Trigonometrical Ratios, Associated Angles & Standard Angles

1. Trigonometrical Ratios of an Angle

For an angle θ in a right-angled triangle, the six trigonometric ratios are defined using the sides opposite (opp), adjacent (adj) to θ, and the hypotenuse (hyp):

- **sinθ = opp/hyp , cosθ = adj/hyp , tanθ = opp/adj**
- **cosecθ = 1/sinθ , secθ = 1/cosθ , cotθ = 1/tanθ**

Note: tanθ = sinθ/cosθ, and cotθ = cosθ/sinθ.

2. Fundamental (Pythagorean) Identities

- **$\sin^2\theta + \cos^2\theta = 1$**
- **$1 + \tan^2\theta = \sec^2\theta$**
- **$1 + \cot^2\theta = \text{cosec}^2\theta$**

3. Signs of Trigonometric Ratios in the Four Quadrants (ASTC Rule)

- 1st Quadrant (0°–90°): ALL six ratios are positive.
- 2nd Quadrant (90°–180°): only SIN (and cosec) are positive.
- 3rd Quadrant (180°–270°): only TAN (and cot) are positive.
- 4th Quadrant (270°–360°): only COS (and sec) are positive.

(Remembered by the phrase: 'All Silver Tea Cups' — A, S, T, C for Quadrants I, II, III, IV.)

4. Associated (Allied) Angles

These formulas express the trig ratios of 90°±θ, 180°±θ, 270°±θ, 360°±θ, and −θ in terms of θ:

- **sin(90°−θ)=cosθ , cos(90°−θ)=sinθ , tan(90°−θ)=cotθ**
- **sin(90°+θ)=cosθ , cos(90°+θ)=−sinθ , tan(90°+θ)=−cotθ**
- **sin(180°−θ)=sinθ , cos(180°−θ)=−cosθ , tan(180°−θ)=−tanθ**
- **sin(180°+θ)=−sinθ , cos(180°+θ)=−cosθ , tan(180°+θ)=tanθ**
- **sin(270°−θ)=−cosθ , cos(270°−θ)=−sinθ , tan(270°−θ)=cotθ**
- **sin(270°+θ)=−cosθ , cos(270°+θ)=sinθ , tan(270°+θ)=−cotθ**
- **sin(360°−θ)=−sinθ , cos(360°−θ)=cosθ , tan(360°−θ)=−tanθ**
- **sin(−θ)=−sinθ , cos(−θ)=cosθ , tan(−θ)=−tanθ**

5. Trigonometric Ratios of Standard Angles

- 0°: sin=0, cos=1, tan=0
- 30°: sin=1/2, cos=√3/2, tan=1/√3
- 45°: sin=1/√2, cos=1/√2, tan=1
- 60°: sin=√3/2, cos=1/2, tan=√3
- 90°: sin=1, cos=0, tan=undefined

6. Important Points for Exams

- When simplifying an angle like 90°±θ or 270°±θ, the ratio CHANGES (sin↔cos, tan↔cot); for 180°±θ or 360°±θ, the ratio STAYS THE SAME (sin stays sin, etc.) — only the sign may change.
- To determine the sign after applying an associated-angle formula, imagine θ as a small acute angle and check which quadrant the original angle (e.g. 180°+θ) falls into, then apply the ASTC rule.
- Standard angle values (0°,30°,45°,60°,90°) must be memorised cold — nearly every numerical problem in this topic depends on them.
- For 'find the other ratios given one' problems, always draw a right triangle mentally using the given ratio, find the third side using Pythagoras, then read off the rest.

7. 30 Important Questions & Answers (With Step-by-Step Solutions)

The following questions are compiled based on the WBSCTE Mathematics I syllabus for this topic (Trigonometrical Ratios, Associated Angles, and Standard Angles) and repeatedly tested question patterns seen in external examination papers. Practising all of these thoroughly gives very strong exam coverage.

Q1. Define the six trigonometric ratios of an angle θ in a right-angled triangle.

[Basic definitions — asked almost every year]

Solution:

In a right triangle with angle θ, let 'opp' = side opposite θ, 'adj' = side adjacent to θ, 'hyp' = hypotenuse.

sinθ = opp/hyp, cosθ = adj/hyp, tanθ = opp/adj.

cosecθ = 1/sinθ = hyp/opp, secθ = 1/cosθ = hyp/adj, cotθ = 1/tanθ = adj/opp.

Answer: sinθ=opp/hyp, cosθ=adj/hyp, tanθ=opp/adj, and their reciprocals cosecθ, secθ, cotθ

Q2. Prove that $\sin^2\theta + \cos^2\theta = 1$.

[Fundamental identity — very important, proved almost every year]

Solution:

In a right triangle, let opp=a, adj=b, hyp=c. By Pythagoras' theorem: $a^2+b^2 = c^2$.

sinθ = a/c, cosθ = b/c.

$\sin^2\theta+\cos^2\theta = a^2/c^2 + b^2/c^2 = (a^2+b^2)/c^2 = c^2/c^2 = 1$. (Proved)

Answer: $\sin^2\theta + \cos^2\theta = 1$ (proved)

Q3. Prove that $1 + \tan^2\theta = \sec^2\theta$.

[Fundamental identity — very important]

Solution:

Start from $\sin^2\theta+\cos^2\theta = 1$.

Divide throughout by $\cos^2\theta$: $\sin^2\theta/\cos^2\theta + 1 = 1/\cos^2\theta$.

$\tan^2\theta + 1 = \sec^2\theta$. (Proved)

Answer: $1 + \tan^2\theta = \sec^2\theta$ (proved)

Q4. Prove that $1 + \cot^2\theta = \text{cosec}^2\theta$.

[Fundamental identity — very important]

Solution:

Start from $\sin^2\theta+\cos^2\theta = 1$.

Divide throughout by $\sin^2\theta$: $1 + \cos^2\theta/\sin^2\theta = 1/\sin^2\theta$.

$1 + \cot^2\theta = \text{cosec}^2\theta$. (Proved)

Answer: $1 + \cot^2\theta = \text{cosec}^2\theta$ (proved)

Q5. If $\sin\theta = 3/5$ and θ is acute, find $\cos\theta$ and $\tan\theta$.

[Basic application of Pythagorean identity — very common]

Solution:

$\cos\theta = \sqrt{1-\sin^2\theta} = \sqrt{1-9/25} = \sqrt{16/25} = 4/5$.

$\tan\theta = \sin\theta/\cos\theta = (3/5)/(4/5) = 3/4$.

Answer: $\cos\theta = 4/5$, $\tan\theta = 3/4$

Q6. If $\cos\theta = 12/13$ and θ is acute, find $\sin\theta$ and $\tan\theta$.

[Basic application of Pythagorean identity — very common]

Solution:

$\sin\theta = \sqrt{1-\cos^2\theta} = \sqrt{1-144/169} = \sqrt{25/169} = 5/13$.

$\tan\theta = \sin\theta/\cos\theta = (5/13)/(12/13) = 5/12$.

Answer: $\sin\theta = 5/13$, $\tan\theta = 5/12$

Q7. If $\tan\theta = 5/12$ and θ is acute, find $\sec\theta$ and $\sin\theta$.

[Basic application using $1+\tan^2\vartheta=\sec^2\vartheta$]

Solution:

$\sec\theta = \sqrt{1+\tan^2\theta} = \sqrt{1+25/144} = \sqrt{169/144} = 13/12$.

So $\cos\theta = 12/13$.

$\sin\theta = \sqrt{1-\cos^2\theta} = \sqrt{1-144/169} = \sqrt{25/169} = 5/13$.

Answer: $\sec\theta = 13/12$, $\sin\theta = 5/13$

Q8. Write the values of sin30°, cos30°, and tan30°.

[Standard angle values — must memorise]

Solution:

These are standard values from the trigonometric ratio table.

Answer: $\sin 30° = 1/2$, $\cos 30° = \sqrt{3}/2$, $\tan 30° = 1/\sqrt{3}$

Q9. Write the values of sin60°, cos60°, and tan60°.

[Standard angle values — must memorise]

Solution:

These are standard values from the trigonometric ratio table.

Answer: $\sin 60° = \sqrt{3}/2$, $\cos 60° = 1/2$, $\tan 60° = \sqrt{3}$

Q10. Write the values of sin45°, cos45°, and tan45°.

[Standard angle values — must memorise]

Solution:

These are standard values from the trigonometric ratio table.

Answer: $\sin 45° = 1/\sqrt{2}$, $\cos 45° = 1/\sqrt{2}$, $\tan 45° = 1$

Q11. Evaluate: $\sin 30° \cdot \cos 60° + \cos 30° \cdot \sin 60°$.

[Standard-value evaluation — very common]

Solution:

$= (1/2)(1/2) + (\sqrt{3}/2)(\sqrt{3}/2)$.

$= 1/4 + 3/4$.

$= 1$.

Answer: 1

Q12. Evaluate: $2\sin^2 45° + 3\cos^2 60° - \tan^2 30°$.

[Standard-value evaluation — common]

Solution:

$= 2(1/\sqrt{2})^2 + 3(1/2)^2 - (1/\sqrt{3})^2$.

$= 2(1/2) + 3(1/4) - 1/3 = 1 + 3/4 - 1/3$.

Converting to a common denominator of 12: $12/12 + 9/12 - 4/12 = 17/12$.

Answer: 17/12

Q13. Evaluate: $\sin 60° \cdot \cos 30° - \cos 60° \cdot \sin 30°$.

[Standard-value evaluation — common]

Solution:

$= (\sqrt{3}/2)(\sqrt{3}/2) - (1/2)(1/2)$.

$= 3/4 - 1/4$.

$= 1/2$.

Answer: 1/2

Q14. Evaluate: $(\tan 45° + \tan 30°) / (1 - \tan 45° \cdot \tan 30°)$.

[Standard-value evaluation — combined fraction]

Solution:

$= (1 + 1/\sqrt{3}) / (1 - 1 \times 1/\sqrt{3}) = (1+1/\sqrt{3})/(1-1/\sqrt{3})$.

Multiply numerator and denominator by $\sqrt{3}$: $= (\sqrt{3}+1)/(\sqrt{3}-1)$.

Rationalize by multiplying by $(\sqrt{3}+1)/(\sqrt{3}+1)$: $= (\sqrt{3}+1)^2/((\sqrt{3})^2-1^2) = (4+2\sqrt{3})/2 = 2+\sqrt{3}$.

Answer: 2 + √3

Q15. In which quadrant does the angle 200° lie, and what are the signs of sinθ, cosθ, tanθ there?

[Quadrant / sign-rule question — very important]

Solution:

Since 180° < 200° < 270°, the angle 200° lies in the 3rd quadrant.

By the ASTC rule, only tan (and cot) are positive in the 3rd quadrant.

So sin200° and cos200° are negative, while tan200° is positive.

Answer: 3rd quadrant; sin and cos negative, tan positive

Q16. In which quadrant does the angle 300° lie, and what are the signs of sinθ, cosθ, tanθ there?

[Quadrant / sign-rule question — very important]

Solution:

Since 270° < 300° < 360°, the angle 300° lies in the 4th quadrant.

By the ASTC rule, only cos (and sec) are positive in the 4th quadrant.

So sin300° and tan300° are negative, while cos300° is positive.

Answer: 4th quadrant; sin and tan negative, cos positive

Q17. Simplify: sin(90°+θ).

[Associated angle formula — very important]

Solution:

Using the standard associated-angle result:

sin(90°+θ) = cosθ.

Answer: cosθ

Q18. Simplify: cos(180°−θ).

[Associated angle formula — very important]

Solution:

Using the standard associated-angle result:

cos(180°−θ) = −cosθ.

Answer: −cosθ

Q19. Simplify: tan(180°+θ).

[Associated angle formula — very important]

Solution:

Using the standard associated-angle result:

$\tan(180° + \theta) = \tan\theta$.

Answer: $\tan\theta$

Q20. Simplify: $\sin(270° - \theta)$.

[Associated angle formula — important]

Solution:

Using the standard associated-angle result:

$\sin(270° - \theta) = -\cos\theta$.

Answer: $-\cos\theta$

Q21. Simplify: $\cos(270° + \theta)$.

[Associated angle formula — important]

Solution:

Using the standard associated-angle result:

$\cos(270° + \theta) = \sin\theta$.

Answer: $\sin\theta$

Q22. Simplify: $\sin(360° - \theta)$.

[Associated angle formula — important]

Solution:

Using the standard associated-angle result:

$\sin(360° - \theta) = -\sin\theta$.

Answer: $-\sin\theta$

Q23. Simplify: $\sin(-\theta) + \cos(-\theta)$.

[Associated angle formula combined — important]

Solution:

$\sin(-\theta) = -\sin\theta$ (sine is an odd function).

$\cos(-\theta) = \cos\theta$ (cosine is an even function).

Sum $= -\sin\theta + \cos\theta = \cos\theta - \sin\theta$.

Answer: $\cos\theta - \sin\theta$

Q24. Evaluate $\sin 150°$.

[Standard-angle evaluation using associated angles — common]

Solution:

$150° = 180° - 30°$.

$\sin(180° - \theta) = \sin\theta$, so $\sin 150° = \sin 30°$.

$= 1/2$.

Answer: $\sin 150° = 1/2$

Q25. Evaluate cos120°.

[Standard-angle evaluation using associated angles — common]

Solution:

$120° = 180° - 60°$.

$\cos(180° - \theta) = -\cos\theta$, so $\cos 120° = -\cos 60°$.

$= -1/2$.

Answer: $\cos 120° = -1/2$

Q26. Evaluate tan135°.

[Standard-angle evaluation using associated angles — common]

Solution:

$135° = 180° - 45°$.

$\tan(180° - \theta) = -\tan\theta$, so $\tan 135° = -\tan 45°$.

$= -1$.

Answer: $\tan 135° = -1$

Q27. Evaluate sin210°.

[Standard-angle evaluation using associated angles — common]

Solution:

$210° = 180° + 30°$.

$\sin(180° + \theta) = -\sin\theta$, so $\sin 210° = -\sin 30°$.

$= -1/2$.

Answer: $\sin 210° = -1/2$

Q28. Evaluate cos300°.

[Standard-angle evaluation using associated angles — common]

Solution:

$300° = 360° - 60°$.

$\cos(360° - \theta) = \cos\theta$, so $\cos 300° = \cos 60°$.

$= 1/2$.

Answer: $\cos 300° = 1/2$

Q29. Prove that $\sin(90°+\theta) = \cos(-\theta)$.

[Combined associated-angle proof — important]

Solution:

LHS: $\sin(90°+\theta) = \cos\theta$ (standard associated-angle result).

RHS: $\cos(-\theta) = \cos\theta$ (cosine is an even function).

Since LHS = RHS = $\cos\theta$, we have $\sin(90°+\theta) = \cos(-\theta)$. (Proved)

Answer: $\sin(90°+\theta) = \cos(-\theta) = \cos\theta$ (proved)

Q30. Simplify: $[\sin(180°+\theta)\cdot\cos(90°+\theta)] / [\tan(270°-\theta)\cdot\sin(360°-\theta)]$.

[Combined associated-angle simplification — classic important closing question]

Solution:

$\sin(180°+\theta) = -\sin\theta$. $\cos(90°+\theta) = -\sin\theta$. Numerator = $(-\sin\theta)(-\sin\theta) = \sin^2\theta$.

$\tan(270°-\theta) = \cot\theta$. $\sin(360°-\theta) = -\sin\theta$. Denominator = $\cot\theta\times(-\sin\theta) = (\cos\theta/\sin\theta)\times(-\sin\theta) = -\cos\theta$.

Expression = $\sin^2\theta / (-\cos\theta) = -\sin^2\theta/\cos\theta$.

Answer: $-\sin^2\theta / \cos\theta$
