Applied Physics 1

Circular Motion

1. Circular Motion

Circular motion is the motion of a body along the circumference of a circle, in which every point of the body moves at a fixed distance (the radius) from a central point called the centre of the circular path. Circular motion is a common feature of both natural and engineering systems — examples include the motion of the Moon around the Earth, a stone tied to a string and whirled around, and the motion of a vehicle taking a turn on a curved road.

Circular motion can be uniform, where the body moves with constant speed along the circular path, or non-uniform, where the speed of the body changes continuously as it moves along the path. In uniform circular motion, although the speed remains constant, the direction of velocity keeps changing at every instant, which means the motion is always accelerated.

2. Angular Displacement, Angular Velocity and Angular Acceleration

2.1 Angular Displacement (θ)

Angular displacement is defined as the angle swept out by the radius vector joining a moving body to the centre of its circular path, in a given interval of time. It describes how far a body has rotated about the centre, rather than how far it has travelled along a straight line.

Angular displacement is measured in radians (rad), where one radian is the angle subtended at the centre of a circle by an arc whose length is equal to the radius of the circle. Angular displacement is a vector quantity for infinitesimally small rotations, with its direction given by the right-hand rule along the axis of rotation.

2.2 Angular Velocity (ω)

Angular velocity is defined as the rate of change of angular displacement with respect to time. It describes how fast a body is rotating about the centre of its circular path:

- $\omega = \Delta\theta / \Delta t$

The SI unit of angular velocity is radian per second (rad/s). Angular velocity is a vector quantity, directed along the axis of rotation, and its magnitude indicates the speed of rotation.

2.3 Angular Acceleration (α)

Angular acceleration is defined as the rate of change of angular velocity with respect to time. It indicates how quickly the angular velocity of a rotating body is increasing or decreasing:

- $\alpha = \Delta\omega / \Delta t$

The SI unit of angular acceleration is radian per second squared (rad/s²). For a body performing uniform circular motion, since the angular velocity remains constant, the angular acceleration is zero; angular acceleration becomes significant only in non-uniform circular motion.

2.4 Frequency and Time Period

Frequency (f or n) is defined as the number of complete revolutions made by a body in one second, while time period (T) is defined as the time taken by the body to complete one full revolution around the circular path. Frequency and time period are reciprocals of each other:

- $f = 1 / T$ and $T = 1 / f$

The SI unit of frequency is hertz (Hz), and the SI unit of time period is second (s).

A summary of these quantities is given below:

Quantity	Symbol	Definition	SI Unit
Angular Displacement	θ	Angle swept by the radius vector about the centre	radian (rad)
Angular Velocity	ω	Rate of change of angular displacement with time	rad/s
Angular Acceleration	α	Rate of change of angular velocity with time	rad/s²
Frequency	f (or n)	Number of complete revolutions made per unit time	hertz (Hz)
Time Period	T	Time taken to complete one full revolution	second (s)

3. Relation Between Linear and Angular Quantities

3.1 Relation Between Linear Velocity and Angular Velocity

For a body moving along a circular path of radius r with angular velocity ω, the linear (tangential) velocity v of the body is related to the angular velocity by the formula:

- $v = \omega r$

This shows that for a given angular velocity, points farther from the centre of rotation (larger r) move with greater linear speed than points closer to the centre.

3.2 Relation Between Linear Acceleration and Angular Acceleration

Similarly, the linear (tangential) acceleration a of a body moving along a circular path of radius r, with angular acceleration α, is related by the formula:

- $a = \alpha r$

Also, the relation between angular velocity, frequency, and time period is given by:

- $\omega = 2\pi f = 2\pi / T$

Worked Example 1:
A wheel completes 300 revolutions in one minute. Calculate its frequency, time period, and angular velocity.

Solution: Number of revolutions per second (frequency) $f = 300 / 60 = 5$ Hz. Time period $T = 1/f = 1/5 = 0.2$ s. Angular velocity $\omega = 2\pi f = 2\pi \times 5 = 10\pi \approx 31.4$ rad/s.

Worked Example 2:
A particle moves along a circular path of radius 2 m with an angular velocity of 4 rad/s. Find its linear velocity.

Solution: Using $v = \omega r$, $v = 4 \times 2 = 8$ m/s.

Worked Example 3:
A rotating wheel of radius 0.5 m has an angular acceleration of 6 rad/s². Find the linear (tangential) acceleration of a point on its rim.

Solution: Using $a = \alpha r$, $a = 6 \times 0.5 = 3$ m/s².

4. Centripetal and Centrifugal Force

4.1 Centripetal Force

Centripetal force is the force that acts on a body moving along a circular path, directed always towards the centre of the circle. It is this force that continuously changes the direction of the velocity of the body, keeping it moving along the curved path instead of a straight line. Without a centripetal force, a moving body would travel in a straight line, in accordance with Newton's first law of motion.

The formula for centripetal force (no derivation, formula only) is:

- $F = m v^2 / r = m \omega^2 r$

where m is the mass of the body, v is its linear velocity, ω is its angular velocity, and r is the radius of the circular path.

Examples of centripetal force in everyday life:

- The tension in a string when a stone tied to it is whirled in a horizontal circle acts as the centripetal force.
- The gravitational force of the Earth on the Moon provides the centripetal force that keeps the Moon in its orbit.
- The frictional force between the tyres of a car and the road provides the centripetal force needed for the car to turn on a flat curved road.
- The normal reaction from the walls of a rotor (in an amusement park ride) provides the centripetal force on the passengers.

4.2 Centrifugal Force

Centrifugal force is a pseudo (apparent) force that appears to act on a body moving in a circular path, directed away from the centre, when the motion is observed from a rotating (non-inertial) frame of reference. Unlike centripetal force, centrifugal force is not a real, physically applied force; it arises only because of the acceleration of the observer's own frame of reference and is used to explain the outward tendency felt by a body in circular motion when viewed from within that rotating system.

Examples of centrifugal effects in everyday life:

- A person standing in a fast-turning bus feels thrown outward, away from the centre of the turn.
- Mud flies off outward from a rapidly rotating bicycle wheel.
- Water is thrown outward from wet clothes in a spinning washing machine drum, which works on this principle.
- A passenger on a merry-go-round feels an outward pull as the ride spins faster.

5. Banking of Roads and Bending of a Cyclist

5.1 Banking of Roads

When a vehicle moves along a curved road, it requires a centripetal force directed towards the centre of the curve to keep it moving along the curved path. On a flat road, this centripetal force is provided only by the friction between the tyres and the road surface, which is often insufficient at higher speeds and can result in skidding.

To overcome this limitation, curved roads are banked, meaning the outer edge of the road is raised above the level of the inner edge. This banking allows a component of the normal reaction force from the road surface to contribute towards the required centripetal force, reducing dependence on friction and allowing vehicles to negotiate the curve safely even at higher speeds.

The formula for the angle of banking (concept and formula only, no derivation) is:

- $\tan\theta = v^2 / (r g)$

where θ is the angle of banking, v is the speed of the vehicle, r is the radius of the curved road, and g is the acceleration due to gravity. This formula gives the ideal angle of banking at which a vehicle can safely take the turn at speed v without depending on friction.

Worked Example 4:
A road is banked for a speed of 20 m/s on a curve of radius 100 m. Find the required angle of banking. (Take $g = 10\ m/s^2$)

Solution: Using $\tan\theta = v^2/(rg)$, $\tan\theta = (20)^2 / (100 \times 10) = 400/1000 = 0.4$. Therefore $\theta = \tan^{-1}(0.4) \approx 21.8°$.

5.2 Bending of a Cyclist

When a cyclist takes a turn on a circular path, they must lean (bend) inward, towards the centre of the curve, in order to obtain the necessary centripetal force safely and maintain balance. If the cyclist remains upright while turning, the required centripetal force must come entirely from friction, and the cyclist risks toppling outward due to the effect of centrifugal force experienced in their own rotating frame of reference.

By leaning inward at an appropriate angle, the cyclist allows a component of the normal reaction and friction to combine and provide exactly the centripetal force required, keeping the cyclist balanced and preventing outward toppling. The formula for the angle of bending (concept and formula only) is the same form as the angle of banking:

- $\tan\theta = v^2 / (r g)$

where θ is the angle through which the cyclist must lean from the vertical, v is the speed of the cyclist, r is the radius of the circular turn, and g is the acceleration due to gravity. This shows that a cyclist must lean more sharply either at higher speeds or while taking sharper (smaller radius) turns.

Worked Example 5:
A cyclist rides at a speed of 9 m/s around a curve of radius 12 m. Find the angle through which the cyclist must lean from the vertical. (Take $g = 10\ m/s^2$)

Solution: Using $\tan\theta = v^2/(rg)$, $\tan\theta = (9)^2 / (12 \times 10) = 81/120 = 0.675$. Therefore $\theta = \tan^{-1}(0.675) \approx 34.1°$.

6. Multiple Choice Questions (MCQs)

Answer the following questions by selecting the most appropriate option.

1. The SI unit of angular displacement is:

A) Degree
B) Radian
C) Metre
D) Hertz

2. Angular velocity is defined as the rate of change of:

A) Linear displacement
B) Angular displacement
C) Linear velocity
D) Angular acceleration

3. The relation between linear velocity v and angular velocity ω is:

A) $v = \omega/r$
B) $v = \omega r$
C) $v = r/\omega$
D) $v = \omega^2 r$

4. The SI unit of frequency is:

A) Radian per second
B) Second
C) Hertz
D) Newton

5. The relation between angular velocity ω and time period T is:

A) $\omega = 2\pi T$
B) $\omega = 2\pi/T$
C) $\omega = T/2\pi$

D) $\omega = \pi/T$

6. The relation between linear acceleration a and angular acceleration α (for tangential acceleration) is:

A) $a = \alpha/r$

B) $a = \alpha r$

C) $a = r/\alpha$

D) $a = \alpha^2 r$

7. Centripetal force acts on a body moving in a circular path in a direction:

A) Tangent to the circle

B) Away from the centre

C) Towards the centre

D) Along the axis of rotation

8. The formula for centripetal force is:

A) $F = mv/r$

B) $F = mv^2/r$

C) $F = mv^2r$

D) $F = m\omega/r$

9. Centrifugal force is best described as:

A) A real force acting towards the centre

B) A pseudo (apparent) force acting away from the centre in a rotating frame

C) The same as gravitational force

D) A force that increases linear velocity

10. Which of the following is an everyday example of centripetal force in action?

A) A ball rolling on a flat table

B) The Moon revolving around the Earth

C) A book resting on a shelf

D) A stone falling freely under gravity

11. Banking of roads is done mainly to:

A) Reduce the length of the road

B) Provide the necessary centripetal force for safe turning at higher speeds

C) Reduce air resistance on vehicles

D) Increase the frictional force to a maximum

12. The formula for the angle of banking of a road (ignoring friction) is:

A) $\tan\theta = v^2/rg$

B) $\tan\theta = rg/v^2$

C) $\tan\theta = v/rg$

D) $\tan\theta = vg/r$

13. A cyclist bends while going around a curve mainly to:

A) Reduce speed
B) Provide the necessary centripetal force and maintain balance
C) Increase the radius of the curve
D) Reduce friction with the ground

14. The formula for the angle of bending of a cyclist is:

A) $\tan\theta = rg/v^2$
B) $\tan\theta = v^2/rg$
C) $\tan\theta = v^2g/r$
D) $\tan\theta = r/v^2g$

15. For a body performing uniform circular motion, which quantity remains constant?

A) Velocity
B) Speed
C) Centripetal force direction
D) Acceleration direction

Answer Key

1. B) Radian | 2. B) Angular displacement | 3. B) $v = \omega r$ | 4. C) Hertz | 5. B) $\omega = 2\pi/T$ | 6. B) $a = \alpha r$ | 7. C) Towards the centre | 8. B) $F = mv^2/r$ | 9. B) A pseudo (apparent) force acting away from the centre in a rotating frame | 10. B) The Moon revolving around the Earth | 11. B) Provide the necessary centripetal force for safe turning at higher speeds | 12. A) $\tan\theta = v^2/rg$ | 13. B) Provide the necessary centripetal force and maintain balance | 14. B) $\tan\theta = v^2/rg$ | 15. B) Speed

7. Broad / Long Answer and Numerical Questions

1. Define angular displacement, angular velocity, and angular acceleration, giving the SI unit of each.

2. Define frequency and time period of a body performing circular motion. Derive the relation between angular velocity, frequency, and time period.

3. Establish the relation between linear velocity and angular velocity for a body moving in a circular path.

4. Establish the relation between linear (tangential) acceleration and angular acceleration for a body moving in a circular path.

5. Explain the concept of centripetal force with two suitable examples from everyday life, stating its formula (no derivation required).

6. Explain the concept of centrifugal force with two suitable examples, and distinguish it from centripetal force.

7. Explain the necessity of banking of roads. State the formula for the angle of banking and the factors on which it depends.

8. Explain why a cyclist bends while taking a turn on a circular path. State the formula for the angle of bending.

9. A wheel completes 300 revolutions in one minute. Calculate its frequency, time period, and angular velocity.

10. A car of mass 1000 kg moves on a circular track of radius 50 m with a speed of 10 m/s. Calculate the centripetal force acting on the car. Also, find the angle of banking required for the same speed if the track is frictionless.