Concept of Trigonometrical Angles & Measurement of Angles (Degree, Radian, Grade)

1. Concept of a Trigonometrical (Generalized) Angle

In elementary geometry, an angle is always taken between 0° and 360°. In trigonometry, however, an angle is defined in a more general way: it is generated by the ROTATION of a revolving line OP about a fixed point O, starting from a fixed initial position OA (called the initial side) and ending at a final position OP (called the terminal side). The angle so generated is called a trigonometrical angle (or generalized angle), and it can take ANY real value — positive, negative, or greater than 360° — depending on the amount and direction of rotation.

- If the rotation is anticlockwise, the angle generated is taken as POSITIVE.
- If the rotation is clockwise, the angle generated is taken as NEGATIVE.
- A full rotation (back to the initial position) generates an angle of 360° (or 2π radians); repeated rotations can generate angles of any magnitude.

2. Systems of Measurement of Angles

There are three commonly used systems for measuring an angle:

- Sexagesimal System (Degree measure): 1 right angle = 90 degrees (90°). 1° = 60 minutes (60'), 1' = 60 seconds (60").
- Centesimal System (Grade measure): 1 right angle = 100 grades (100^g). 1 grade = 100 minutes (100'), 1 minute = 100 seconds (100").
- Circular System (Radian measure): the angle subtended at the center of a circle by an arc whose length equals the radius of the circle is called ONE RADIAN. 1 right angle = π/2 radians; a full circle = 2π radians = 360°.

3. Relation Between Degree, Grade and Radian

If an angle contains D degrees, G grades, and R radians, then since 1 right angle = 90° = 100 grades = π/2 radians, the following fundamental relation holds:

- **D/90 = G/100 = 2R/π**

From this relation, the following direct conversion formulas are obtained:

- **Degree → Radian: R = D × π/180**
- **Radian → Degree: D = R × 180/π**
- **Degree → Grade: G = D × 10/9**
- **Grade → Degree: D = G × 9/10**
- **Grade → Radian: R = G × π/200**

- **Radian → Grade: $G = R \times 200/\pi$**

4. Arc Length Formula (Application of Radian Measure)

If an arc of length s subtends an angle θ (in RADIANS) at the center of a circle of radius r, then:

- **$s = r \cdot \theta$ (θ must always be in radians for this formula)**

5. Important Points for Exams

- Always convert an angle to RADIANS before using the arc-length formula $s=r\theta$ — this is a very common mistake if left in degrees.
- For 'D and G' relation problems (sum, difference, ratio), the fastest method is to set D=90k and G=100k for a single unknown k, then solve.
- Remember 1 right angle = 90° = 100 grades = $\pi/2$ radians — this single fact generates every conversion formula in this topic.
- Minutes and seconds (in degree measure) must be converted to a decimal degree first (by dividing by 60 or 3600) before converting to radians or grades.
- The sum of the angles of a triangle is 180° (π radians); of a quadrilateral is 360° (2π radians) — very useful for ratio-based angle problems.

6. 30 Important Questions & Answers (With Step-by-Step Solutions)

The following questions are compiled based on the WBSCTE Mathematics I syllabus for this topic (Concept of Trigonometrical Angles and their Measurement) and repeatedly tested question patterns seen in external examination papers. Practising all of these thoroughly gives very strong exam coverage.

Q1. Convert 60° into radians.

[Basic degree-to-radian conversion — asked almost every year]

Solution:

Formula: Radian = Degree × $\pi/180$.

= $60 \times \pi/180$.

= $\pi/3$.

Answer: 60° = $\pi/3$ radians

Q2. Convert $\pi/6$ radians into degrees.

[Basic radian-to-degree conversion — asked almost every year]

Solution:

Formula: Degree = Radian × $180/\pi$.

= $(\pi/6) \times (180/\pi)$.

= 30°.

Answer: $\pi/6$ radians = 30°

Q3. Convert 45° into grades.

[Basic degree-to-grade conversion]

Solution:

Formula: Grade = Degree × 10/9.

= 45 × 10/9.

= 50 grades.

Answer: 45° = 50 grades

Q4. Convert 54 grades into degrees.

[Basic grade-to-degree conversion]

Solution:

Formula: Degree = Grade × 9/10.

= 54 × 9/10.

= 48.6°.

Answer: 54 grades = 48.6°

Q5. Convert 40 grades into radians.

[Basic grade-to-radian conversion]

Solution:

Formula: Radian = Grade $\times \pi/200$.

$= 40 \times \pi/200$.

$= \pi/5$.

Answer: 40 grades = $\pi/5$ radians

Q6. Convert $3\pi/4$ radians into grades.

[Basic radian-to-grade conversion]

Solution:

Formula: Grade = Radian $\times 200/\pi$.

$= (3\pi/4) \times (200/\pi)$.

$= 150$ grades.

Answer: $3\pi/4$ radians = 150 grades

Q7. Convert 150° into both radians and grades.

[Combined conversion — common exam pattern]

Solution:

Radian $= 150 \times \pi/180 = 5\pi/6$.

Grade $= 150 \times 10/9 = 1500/9 = 166\frac{2}{3}$ grades.

Answer: $150° = 5\pi/6$ radians $= 166\frac{2}{3}$ grades

Q8. If an angle is $2\pi/3$ radians, express it in degrees and grades.

[Combined conversion starting from radians — common]

Solution:

Degree $= (2\pi/3) \times (180/\pi) = 120°$.

Grade $= 120 \times 10/9 = 1200/9 = 133\frac{1}{3}$ grades.

Answer: $2\pi/3$ radians $= 120° = 133\frac{1}{3}$ grades

Q9. The angles of a triangle are in the ratio 2:3:4. Express each angle in radians.

[Ratio-based application — very important]

Solution:

Sum of angles of a triangle $= 180° = \pi$ radians.

Ratio parts sum $= 2+3+4 = 9$. Each part $= \pi/9$.

Angles $= 2\times(\pi/9), 3\times(\pi/9), 4\times(\pi/9) = 2\pi/9, \pi/3, 4\pi/9$.

Answer: Angles = $2\pi/9$, $\pi/3$, $4\pi/9$ radians

Q10. Prove that D/9 = G/10, where D and G denote the number of degrees and number of grades in the same angle.

[Standard proof — very important]

Solution:

Since 1 right angle = 90° = 100 grades, if the angle contains k right angles: $D=90k$, $G=100k$.

$D/90 = k$ and $G/100 = k$, so $D/90 = G/100$.

Dividing both sides by 10: $D/9 = G/10$. (Proved)

Answer: D/9 = G/10 (proved)

Q11. Prove that D/90 = G/100 = 2R/π, where D, G, R denote the degree, grade, and radian measures of the same angle.

[Standard proof — very important, most fundamental identity of this topic]

Solution:

Since 1 right angle = 90° = 100 grades = $\pi/2$ radians, suppose the angle contains k right angles.

Then $D = 90k$, $G = 100k$, $R = (\pi/2)k$.

$D/90 = k$, $G/100 = k$, and $2R/\pi = 2\times(\pi/2)k/\pi = k$.

Hence $D/90 = G/100 = 2R/\pi = k$. (Proved)

Answer: D/90 = G/100 = 2R/π (proved)

Q12. If the number of degrees in an angle added to the number of grades in it is 152, find the angle in degrees.

[Classic important question using D/90 = G/100 = k]

Solution:

Let $D=90k$ and $G=100k$ (k = number of right angles in the angle).

$D+G = 190k = 152 \implies k = 152/190 = 0.8$.

$D = 90 \times 0.8 = 72°$.

Answer: The angle = 72° (and 80 grades)

Q13. If the number of grades exceeds the number of degrees in an angle by 25, find the angle in radians.

[Classic important question using D/90 = G/100 = k]

Solution:

Let $D=90k$, $G=100k$. Given $G-D=25$: $100k-90k=10k=25 \implies k=2.5$.

Radian measure $R = (\pi/2)k = (\pi/2)(2.5) = 1.25\pi = 5\pi/4$.

(Check: $D=90\times2.5=225°$, and $225\times\pi/180 = 5\pi/4$ ✓)

Answer: The angle = 5π/4 radians (= 225°)

Q14. Express one radian in degrees, minutes, and seconds (approximate).

[Conceptual conversion — important]

Solution:

1 radian = 180/π degrees ≈ 57.2958°.

0.2958° × 60 = 17.75', so ≈ 57°17'.

0.75' × 60 ≈ 45", so ≈ 57°17'45".

Answer: 1 radian ≈ 57°17'45"

Q15. Convert 1° into radians (state the exact value and an approximate decimal value).

[Basic conversion]

Solution:

Radian = Degree × π/180.

1° = π/180 radians.

≈ 0.01745 radians.

Answer: 1° = π/180 radians ≈ 0.01745 radians

Q16. The angle of a sector is 60° and the radius of the circle is 7 cm. Find the length of the arc. (Use π ≈ 22/7)

[Arc-length application (s = rϑ) — very important]

Solution:

Convert the angle to radians (arc-length formula needs θ in radians): 60° = π/3 radians.

Arc length s = r·θ = 7 × π/3.

Using π≈22/7: s = 7 × (22/7)/3 = 22/3 ≈ 7.33 cm.

Answer: Arc length ≈ 7.33 cm

Q17. A pendulum swings through an angle of 30° and describes an arc of length 8.8 cm. Find the length of the pendulum. (Use π ≈ 22/7)

[Reverse arc-length application — classic important question]

Solution:

Convert 30° to radians: θ = π/6.

Using s = rθ: r = s/θ = 8.8/(π/6) = 8.8×6/π = 52.8/π.

Using π≈22/7: r = 52.8 × 7/22 = 16.8 cm.

Answer: Length of the pendulum = 16.8 cm

Q18. Find the radian measure corresponding to 5°37'30".

[Combined degree-minute-second to radian conversion — important]

Solution:

Convert minutes and seconds to a decimal degree: 37'30" = 37.5' = 37.5/60° = 0.625°.

Total angle = 5° + 0.625° = 5.625°.

Radian = 5.625 × π/180 = (5.625/180)π = π/32 [since 5.625/180 = 1/32].

Answer: 5°37'30" = π/32 radians

Q19. Find the degree measure corresponding to 11π/36 radians.

[Basic radian-to-degree conversion]

Solution:

Degree = Radian × 180/π.

= (11π/36) × (180/π) = 11 × (180/36) = 11 × 5.

= 55°.

Answer: 11π/36 radians = 55°

Q20. Two angles of a triangle are 45° and π/3 radians. Find the third angle in radians.

[Mixed-unit application — important]

Solution:

Convert 45° to radians: π/4.

Sum of the two given angles = π/4 + π/3 = 3π/12 + 4π/12 = 7π/12.

Sum of angles of a triangle = π radians, so the third angle = π − 7π/12 = 5π/12.

Answer: Third angle = 5π/12 radians

Q21. The angles of a triangle are in the ratio 1:2:3. Find the angles in both degrees and radians.

[Ratio-based application — very common]

Solution:

Sum of angles = 180°, ratio parts = 1+2+3 = 6. Each part = 30°.

Angles in degrees: 30°, 60°, 90°.

In radians: π/6, π/3, π/2.

Answer: 30°(π/6), 60°(π/3), 90°(π/2)

Q22. Convert 320 grades into degrees and radians.

[Combined conversion from grades]

Solution:

Degree = 320 × 9/10 = 288°.

Radian = 320 × π/200 = 1.6π = 8π/5.

Answer: 320 grades = 288° = 8π/5 radians

Q23. Find the angle whose numerical value in degrees equals its numerical value in grades.

[Conceptual — interesting edge-case question]

Solution:

Let D=90k and G=100k. If D = G numerically: 90k = 100k $\Rightarrow$ −10k = 0 $\Rightarrow$ k = 0.

So the only such angle is the zero angle.

Answer: The angle is 0° (the only angle where D and G are numerically equal)

Q24. The angles of a quadrilateral are in the ratio 2:3:4:6. Express each angle in radians.

[Ratio-based application (quadrilateral) — important]

Solution:

Sum of angles of a quadrilateral = 360° = 2π radians.

Ratio parts sum = 2+3+4+6 = 15. Each part = 2π/15.

Angles: 4π/15, 6π/15=2π/5, 8π/15, 12π/15=4π/5.

Answer: 4π/15, 2π/5, 8π/15, 4π/5 radians

Q25. The sum of two angles is 5π/9 radians and their difference is 20°. Find the two angles in degrees.

[Mixed-unit sum-difference application — classic important]

Solution:

Convert 5π/9 radians to degrees: (5π/9)×(180/π) = 5×20 = 100°.

Sum = 100°, Difference = 20°.

Larger angle = (100+20)/2 = 60°. Smaller angle = (100−20)/2 = 40°.

Answer: The two angles are 60° and 40°

Q26. Find, in radians, the angle between the hour hand and the minute hand of a clock at 4 o'clock.

[Classic real-life application question]

Solution:

At 4 o'clock, the hour hand is at 4×30° = 120° from the 12 mark, and the minute hand is at 0°.

Angle between them = 120°.

In radians: 120 × π/180 = 2π/3.

Answer: Angle = 2π/3 radians (120°)

Q27. A wheel makes 360 revolutions per minute. Find the angle (in radians) it turns through in one second.

[Practical application — angular speed]

Solution:

360 revolutions/minute = 360/60 = 6 revolutions/second.

Each revolution corresponds to an angle of 2π radians.

Angle turned per second = $6 \times 2\pi = 12\pi$ radians.

Answer: 12π radians per second

Q28. Find the angle in grades which is equivalent to $3\pi/10$ radians.

[Basic radian-to-grade conversion]

Solution:

Grade = Radian × $200/\pi$.

= $(3\pi/10) \times (200/\pi) = 3 \times 20$.

= 60 grades.

Answer: $3\pi/10$ radians = 60 grades

Q29. The difference between the two acute angles of a right-angled triangle is $\pi/9$ radians. Find the two angles in degrees.

[Combined application — classic closing question]

Solution:

Since the triangle is right-angled, the two acute angles sum to 90°.

Convert $\pi/9$ radians to degrees: $(\pi/9)\times(180/\pi) = 20°$.

Sum = 90°, Difference = 20°. Larger = (90+20)/2 = 55°. Smaller = (90−20)/2 = 35°.

Answer: The two acute angles are 55° and 35°
