WBSCTE — Diploma / Polytechnic, 1st Semester

Mathematics I

Chapter: CO-ORDINATE SYSTEM

---

1. Introduction

The Co-ordinate System is the branch of mathematics that connects algebra with geometry, allowing the position of a point in a plane to be described using numbers. This chapter (Unit 4.1 of the WBSCTE syllabus, under Co-ordinate Geometry & Mensuration) covers the Cartesian and Polar co-ordinate systems, the distance formula, the section formula, and the area of a triangle along with the condition for collinearity of points. These concepts form the foundation for later topics like straight lines, circles, and conic sections, and are tested almost every year in the examination.

2. Cartesian Co-ordinate System

A point in a plane is located using two mutually perpendicular lines: the horizontal x-axis and the vertical y-axis, meeting at the origin O(0,0). Any point is written as an ordered pair (x, y), where x (the abscissa) is the distance from the y-axis, and y (the ordinate) is the distance from the x-axis. The two axes divide the plane into four quadrants:

- Quadrant I: $x > 0, y > 0$
- Quadrant II: $x < 0, y > 0$
- Quadrant III: $x < 0, y < 0$
- Quadrant IV: $x > 0, y < 0$

3. Polar Co-ordinate System

In the polar system, a point is located by its distance r from a fixed point (the pole/origin) and the angle θ made with a fixed line (the polar axis/positive x-axis), written as (r, θ). The relation between Cartesian (x, y) and Polar (r, θ) coordinates is:

- **$x = r\cos\theta$, $y = r\sin\theta$ (Polar → Cartesian)**
- **$r = \surd(x^{2+y^2})$, $\theta = \tan^{-1}(y/x)$ (Cartesian → Polar)**

4. Distance Formula

The distance between two points $A(x_1,y_1)$ and $B(x_2,y_2)$ is given by:

- **$d = \surd[(x_2-x_1)^2 + (y_2-y_1)^2]$**

Distance of a point (x, y) from the origin:

- **$d = \surd(x^{2+y^2})$**

5. Section Formula, Midpoint & Centroid

If a point P(x, y) divides the line segment joining $A(x_1,y_1)$ and $B(x_2,y_2)$ in the ratio m:n:

- **Internal division: $P(x,y) = ((m \cdot x_{2+n} \cdot x_1)/(m+n) , (m \cdot y_{2+n} \cdot y_1)/(m+n))$**
- **External division: $P(x,y) = ((m \cdot x_2 - n \cdot x_1)/(m-n) , (m \cdot y_2 - n \cdot y_1)/(m-n))$**
- **Midpoint (ratio 1:1): $M = ((x_{1+x2})/2 , (y_{1+y2})/2)$**
- **Centroid of a triangle: $G = ((x_{1+x2+x3})/3 , (y_{1+y2+y3})/3)$**

6. Area of a Triangle & Condition for Collinearity

The area of a triangle with vertices $A(x_1,y_1)$, $B(x_2,y_2)$, $C(x_3,y_3)$ is:

- **Area = $(1/2)\ |x_1(y_2-y_3) + x_2(y_3-y_1) + x_3(y_1-y_2)|$**

Three points A, B, C are collinear (lie on the same straight line) if and only if the area of the triangle they form is zero:

- **$x_1(y_2-y_3) + x_2(y_3-y_1) + x_3(y_1-y_2) = 0$**

7. Important Points for Exams

- Always take the square root only at the final step in the distance formula — do not simplify prematurely.
- For 'equidistant point' problems, set up PA = PB and square both sides to remove the square roots, then solve the resulting linear equation.
- Remember: area = 0 is both a way to prove collinearity AND a way to find an unknown coordinate that makes three points collinear.
- For section-formula 'ratio' questions, always assume ratio k:1 and solve using either the x or y coordinate, then verify with the other.
- For quadrilateral proofs (parallelogram, rhombus, square, rectangle), use a strategic combination of distance formula (sides/diagonals) and midpoint (diagonal bisection) — pick whichever proves the property fastest.

8. 30 Important Questions & Answers (With Step-by-Step Solutions)

The following questions are compiled based on the WBSCTE Mathematics I syllabus for the Co-ordinate System chapter and repeatedly tested question patterns seen in external examination papers. Practising all of these thoroughly gives very strong exam coverage.

Q1. Find the distance between the points A(3, 4) and B(7, 1).

[Basic distance formula — asked almost every year]

Solution:

Distance formula: $d = \sqrt{[(x_2-x_1)^2 + (y_2-y_1)^2]}$.

Here $(x_1,y_1)=(3,4)$, $(x_2,y_2)=(7,1)$.

$d = \sqrt{[(7-3)^2 + (1-4)^2]} = \sqrt{[16 + 9]} = \sqrt{25} = 5$.

Answer: AB = 5 units

Q2. Find the distance of the point P(−5, 12) from the origin.

[Basic — distance from origin]

Solution:

Distance from origin: $d = \sqrt{(x^2 + y^2)}$.

$d = \sqrt{[(-5)^2 + 12^2]} = \sqrt{[25 + 144]} = \sqrt{169}$.

$= 13$.

Answer: OP = 13 units

Q3. Show that the points A(0,0), B(5,0), C(5,12) form a right-angled triangle.

[Standard proof-type question using distance formula]

Solution:

$AB = \sqrt{[(5-0)^{2+}(0-0)^2]} = \sqrt{25} = 5$.

$BC = \sqrt{[(5-5)^{2+}(12-0)^2]} = \sqrt{144} = 12$.

$AC = \sqrt{[(5-0)^{2+}(12-0)^2]} = \sqrt{[25+144]} = \sqrt{169} = 13$.

Since $AB^2 + BC^2 = 25+144 = 169 = AC^2$, by the converse of Pythagoras' theorem, the triangle is right-angled at B.

Answer: Right angle at B (proved, since $AB^2 + BC^2 = AC^2$)

Q4. Show that the points A(1,2), B(4,2), C(4,6) form a right-angled triangle, and find its area.

[Combines distance formula with area — common exam pattern]

Solution:

$AB = \sqrt{[(4-1)^{2+}(2-2)^2]} = \sqrt{9} = 3$ (horizontal segment).

$BC = \sqrt{[(4-4)^{2+}(6-2)^2]} = \sqrt{16} = 4$ (vertical segment).

$AC = \sqrt{[(4-1)^{2+}(6-2)^2]} = \sqrt{[9+16]} = \sqrt{25} = 5$.

Since AB^{2+BC2} = 9+16 = 25 = AC^2, the triangle is right-angled at B.

Area = (1/2) × base × height = (1/2) × 3 × 4 = 6 sq. units.

Answer: Right angled at B; Area = 6 sq. units

Q5. Find the point on the x-axis which is equidistant from A(2,3) and B(4,1).

[Classic equidistant-point question]

__Solution:__

Let the required point be (x, 0). Then PA = PB.

$(x-2)^2 + (0-3)^2 = (x-4)^2 + (0-1)^2$.

$x^2-4x+4+9 = x^2-8x+16+1 \implies -4x+13 = -8x+17$.

$4x = 4 \implies x = 1$.

Answer: Point = (1, 0)

Q6. Find the point on the y-axis which is equidistant from A(3,2) and B(−1,4).

[Classic equidistant-point question]

__Solution:__

Let the required point be (0, y). Then PA = PB.

$(0-3)^{2+}(y-2)^2 = (0+1)^{2+}(y-4)^2$.

$9+y^2-4y+4 = 1+y^2-8y+16 \implies 13-4y = 17-8y$.

$4y = 4 \implies y = 1$.

Answer: Point = (0, 1)

Q7. Find the coordinates of the point dividing the line segment joining A(2,3) and B(8,9) internally in the ratio 2:1.

[Section formula (internal division) — very important]

__Solution:__

Internal section formula: $P(x,y) = ((m\cdot x_{2+n}\cdot x_1)/(m+n)\ ,\ (m\cdot y_{2+n}\cdot y_1)/(m+n))$, ratio m:n.

Here m=2, n=1, (x_1,y_1)=(2,3), (x_2,y_2)=(8,9).

x = (2·8+1·2)/3 = 18/3 = 6.   y = (2·9+1·3)/3 = 21/3 = 7.

Answer: Point = (6, 7)

Q8. Find the coordinates of the point dividing the line segment joining A(−3,5) and B(4,−9) externally in the ratio 3:2.

[Section formula (external division) — important]

__Solution:__

External section formula: $P(x,y) = ((m\cdot x_2-n\cdot x_1)/(m-n)\ ,\ (m\cdot y_2-n\cdot y_1)/(m-n))$, ratio m:n.

Here m=3, n=2, (x_1,y_1)=(−3,5), (x_2,y_2)=(4,−9).

$x = (3\cdot4 - 2\cdot(-3))/(3-2) = (12+6)/1 = 18.$

$y = (3\cdot(-9) - 2\cdot5)/(3-2) = (-27-10)/1 = -37.$

Answer: Point = (18, −37)

Q9. Find the midpoint of the line segment joining A(6,−3) and B(−2,7).

[Basic midpoint formula]

Solution:

Midpoint formula: $M = ((x_{1+x2})/2 , (y_{1+y2})/2).$

$M = ((6-2)/2 , (-3+7)/2) = (4/2, 4/2).$

$= (2, 2).$

Answer: Midpoint = (2, 2)

Q10. Find the coordinates of the centroid of the triangle with vertices A(2,3), B(4,−1), C(−3,5).

[Centroid formula — common short question]

Solution:

Centroid formula: $G = ((x_{1+x2+x3})/3 , (y_{1+y2+y3})/3).$

$G = ((2+4-3)/3 , (3-1+5)/3) = (3/3, 7/3).$

$= (1, 7/3).$

Answer: Centroid = (1, 7/3)

Q11. In what ratio does the point P(2,5) divide the line segment joining A(1,3) and B(4,9)?

[Finding the ratio — important reverse-application question]

Solution:

Let P divide AB in the ratio k:1 (from A to B).

x-coordinate: $(4k+1)/(k+1) = 2 \implies 4k+1 = 2k+2 \implies 2k = 1 \implies k = 1/2.$

Check with y: $(9k+3)/(k+1)$ with $k=1/2$ gives $(4.5+3)/1.5 = 7.5/1.5 = 5$, which matches P's y-coordinate. ✓

So the ratio $k:1 = (1/2):1 = 1:2.$

Answer: Ratio = 1 : 2

Q12. Find the area of the triangle with vertices A(1,1), B(4,5), C(6,3).

[Area of triangle — very important, asked every year]

Solution:

$\text{Area} = (1/2)\ |x_1(y_2-y_3) + x_2(y_3-y_1) + x_3(y_1-y_2)|.$

$= (1/2)\ |1(5-3) + 4(3-1) + 6(1-5)|.$

$= (1/2)\ |2 + 8 - 24| = (1/2)|-14| = 7.$

Answer: Area = 7 sq. units

Q13. Show that the points A(1,2), B(3,6), C(5,10) are collinear.

[Collinearity — very important, asked every year]

Solution:

Points are collinear if Area of triangle ABC = 0.

Area = $(1/2)|1(6-10) + 3(10-2) + 5(2-6)|$.

= $(1/2)|-4 + 24 - 20| = (1/2)|0| = 0$.

Since area = 0, the points A, B, C are collinear.

Answer: Area = 0, hence A, B, C are collinear (proved)

Q14. Find the value of k for which the points A(2,3), B(4,k), C(6,−3) are collinear.

[Classic important long-answer question]

Solution:

For collinear points: $x_1(y_2-y_3) + x_2(y_3-y_1) + x_3(y_1-y_2) = 0$.

$2(k-(-3)) + 4(-3-3) + 6(3-k) = 0$.

$2k+6 - 24 + 18-6k = 0 \implies -4k + 0 = 0$.

$k = 0$.

Answer: k = 0

Q15. Find the area of the quadrilateral with vertices A(0,0), B(4,0), C(4,3), D(0,3) using the coordinate (shoelace) method.

[Area of quadrilateral — application question]

Solution:

Shoelace formula: Area = $(1/2)|x_1y_2-x_2y_1 + x_2y_3-x_3y_2 + x_3y_4-x_4y_3 + x_4y_1-x_1y_4|$.

= $(1/2)|(0\cdot0-4\cdot0) + (4\cdot3-4\cdot0) + (4\cdot3-0\cdot3) + (0\cdot0-0\cdot3)|$.

= $(1/2)|0+12+12+0| = (1/2)(24) = 12$.

Answer: Area = 12 sq. units

Q16. Prove that the points A(2,1), B(0,3), C(−2,1), D(0,−1) are the vertices of a square.

[Standard proof-type question — properties of quadrilaterals]

Solution:

$AB=\sqrt{[(2-0)^2+(1-3)^2]}=\sqrt{8}=2\sqrt{2}$. $BC=\sqrt{[(0+2)^2+(3-1)^2]}=\sqrt{8}=2\sqrt{2}$.

$CD=\sqrt{[(-2-0)^2+(1+1)^2]}=\sqrt{8}=2\sqrt{2}$. $DA=\sqrt{[(0-2)^2+(-1-1)^2]}=\sqrt{8}=2\sqrt{2}$. (All 4 sides equal)

$AC=\sqrt{[(2+2)^2+(1-1)^2]}=\sqrt{16}=4$. $BD=\sqrt{[(0-0)^2+(3+1)^2]}=\sqrt{16}=4$. (Diagonals equal)

Since all sides are equal, both diagonals are equal, ABCD is a square.

Answer: ABCD is a square (proved)

Q17. Convert the Cartesian coordinates (3, 4) into Polar coordinates.

[Cartesian to Polar conversion — important]

Solution:

$r = \sqrt{(x^{2+y2})} = \sqrt{(3^{2+42})} = \sqrt{25} = 5$.

$\theta = \tan^{-1}(y/x) = \tan^{-1}(4/3) \approx 53.13°$.

Answer: Polar coordinates ≈ (5, 53.13°)

Q18. Convert the Cartesian coordinates (−1, √3) into Polar coordinates.

[Cartesian to Polar conversion — important]

Solution:

$r = \sqrt{[(-1)^{2+}(\sqrt{3})^2]} = \sqrt{(1+3)} = 2$.

Reference angle: $\tan^{-1}(\sqrt{3}/1) = 60°$. Since x is negative and y is positive, the point lies in the 2nd quadrant.

$\theta = 180° - 60° = 120°$.

Answer: Polar coordinates = (2, 120°)

Q19. Convert the Polar coordinates (4, 30°) into Cartesian coordinates.

[Polar to Cartesian conversion — important]

Solution:

$x = r \cos\theta = 4 \cos 30° = 4 \times (\sqrt{3}/2) = 2\sqrt{3}$.

$y = r \sin\theta = 4 \sin 30° = 4 \times (1/2) = 2$.

Answer: Cartesian coordinates = (2√3, 2)

Q20. Convert the Polar coordinates (6, 90°) into Cartesian coordinates.

[Polar to Cartesian conversion — basic]

Solution:

$x = r \cos\theta = 6 \cos 90° = 6 \times 0 = 0$.

$y = r \sin\theta = 6 \sin 90° = 6 \times 1 = 6$.

Answer: Cartesian coordinates = (0, 6)

Q21. If the point (x, 3) is equidistant from A(3,4) and B(−2,3), find x.

[Standard equidistant-point algebra question]

Solution:

$(x-3)^2 + (3-4)^2 = (x+2)^2 + (3-3)^2$.

$x^2-6x+9+1 = x^{2+4x+4}$.

$-6x+10 = 4x+4 \implies 6 = 10x$.

$x = 3/5$.

Answer: $x = 3/5$

Q22. Find the fourth vertex D of a parallelogram ABCD whose three vertices are A(1,2), B(4,3), C(6,7).

[Classic important long-answer question — diagonal bisection property]

Solution:

In parallelogram ABCD, diagonals AC and BD bisect each other, so their midpoints are equal.

Midpoint of AC = $((1+6)/2, (2+7)/2) = (3.5, 4.5)$.

Let D=(x,y). Midpoint of BD = $((4+x)/2, (3+y)/2) = (3.5, 4.5)$.

$(4+x)/2=3.5 \Rightarrow x=3$. $(3+y)/2=4.5 \Rightarrow y=6$.

Answer: D = (3, 6)

Q23. Find the ratio in which the x-axis divides the line segment joining A(2,−3) and B(5,6). Also find the point of intersection.

[Ratio + section formula combined — important]

Solution:

On the x-axis, y=0. Let the ratio be k:1 (A to B).

$y = (6k-3)/(k+1) = 0 \Rightarrow 6k-3=0 \Rightarrow k=1/2$, i.e. ratio = 1:2.

$x = (5k+2)/(k+1)$ with $k=1/2$: $(2.5+2)/1.5 = 4.5/1.5 = 3$.

Answer: Ratio = 1:2, point of intersection = (3, 0)

Q24. Prove that A(−2,−1), B(1,0), C(4,3), D(1,2) are the vertices of a parallelogram.

[Standard proof-type question — diagonal bisection property]

Solution:

For a parallelogram, the midpoints of both diagonals must coincide.

Midpoint of AC = $((-2+4)/2, (-1+3)/2) = (1,1)$.

Midpoint of BD = $((1+1)/2, (0+2)/2) = (1,1)$.

Since both midpoints are equal, the diagonals bisect each other, so ABCD is a parallelogram.

Answer: ABCD is a parallelogram (proved)

Q25. Find the distance between the points (a cosθ, a sinθ) and (a cosϕ, a sinϕ).

[Trigonometric application of distance formula — important]

Solution:

$d^2 = a^2(\cos\theta-\cos\phi)^2 + a^2(\sin\theta-\sin\phi)^2 = a^2[(\cos^2\theta+\sin^2\theta)+(\cos^2\phi+\sin^2\phi) - 2(\cos\theta\cos\phi+\sin\theta\sin\phi)]$.

$= a^2[1+1-2\cos(\theta-\phi)] = 2a^2[1-\cos(\theta-\phi)]$.

Using $1-\cos X = 2\sin^2(X/2)$: $d^2 = 4a^2 \sin^2((\theta-\phi)/2)$.

So $d = 2a\ |\sin((\theta-\phi)/2)|$.

Answer: $d = 2a\ |\sin((\theta-\phi)/2)|$

Q26. Show that A(3,0), B(6,4), C(−1,3) are the vertices of a right-angled isosceles triangle.

[Combined distance + Pythagoras question]

Solution:

$AB=\sqrt{[(6-3)^{2+}(4-0)^2]}=\sqrt{25}=5$. $AC=\sqrt{[(-1-3)^{2+}(3-0)^2]}=\sqrt{25}=5$. (AB=AC, isosceles)

$BC=\sqrt{[(-1-6)^{2+}(3-4)^2]}=\sqrt{50}=5\sqrt{2}$.

Check: $AB^{2+AC2} = 25+25 = 50 = BC^2$, so the right angle is at A.

Answer: Right-angled isosceles triangle, right angle at A (proved)

Q27. Find k if the distance between the points (k,3) and (2,7) is 5 units.

[Reverse application of distance formula]

Solution:

$\sqrt{[(k-2)^{2+}(3-7)^2]} = 5$.

$(k-2)^{2+16} = 25 \implies (k-2)^2 = 9$.

$k-2 = \pm 3 \implies k = 5$ or $k = -1$.

Answer: $k = 5$ or $k = -1$

Q28. Find the area of the triangle formed by the origin O(0,0) and points $A(x_1,y_1)$, $B(x_2,y_2)$. (General result)

[Standard derivation-type question]

Solution:

Using the area formula with $(x_3,y_3)=(0,0)$:

Area $= (1/2)|x_1(y_2-0) + x_2(0-y_1) + 0(y_1-y_2)|$.

$= (1/2)|x_{1y2} - x_{2y1}|$.

Answer: Area of $\Delta OAB = (1/2)|x_{1y2} - x_{2y1}|$

Q29. Using the section formula, find the coordinates of the points which trisect the line segment joining A(2,−2) and B(−7,4).

[Trisection — important application of section formula]

Solution:

The two trisection points P and Q divide AB into 3 equal parts: P divides AB in ratio 1:2, and Q divides AB in ratio 2:1.

P: $x=(1\cdot(-7)+2\cdot 2)/3=(-7+4)/3=-1$. $y=(1\cdot 4+2\cdot(-2))/3=(4-4)/3=0$. $P=(-1,0)$.

Q: $x=(2\cdot(-7)+1\cdot 2)/3=(-14+2)/3=-4$. $y=(2\cdot 4+1\cdot(-2))/3=(8-2)/3=2$. $Q=(-4,2)$.

Answer: Trisection points: $P=(-1,0)$ and $Q=(-4,2)$

Q30. If the centroid of a triangle with vertices A(2,p), B(3,−4), C(q,6) is (5,2), find p and q.

[Centroid — reverse application, classic exam question]

Solution:

Centroid x-coordinate: $(2+3+q)/3 = 5 \implies 5+q = 15 \implies q = 10$.

Centroid y-coordinate: $(p-4+6)/3 = 2 \implies p+2 = 6 \implies p = 4$.

Answer: p = 4, q = 10