Chapter: QUADRATIC EQUATION

1. Introduction

A quadratic equation is a polynomial equation of degree 2, of the standard form $ax^2+bx+c=0$, where a, b, c are real numbers and $a\neq0$. This chapter covers the methods of solving quadratic equations (factorization and the quadratic formula), the discriminant and nature of roots, the relationship between the roots and the coefficients (sum and product of roots), forming a quadratic equation from given roots, and finding symmetric functions of the roots (like $\alpha^2+\beta^2$, $\alpha^3+\beta^3$) without actually solving the equation. This is one of the most fundamental and frequently tested chapters, forming the basis for many later algebra and calculus topics.

2. Methods of Solving a Quadratic Equation

For $ax^2+bx+c=0$ ($a\neq0$):

- Factorization Method: split the middle term so the equation factors as $(x-p)(x-q)=0$, giving $x=p$ or $x=q$.
  - **Quadratic Formula: $x = [-b \pm \sqrt{(b^2-4ac)}] / 2a$**

3. The Discriminant and Nature of Roots

The discriminant $D = b^2-4ac$ determines the nature of the roots:

- $D > 0$: roots are real and distinct (and rational if D is a perfect square with rational a,b,c).
- $D = 0$: roots are real and equal.
- $D < 0$: roots are complex (imaginary) and unequal.

4. Sum and Product of Roots

If α, β are the roots of $ax^2+bx+c=0$:

- **Sum of roots: $\alpha + \beta = -b/a$**
- **Product of roots: $\alpha\cdot\beta = c/a$**

5. Formation of a Quadratic Equation from Given Roots

If α and β are the required roots, the quadratic equation is:

- **$x^2 - (\alpha+\beta)x + \alpha\beta = 0$**

6. Useful Symmetric Function Formulas

- **$\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$**

- $\alpha^3 + \beta^3 = (\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta)$
- $(\alpha-\beta)^2 = (\alpha+\beta)^2 - 4\alpha\beta$
- $1/\alpha + 1/\beta = (\alpha+\beta) / (\alpha\beta)$

7. Important Points for Exams

- Always compute the discriminant FIRST if only the 'nature of roots' is asked — there is no need to actually solve the equation.
- For 'equal roots' or 'real and equal roots' conditions, always set D=0 and solve for the unknown parameter.
- For 'roots reciprocal to each other', set product of roots = 1; for 'roots equal in magnitude but opposite in sign', set sum of roots = 0.
- Symmetric functions like $\alpha^2+\beta^2$, $\alpha^3+\beta^3$ should always be expressed in terms of $(\alpha+\beta)$ and $\alpha\beta$ using the standard identities — never attempt to find α and β individually unless explicitly asked.
- For word problems, carefully translate the given condition into an equation in one variable, simplify to standard quadratic form, then solve and reject any answer that doesn't fit the context (e.g. negative length, negative count).

8. 30 Important Questions & Answers (With Step-by-Step Solutions)

The following questions are compiled based on the WBSCTE Mathematics I syllabus for the Quadratic Equation chapter and repeatedly tested question patterns seen in external examination papers. Practising all of these thoroughly gives very strong exam coverage.

Q1. Solve by factorization: $x^2-5x+6=0$.

[Basic factorization method — asked almost every year]

Solution:

Find two numbers whose product is 6 and sum is −5: these are −2 and −3.

$x^2-5x+6 = (x-2)(x-3) = 0$.

$x = 2$ or $x = 3$.

Answer: x = 2, 3

Q2. Solve by factorization: $x^2+2x-15=0$.

[Basic factorization method — common]

Solution:

Find two numbers whose product is −15 and sum is 2: these are 5 and −3.

$x^2+2x-15 = (x+5)(x-3) = 0$.

$x = -5$ or $x = 3$.

Answer: x = −5, 3

Q3. Solve using the Quadratic Formula: $2x^2-7x+3=0$.

[Quadratic formula method — very important]

Solution:

$a=2$, $b=-7$, $c=3$. Discriminant $D = b^2-4ac = 49-24 = 25$.

$x = [-b\pm\sqrt{D}] / 2a = [7\pm5] / 4$.

$x = 12/4 = 3$ or $x = 2/4 = 1/2$.

Answer: x = 3, 1/2

Q4. Solve using the Quadratic Formula: $x^2-4x+5=0$.

[Quadratic formula with complex roots — important]

Solution:

$a=1$, $b=-4$, $c=5$. Discriminant $D = 16-20 = -4$.

$x = [4\pm\sqrt{(-4)}] / 2 = [4\pm2i] / 2$.

$x = 2+i$ or $x = 2-i$.

Answer: x = 2+i, 2−i

Q5. Find the nature of the roots of $2x^2-3x+4=0$, without solving the equation.

[Discriminant / nature-of-roots — very important]

Solution:

$D = b^2-4ac = (-3)^2-4(2)(4) = 9-32.$

$= -23.$

Since $D<0$, the roots are complex (imaginary) and unequal.

Answer: Roots are complex and unequal ($D=-23<0$)

Q6. Find the nature of the roots of $x^2-6x+9=0$.

[Discriminant / nature-of-roots — common]

Solution:

$D = (-6)^2-4(1)(9) = 36-36.$

$= 0.$

Since $D=0$, the roots are real and equal.

Answer: Roots are real and equal ($D=0$)

Q7. Find the nature of the roots of $3x^2+5x-2=0$.

[Discriminant / nature-of-roots — common]

Solution:

$D = 5^2-4(3)(-2) = 25+24 = 49.$

Since $D=49>0$ and is a perfect square, the roots are real, distinct, and rational.

Answer: Roots are real, distinct, and rational ($D=49$, a perfect square)

Q8. Find the sum and product of the roots of $3x^2-7x+2=0$, without solving the equation.

[Sum & product of roots — very important]

Solution:

For $ax^2+bx+c=0$: Sum of roots $= -b/a$, Product of roots $= c/a$.

Sum $= -(-7)/3 = 7/3$.

Product $= 2/3$.

Answer: Sum = 7/3, Product = 2/3

Q9. Find the sum and product of the roots of $x^2+5x-6=0$.

[Sum & product of roots — common]

Solution:

Sum $= -b/a = -5$.

Product $= c/a = -6$.

Answer: Sum = −5, Product = −6

Q10. Form the quadratic equation whose roots are 4 and −3.

[Formation of quadratic equation from roots — very important]

Solution:

Sum of roots = $4+(-3) = 1$. Product of roots = $4\times(-3) = -12$.

Standard form: $x^2 - (\text{sum})x + (\text{product}) = 0$.

$x^2 - x - 12 = 0$.

Answer: $x^2 - x - 12 = 0$

Q11. Form the quadratic equation whose roots are $2+\sqrt{3}$ and $2-\sqrt{3}$.

[Formation of quadratic equation (irrational roots) — important]

Solution:

Sum = $(2+\sqrt{3})+(2-\sqrt{3}) = 4$. Product = $(2+\sqrt{3})(2-\sqrt{3}) = 4-3 = 1$.

$x^2 - (\text{sum})x + (\text{product}) = 0$.

$x^2 - 4x + 1 = 0$.

Answer: $x^2 - 4x + 1 = 0$

Q12. Find the value(s) of k for which the equation $x^2-kx+9=0$ has equal roots.

[Classic important application (equal roots) — very common]

Solution:

For equal roots, D=0: $k^2-4(1)(9)=0$.

$k^2 = 36$.

$k = \pm 6$.

Answer: $k = 6$ or $k = -6$

Q13. Find the value of k for which the equation $kx^2-6x+1=0$ has real and equal roots.

[Classic important application (equal roots) — very common]

Solution:

For equal roots, D=0: $(-6)^2-4(k)(1)=0$.

$36-4k = 0$.

$k = 9$.

Answer: $k = 9$

Q14. If α, β are the roots of $x^2-5x+6=0$, find $\alpha^2+\beta^2$.

[Symmetric functions of roots — very important]

Solution:

Sum $\alpha+\beta=5$, Product $\alpha\beta=6$.

$\alpha^2+\beta^2 = (\alpha+\beta)^2 - 2\alpha\beta = 25 - 12$.

$= 13$.

Answer: $\alpha^2+\beta^2 = 13$

Q15. If α, β are the roots of $x^2-3x-4=0$, find $\alpha^3+\beta^3$.

[Symmetric functions of roots — important]

Solution:

Sum $\alpha+\beta=3$, Product $\alpha\beta=-4$.

$\alpha^3+\beta^3 = (\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta) = 27 - 3(-4)(3)$.

$= 27+36 = 63$.

Answer: $\alpha^3+\beta^3 = 63$

Q16. If α, β are the roots of $x^2-4x+3=0$, find $1/\alpha + 1/\beta$.

[Symmetric functions of roots — common]

Solution:

Sum $\alpha+\beta=4$, Product $\alpha\beta=3$.

$1/\alpha+1/\beta = (\alpha+\beta)/(\alpha\beta)$.

$= 4/3$.

Answer: $1/\alpha + 1/\beta = 4/3$

Q17. If α, β are the roots of $x^2-6x+8=0$, find $(\alpha-\beta)^2$.

[Symmetric functions of roots — important]

Solution:

Sum $\alpha+\beta=6$, Product $\alpha\beta=8$.

$(\alpha-\beta)^2 = (\alpha+\beta)^2 - 4\alpha\beta = 36 - 32$.

$= 4$.

Answer: $(\alpha-\beta)^2 = 4$

Q18. If α, β are the roots of $2x^2-3x-5=0$, find $\alpha^2\beta + \alpha\beta^2$.

[Symmetric functions of roots — common]

Solution:

Sum $\alpha+\beta=3/2$, Product $\alpha\beta=-5/2$.

$\alpha^2\beta+\alpha\beta^2 = \alpha\beta(\alpha+\beta) = (-5/2)(3/2)$.

$= -15/4$.

Answer: $\alpha^2\beta + \alpha\beta^2 = -15/4$

Q19. If one root of $x^2-7x+k=0$ is 3, find k and the other root.

[Classic important application (given one root) — very common]

Solution:

Since $x=3$ is a root: $9-21+k=0 \Rightarrow k=12$.

Sum of roots = 7 (= $-b/a$). So the other root = $7-3 = 4$.

Answer: k = 12, other root = 4

Q20. If one root of $x^2+px-8=0$ is 4, find p and the other root.

[Classic important application (given one root) — common]

Solution:

Since $x=4$ is a root: $16+4p-8=0 \Rightarrow 4p=-8 \Rightarrow p=-2$.

Sum of roots = $-p$ = 2. So the other root = $2-4 = -2$.

(Check: product = $4\times(-2)=-8$, matches $c/a=-8$ ✓)

Answer: p = −2, other root = −2

Q21. If the roots of $x^2-(k+6)x+2k+9=0$ are equal, find k.

[Classic important application (equal roots with parameter) — important]

Solution:

For equal roots, D=0: $(k+6)^2-4(2k+9)=0$.

$k^2+12k+36-8k-36=0 \Rightarrow k^2+4k=0$.

$k(k+4)=0 \Rightarrow k=0$ or $k=-4$.

Answer: k = 0 or k = −4

Q22. Find the value of m so that the roots of $x^2-2(m+1)x+m(m-1)=0$ are equal in magnitude and opposite in sign.

[Classic important condition-based question — sum of roots = 0]

Solution:

For roots equal in magnitude but opposite in sign, their sum must be zero.

Sum of roots = $2(m+1)$. Set = 0: $m+1 = 0$.

$m = -1$.

Answer: m = −1

Q23. If the roots of $2x^2-(k+1)x+(k-1)=0$ are reciprocals of each other, find k.

[Classic important condition-based question — product of roots = 1]

Solution:

If the roots are reciprocal, their product must equal 1.

Product of roots = $(k-1)/2$. Set = 1: $k-1 = 2$.

$k = 3$.

Answer: $k = 3$

Q24. Find the value(s) of p for which the equation $x^2+px+16=0$ has real roots.

[Condition for real roots — important]

Solution:

For real roots, $D \geq 0$: $p^2-4(1)(16) \geq 0$.

$p^2 \geq 64$.

$p \geq 8$ or $p \leq -8$.

Answer: $p \geq 8$ or $p \leq -8$

Q25. If the sum of the roots of $3x^2+(2k+1)x-(k+5)=0$ is equal to the product of the roots, find k.

[Classic important condition-based question]

Solution:

Sum of roots = $-(2k+1)/3$. Product of roots = $-(k+5)/3$.

Setting them equal: $-(2k+1)/3 = -(k+5)/3 \implies 2k+1 = k+5$.

$k = 4$.

Answer: $k = 4$

Q26. If α and β are the roots of $x^2-px-(p+q)=0$, show that $(\alpha+1)(\beta+1) = 1-q$.

[Classic important identity-manipulation proof]

Solution:

From the equation: Sum $\alpha+\beta=p$, Product $\alpha\beta=-(p+q)$.

$(\alpha+1)(\beta+1) = \alpha\beta+\alpha+\beta+1 = -(p+q)+p+1$.

$= -p-q+p+1 = 1-q$. (Proved)

Answer: $(\alpha+1)(\beta+1) = 1-q$ (proved)

Q27. Solve for x: $x^4-13x^2+36=0$.

[Bi-quadratic reducible to quadratic (substitution method) — important]

Solution:

Let $y=x^2$. The equation becomes $y^2-13y+36=0$.

Factorising: $(y-4)(y-9)=0 \implies y=4$ or $y=9$.

$x^2=4 \implies x=\pm2$. $x^2=9 \implies x=\pm3$.

Answer: $x = \pm 2, \pm 3$

Q28. The sum of a number and its reciprocal is 26/5. Find the number.

[Classic important word problem — very common]

Solution:

Let the number be x. $x + 1/x = 26/5$.

Multiplying by 5x: $5x^2+5 = 26x \implies 5x^2-26x+5=0$.

$D = 676-100 = 576 = 24^2$. $x = (26\pm24)/10 \implies x=5$ or $x=1/5$.

Answer: The number is 5 or 1/5

Q29. The product of two consecutive positive integers is 240. Find the integers.

[Classic important word problem — very common]

Solution:

Let the integers be x and x+1. $x(x+1) = 240$.

$x^2+x-240 = 0$. Factorising: $(x-15)(x+16)=0$.

$x=15$ (rejecting the negative value $x=-16$). The integers are 15 and 16.

Answer: The integers are 15 and 16

Q30. A number is such that when 3 times its square is subtracted from 4 times the number, the result is −1. Find the number.

[Classic important word problem — important]

Solution:

Let the number be x. $4x - 3x^2 = -1$.

Rearranging: $3x^2-4x-1 = 0$.

Using the quadratic formula: $D=16+12=28$. $x = (4\pm\sqrt{28})/6 = (2\pm\sqrt{7})/3$.

Answer: $x = (2+\sqrt{7})/3$ or $x = (2-\sqrt{7})/3$
