Chapter: 1ST ORDER DERIVATIVES

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1. Introduction

The derivative of a function $y=f(x)$ at a point measures the instantaneous rate of change of y with respect to x — geometrically, it is the slope of the tangent to the curve at that point. This chapter follows directly from Limits, since the derivative is itself defined as a limit. It covers differentiation from first principles, the standard derivative formulas for algebraic, trigonometric, exponential, and logarithmic functions, the rules for differentiating combinations of functions (sum, product, quotient, chain rule), and implicit and parametric differentiation. This is one of the most heavily-weighted and frequently examined chapters in Mathematics.

2. Derivative from First Principles

The derivative of $y=f(x)$ with respect to x is defined as:

- **$dy/dx = f'(x) = \lim_{h\to 0} [f(x+h) - f(x)] / h$**

3. Standard Derivative Formulas

- **$d/dx(x^n) = n\cdot x^{n-1}$, $d/dx(\text{constant}) = 0$**
- **$d/dx(\sin x) = \cos x$, $d/dx(\cos x) = -\sin x$**
- **$d/dx(\tan x) = \sec^2 x$, $d/dx(\cot x) = -\mathrm{cosec}^2 x$**
- **$d/dx(\sec x) = \sec x\cdot\tan x$, $d/dx(\mathrm{cosec}\,x) = -\mathrm{cosec}\,x\cdot\cot x$**
- **$d/dx(e^x) = e^x$, $d/dx(a^x) = a^x\cdot\ln a$**
- **$d/dx(\ln x) = 1/x$**
- **$d/dx(\sin^{-1}x) = 1/\sqrt{(1-x^2)}$, $d/dx(\tan^{-1}x) = 1/(1+x^2)$**

4. Rules of Differentiation

- **Sum/Difference Rule: $d/dx(u\pm v) = du/dx \pm dv/dx$**
- **Product Rule: $d/dx(uv) = u\cdot(dv/dx) + v\cdot(du/dx)$**
- **Quotient Rule: $d/dx(u/v) = [v\cdot(du/dx) - u\cdot(dv/dx)] / v^2$**
- **Chain Rule: if $y=f(u)$ and $u=g(x)$, then $dy/dx = (dy/du)\times(du/dx)$**

5. Implicit and Parametric Differentiation

Implicit differentiation: when y is not isolated (e.g. $x^2+y^2=25$), differentiate both sides with respect to x, treating y as a function of x and applying the Chain Rule to every y-term (this introduces a dy/dx term to solve for).

Parametric differentiation: when x and y are both given in terms of a parameter t (e.g. x=f(t), y=g(t)):

- **dy/dx = (dy/dt) / (dx/dt)**

6. Important Points for Exams

- For first-principles questions, algebraic functions usually need factorization or expansion; square-root functions need rationalization; trigonometric functions need the compound-angle expansion followed by standard limits.
- Always identify u and v clearly before applying the Product or Quotient Rule — mixing up the order in the Quotient Rule (which is NOT symmetric) is a very common mistake.
- For the Chain Rule, always differentiate the 'outer' function first (keeping the inner function unchanged), then multiply by the derivative of the 'inner' function.
- For expressions of the form $(\text{function})^{\text{function}}$, such as x^x, take the natural log of both sides first (logarithmic differentiation) before differentiating.
- In implicit differentiation, every time you differentiate a term containing y, you must multiply by dy/dx (Chain Rule) — this is the step most students forget.

7. 30 Important Questions & Answers (With Step-by-Step Solutions)

The following questions are compiled based on the WBSCTE Mathematics syllabus for the 1st Order Derivatives chapter and repeatedly tested question patterns seen in external examination papers. Practising all of these thoroughly gives very strong exam coverage.

Q1. Find dy/dx from first principles for $y = x^2$.

[First-principles derivation — very important, asked almost every year]

Solution:

$f(x+h)-f(x) = (x+h)^2-x^2 = 2xh+h^2$.

$[f(x+h)-f(x)]/h = (2xh+h^2)/h = 2x+h$.

$dy/dx = \lim_{h\to 0} (2x+h) = 2x$.

Answer: $dy/dx = 2x$

Q2. Find dy/dx from first principles for $y = x^3$.

[First-principles derivation — very important]

Solution:

$(x+h)^3-x^3 = 3x^2h+3xh^2+h^3$.

$[(x+h)^3-x^3]/h = 3x^2+3xh+h^2$.

$dy/dx = \lim_{h\to 0} (3x^2+3xh+h^2) = 3x^2$.

Answer: $dy/dx = 3x^2$

Q3. Find dy/dx from first principles for $y = \sqrt{x}$.

[First-principles derivation (rationalization method) — important]

Solution:

$[\sqrt{(x+h)}-\sqrt{x}]/h$. Multiply by the conjugate $(\sqrt{(x+h)}+\sqrt{x})$:

$= [(x+h)-x] / [h(\sqrt{(x+h)}+\sqrt{x})] = h / [h(\sqrt{(x+h)}+\sqrt{x})] = 1/(\sqrt{(x+h)}+\sqrt{x})$.

$dy/dx = \lim_{h\to 0} 1/(\sqrt{(x+h)}+\sqrt{x}) = 1/(2\sqrt{x})$.

Answer: $dy/dx = 1/(2\sqrt{x})$

Q4. Find dy/dx from first principles for $y = 1/x$.

[First-principles derivation — important]

Solution:

$[1/(x+h) - 1/x]/h = [x-(x+h)] / [hx(x+h)] = -h / [hx(x+h)] = -1/[x(x+h)]$.

$dy/dx = \lim_{h\to 0} -1/[x(x+h)] = -1/x^2$.

Answer: $dy/dx = -1/x^2$

Q5. Find dy/dx from first principles for y = sinx.

[First-principles derivation (trigonometric) — very important]

Solution:

$[\sin(x+h)-\sin x]/h = [\sin x\cdot\cosh + \cos x\cdot\sinh - \sin x]/h = \sin x\cdot(\cosh-1)/h + \cos x\cdot(\sinh/h)$.

As $h\to0$: $(\cosh-1)/h \to 0$, and $\sinh/h \to 1$.

$dy/dx = \sin x(0) + \cos x(1) = \cos x$.

Answer: $dy/dx = \cos x$

Q6. Differentiate: $y = x^7 + 5x^4 - 3x^2 + 2x - 9$.

[Power rule + sum/difference rule — basic application]

Solution:

Differentiate each term separately using $d/dx(x^n)=nx^{n-1}$:

$dy/dx = 7x^6 + 20x^3 - 6x + 2$.

Answer: $dy/dx = 7x^6 + 20x^3 - 6x + 2$

Q7. Differentiate: $y = 3x^{5/2} - 4/x^3 + 7\sqrt{x}$.

[Power rule with fractional/negative exponents — important]

Solution:

Rewrite: $y = 3x^{5/2} - 4x^{-3} + 7x^{1/2}$.

Differentiate each term: $dy/dx = 3\cdot(5/2)x^{3/2} - 4\cdot(-3)x^{-4} + 7\cdot(1/2)x^{-1/2}$.

$= (15/2)x^{3/2} + 12x^{-4} + (7/2)x^{-1/2}$.

Answer: $dy/dx = (15/2)x^{3/2} + 12/x^4 + 7/(2\sqrt{x})$

Q8. Differentiate: $y = (x^2+3)(x^3-2x)$, using the Product Rule.

[Product Rule — very important]

Solution:

Product Rule: $d/dx(uv) = u'v + uv'$. Let $u=x^2+3$, $v=x^3-2x$, so $u'=2x$, $v'=3x^2-2$.

$dy/dx = 2x(x^3-2x) + (x^2+3)(3x^2-2)$.

$= (2x^4-4x^2) + (3x^4+7x^2-6) = 5x^4+3x^2-6$.

Answer: $dy/dx = 5x^4 + 3x^2 - 6$

Q9. Differentiate: $y = (2x+1)/(x-3)$, using the Quotient Rule.

[Quotient Rule — very important]

Solution:

Quotient Rule: $d/dx(u/v) = (vu'-uv')/v^2$. Let $u=2x+1$, $v=x-3$, so $u'=2$, $v'=1$.

$dy/dx = [(x-3)(2) - (2x+1)(1)] / (x-3)^2$.

$= [2x-6-2x-1] / (x-3)^2 = -7/(x-3)^2$.

Answer: $dy/dx = -7/(x-3)^2$

Q10. Differentiate: $y = \sin(3x^2+5x)$, using the Chain Rule.

[Chain Rule (trigonometric composite) — very important]

Solution:

Let $u = 3x^2+5x$, so $y = \sin u$.

$dy/dx = \cos u \cdot du/dx = \cos(3x^2+5x) \cdot (6x+5)$.

Answer: $dy/dx = (6x+5)\cos(3x^2+5x)$

Q11. Differentiate: $y = (x^2+1)^5$, using the Chain Rule.

[Chain Rule (power composite) — very important]

Solution:

$dy/dx = 5(x^2+1)^4 \cdot d/dx(x^2+1) = 5(x^2+1)^4 \cdot 2x$.

$= 10x(x^2+1)^4$.

Answer: $dy/dx = 10x(x^2+1)^4$

Q12. Differentiate: $y = e^{3x}$, using the Chain Rule.

[Chain Rule (exponential composite) — very important]

Solution:

$dy/dx = e^{3x} \cdot d/dx(3x) = e^{3x} \cdot 3$.

$= 3e^{3x}$.

Answer: $dy/dx = 3e^{3x}$

Q13. Differentiate: $y = \log(5x+2)$, using the Chain Rule.

[Chain Rule (logarithmic composite) — very important]

Solution:

$dy/dx = 1/(5x+2) \cdot d/dx(5x+2) = 1/(5x+2) \cdot 5$.

$= 5/(5x+2)$.

Answer: $dy/dx = 5/(5x+2)$

Q14. Differentiate: $y = \tan(4x)$, using the Chain Rule.

[Chain Rule (trigonometric composite) — common]

Solution:

$dy/dx = \sec^2(4x) \cdot d/dx(4x) = \sec^2(4x) \cdot 4$.

$= 4\sec^2(4x)$.

Answer: $dy/dx = 4\sec^2(4x)$

Q15. Differentiate: $y = \sqrt{(x^2+9)}$, using the Chain Rule.

[Chain Rule (square-root composite) — common]

Solution:

Write $y = (x^2+9)^{1/2}$.

$dy/dx = (1/2)(x^2+9)^{-1/2} \cdot 2x = x/(x^2+9)^{1/2}$.

$= x/\sqrt{(x^2+9)}$.

Answer: $dy/dx = x/\sqrt{(x^2+9)}$

Q16. Differentiate: $y = x^2 \cdot \sin x$, using the Product Rule.

[Product Rule (algebraic × trigonometric) — common]

Solution:

Let $u=x^2$, $v=\sin x$, so $u'=2x$, $v'=\cos x$.

$dy/dx = u'v+uv' = 2x \cdot \sin x + x^2 \cdot \cos x$.

Answer: $dy/dx = 2x \cdot \sin x + x^2 \cdot \cos x$

Q17. Differentiate: $y = e^x \cdot \cos x$, using the Product Rule.

[Product Rule (exponential × trigonometric) — common]

Solution:

Let $u=e^x$, $v=\cos x$, so $u'=e^x$, $v'=-\sin x$.

$dy/dx = e^x \cdot \cos x + e^x \cdot (-\sin x) = e^x(\cos x - \sin x)$.

Answer: $dy/dx = e^x(\cos x - \sin x)$

Q18. Differentiate: $y = x/\sin x$, using the Quotient Rule.

[Quotient Rule (algebraic ÷ trigonometric) — common]

Solution:

Let $u=x$, $v=\sin x$, so $u'=1$, $v'=\cos x$.

$dy/dx = (v \cdot u' - u \cdot v')/v^2 = (\sin x \cdot 1 - x \cdot \cos x)/\sin^2 x$.

$= (\sin x - x \cdot \cos x)/\sin^2 x$.

Answer: $dy/dx = (\sin x - x \cdot \cos x)/\sin^2 x$

Q19. If $y = 3x^4-2x^3+7$, find the value of dy/dx at $x=2$.

[Derivative evaluation at a point — important]

Solution:

$dy/dx = 12x^3-6x^2$.

At $x=2$: $dy/dx = 12(8)-6(4) = 96-24$.

$= 72$.

Answer: dy/dx at x=2 is 72

Q20. If $y = x^3-6x^2+9x+5$, find the value(s) of x for which $dy/dx = 0$.

[Setting derivative to zero — very important (basis for maxima/minima)]

Solution:

$dy/dx = 3x^2-12x+9$.

Set $dy/dx = 0$: $3x^2-12x+9=0 \implies x^2-4x+3=0$.

Factorising: $(x-1)(x-3)=0 \implies x=1$ or $x=3$.

Answer: x = 1 or x = 3

Q21. Differentiate: $y = \sin^2 x$.

[Chain Rule (power of trig function) — very important]

Solution:

Write $y = (\sin x)^2$.

$dy/dx = 2\sin x \cdot d/dx(\sin x) = 2\sin x \cdot \cos x$.

(This can also be written as $\sin 2x$, using the double-angle formula.)

Answer: $dy/dx = 2\sin x \cdot \cos x$ (= $\sin 2x$)

Q22. Differentiate: $y = \cos^3 x$.

[Chain Rule (power of trig function) — common]

Solution:

Write $y = (\cos x)^3$.

$dy/dx = 3\cos^2 x \cdot d/dx(\cos x) = 3\cos^2 x \cdot (-\sin x)$.

$= -3\cos^2 x \cdot \sin x$.

Answer: $dy/dx = -3\cos^2 x \cdot \sin x$

Q23. Differentiate: $y = \ln(\sin x)$.

[Chain Rule (log of trig function) — important]

Solution:

$dy/dx = (1/\sin x) \cdot d/dx(\sin x) = (1/\sin x) \cdot \cos x$.

$= \cos x/\sin x = \cot x$.

Answer: $dy/dx = \cot x$

Q24. Differentiate: $y = x^x$ (logarithmic differentiation).

[Classic important technique — logarithmic differentiation]

Solution:

Take natural log of both sides: $\ln y = x \cdot \ln x$.

Differentiate implicitly w.r.t. x: $(1/y)(dy/dx) = \ln x + x \cdot (1/x) = \ln x + 1$.

$dy/dx = y(\ln x + 1) = x^x(\ln x + 1)$.

Answer: $dy/dx = x^x(\ln x + 1)$

Q25. If $x^2+y^2=25$, find dy/dx (implicit differentiation).

[Implicit differentiation — very important]

Solution:

Differentiate both sides with respect to x, remembering y is a function of x:

$2x + 2y \cdot (dy/dx) = 0$.

$dy/dx = -x/y$.

Answer: $dy/dx = -x/y$

Q26. If $x^2+y^2+2xy=10$, find dy/dx (implicit differentiation).

[Implicit differentiation with product term — important]

Solution:

Differentiate: $2x + 2y(dy/dx) + 2[x(dy/dx)+y] = 0$.

$2x+2y(dy/dx)+2x(dy/dx)+2y = 0 \implies (dy/dx)(2y+2x) = -2x-2y$.

$dy/dx = -(2x+2y)/(2x+2y) = -1$.

Answer: $dy/dx = -1$

Q27. If $x=at^2$, $y=2at$, find dy/dx (parametric differentiation).

[Parametric differentiation — very important]

Solution:

$dx/dt = 2at$, $dy/dt = 2a$.

$dy/dx = (dy/dt)/(dx/dt) = 2a/(2at)$.

$= 1/t$.

Answer: $dy/dx = 1/t$

Q28. If $x=a \cdot \cos\theta$, $y=a \cdot \sin\theta$, find dy/dx (parametric differentiation).

[Parametric differentiation — very important]

Solution:

$dx/d\theta = -a \cdot \sin\theta$, $dy/d\theta = a \cdot \cos\theta$.

$dy/dx = (dy/d\theta)/(dx/d\theta) = (a \cdot \cos\theta)/(-a \cdot \sin\theta)$.

$= -\cot\theta$.

Answer: $dy/dx = -\cot\theta$

Q29. Differentiate: $y = \sin^{-1}x$.

[Standard inverse trigonometric derivative — very important]

Solution:

This is a standard result:

$dy/dx = 1/\sqrt{1-x^2}$.

Answer: $dy/dx = 1/\sqrt{1-x^2}$

Q30. Differentiate: $y = \tan^{-1}x$.

[Standard inverse trigonometric derivative — very important]

Solution:

This is a standard result:

$dy/dx = 1/(1+x^2)$.

Answer: $dy/dx = 1/(1+x^2)$
