-angle formula]

Solution:

Half-angle formulas: $\sin(A/2) = \sqrt{[(1-\cos A)/2]}$, $\cos(A/2) = \sqrt{[(1+\cos A)/2]}$ (taking positive root since A/2 is acute).

$\sin(A/2) = \sqrt{[(1-7/25)/2]} = \sqrt{[(18/25)/2]} = \sqrt{(9/25)} = 3/5$.

$\cos(A/2) = \sqrt{[(1+7/25)/2]} = \sqrt{[(32/25)/2]} = \sqrt{(16/25)} = 4/5$.

Answer: $\sin(A/2) = 3/5$, $\cos(A/2) = 4/5$

Q26. If cosA=1/9 and 0<A<90°, find the value of sin(A/2).

[Half-angle formula application — common]

Solution:

$\sin(A/2) = \sqrt{[(1-\cos A)/2]} = \sqrt{[(1-1/9)/2]} = \sqrt{[(8/9)/2]}$.

$= \sqrt{(4/9)} = 2/3$.

Answer: $\sin(A/2) = 2/3$

Q27. Evaluate: 2·sin15°·cos15°.

[Direct application of sin2A formula — common]

Solution:

Using $\sin 2A=2\sin A\cos A$ with $A=15°$: $2\sin 15°\cos 15° = \sin(2\times 15°) = \sin 30°$.

$= 1/2$.

Answer: 1/2

Q28. Evaluate: cos²15° − sin²15°.

[Direct application of cos2A formula — common]

Solution:

Using $\cos 2A=\cos^2 A-\sin^2 A$ with $A=15°$: $\cos^2 15°-\sin^2 15° = \cos(2\times 15°) = \cos 30°$.

$= \sqrt{3}/2$.

Answer: $\sqrt{3}/2$

Q29. Evaluate: $2\tan 15° / (1-\tan^2 15°)$.

[Direct application of tan2A formula — common]

Solution:

Using $\tan 2A = 2\tan A/(1-\tan^2 A)$ with $A=15°$: this expression = $\tan(2\times 15°) = \tan 30°$.

= $1/\sqrt{3}$.

Answer: $1/\sqrt{3}$

Q30. Prove that $(\sin 3A + \sin A)/(\cos 3A + \cos A) = \tan 2A$.

[Combined multiple-angle identity — classic important closing question]

Solution:

Numerator: $\sin 3A+\sin A = (3\sin A-4\sin^3 A)+\sin A = 4\sin A-4\sin^3 A = 4\sin A(1-\sin^2 A) = 4\sin A\cdot\cos^2 A$.

Denominator: $\cos 3A+\cos A = (4\cos^3 A-3\cos A)+\cos A = 4\cos^3 A-2\cos A = 2\cos A(2\cos^2 A-1) = 2\cos A\cdot\cos 2A$.

Ratio = $4\sin A\cdot\cos^2 A / [2\cos A\cdot\cos 2A] = 2\sin A\cdot\cos A / \cos 2A = \sin 2A/\cos 2A = \tan 2A$. (Proved)

Answer: $(\sin 3A+\sin A)/(\cos 3A+\cos A) = \tan 2A$ (proved)
