Applied Physics 1

Elasticity — Stress, Strain, and Hooke's Law

1. Elasticity

Elasticity is the property of a material by virtue of which it tends to regain its original shape and size after the removal of the deforming force that had changed its shape or size. Bodies that possess this property to a high degree, such as steel and rubber (within limits), are called elastic bodies, while bodies that do not regain their original shape after deformation, such as putty or wet clay, are called plastic bodies.

When a deforming force is applied to a body, internal forces develop within the material that oppose the deformation and tend to restore the body to its original condition once the external force is removed. The study of elasticity involves understanding these internal restoring effects through the concepts of stress and strain.

2. Stress

Stress is defined as the internal restoring force developed per unit cross-sectional area within a deformed body, acting in response to an externally applied deforming force. It is a measure of how intensely the internal forces are distributed within the material as it resists deformation.

- Stress = Restoring Force / Area = F / A

The SI unit of stress is the pascal (Pa), equivalent to newton per square metre (N/m^2), which is the same unit used for pressure, since stress and pressure share the same physical dimensions.

Depending on the nature of the deforming force and the type of deformation it produces, stress is classified into three types:

Type of Stress	Description
Tensile / Compressive (Longitudinal) Stress	Restoring force per unit area acting perpendicular to the surface, arising from stretching or compressing a body along its length.
Shearing (Tangential) Stress	Restoring force per unit area acting tangentially (parallel) to the surface, arising from a force that changes the shape of a body without changing its volume.

Volumetric (Bulk) Stress	Restoring force per unit area acting normally over the entire surface of a body, arising from a uniform force applied over the whole surface, changing the volume of the body.

3. Strain

Strain is defined as the ratio of the change produced in a dimension of a body (such as its length, shape, or volume) to its original dimension, as a result of an applied deforming force. Since strain is the ratio of two similar quantities (for example, two lengths, or two volumes), it is a dimensionless quantity and has no units.

Corresponding to the three types of stress, strain is also classified into three types:

Type of Strain	Description
Longitudinal Strain	Ratio of change in length to the original length, produced by tensile or compressive stress.
Shearing Strain	Angle (in radians) through which a face of the body originally perpendicular to the fixed face gets turned, produced by shearing stress.
Volumetric Strain	Ratio of change in volume to the original volume, produced by volumetric (bulk) stress.

4. Hooke's Law

Hooke's law states that, within the elastic limit of a material, the stress developed in the material is directly proportional to the strain produced in it. This can be expressed mathematically as:

- Stress ∝ Strain
- Stress = E × Strain

where E is a constant of proportionality known as the modulus of elasticity of the material. This constant depends on the nature of the material and the type of stress and strain involved, but is independent of the actual dimensions of the specimen being tested.

The term “elastic limit” refers to the maximum stress up to which a material obeys Hooke's law and is able to completely regain its original dimensions once the deforming force is removed. Beyond the elastic limit, the material no longer obeys the simple proportionality of Hooke's law, and some permanent (plastic) deformation begins to occur even after the stress is removed.

5. Moduli of Elasticity

Since three different types of stress and strain exist, three corresponding moduli of elasticity are defined, each applicable within the elastic limit of the material:

Modulus of Elasticity	Applicable Stress / Strain	Formula
Young's Modulus (Y)	Longitudinal stress / Longitudinal strain	$Y = (F/A) / (\Delta L/L)$
Bulk Modulus (K)	Volumetric stress / Volumetric strain	$K = (F/A) / (\Delta V/V)$
Modulus of Rigidity (η or G)	Shearing stress / Shearing strain	$\eta = (F/A) / \phi$

Here, F/A represents the applied stress, $\Delta L/L$ represents the longitudinal strain, $\Delta V/V$ represents the volumetric strain, and ϕ represents the shearing strain (angle of shear, in radians). All three moduli of elasticity have the same SI unit as stress, namely the pascal (N/m^2), since strain itself is dimensionless.

These moduli are important material properties used extensively in engineering design: Young's modulus is used in calculations involving beams, columns, and wires under tension or compression; bulk modulus is used in problems involving fluids and materials under uniform pressure; and the modulus of rigidity is used in problems involving twisting or shearing of shafts and structural members.

Worked Example 1:
A wire of length 2 m and cross-sectional area 1×10^{-6} m^2 is stretched by 0.5 mm under a load. If the Young's modulus of the material is 2×10^{11} N/m^2, calculate the tensile strain, the tensile stress, and the force applied.

Solution: Tensile strain = $\Delta L/L = (0.5 \times 10^{-3}) / 2 = 2.5 \times 10^{-4}$. Tensile stress = Y × strain = $2 \times 10^{11} \times 2.5 \times 10^{-4} = 5 \times 10^{7}$ N/m^2. Force applied, F = stress × area = $5 \times 10^{7} \times 1 \times 10^{-6} = 50$ N.

Worked Example 2:
A metal cube of volume 0.01 m^3 is subjected to a pressure that reduces its volume by 2×10^{-5} m^3. If the bulk modulus of the material is 1.6×10^{10} N/m^2, calculate the volumetric strain and the volumetric stress (pressure) applied.

Solution: Volumetric strain = $\Delta V/V = (2 \times 10^{-5}) / 0.01 = 2 \times 10^{-3}$. Volumetric stress = K × strain = $1.6 \times 10^{10} \times 2 \times 10^{-3} = 3.2 \times 10^{7}$ N/m^2.

6. Significance of the Stress-Strain Curve

A stress-strain curve is a graph obtained by plotting the stress developed in a material against the corresponding strain produced, as a gradually increasing load is applied to a test specimen (typically a wire or rod) until it eventually breaks. This curve provides a complete picture of how a material behaves under load and is of great importance in engineering material selection and design.

The key features and regions of a typical stress-strain curve for a ductile material are as follows:

1. Proportional limit: The initial straight-line portion of the curve, starting from the origin, where stress is directly proportional to strain, in accordance with Hooke's law. The slope of this straight-line portion gives the modulus of elasticity of the material.
2. Elastic limit: The point up to which the material returns completely to its original dimensions once the applied stress is removed. Beyond this point, the material begins to undergo some permanent (plastic) deformation, even after the load is removed.
3. Yield point: The point beyond the elastic limit at which the material begins to undergo a significant increase in strain with little or no further increase in stress, marking the onset of plastic deformation. Many engineering structures are designed to operate with stresses well below the yield point, to avoid permanent deformation.
4. Ultimate (breaking) stress: The maximum stress the material can withstand before failure. This point on the curve represents the maximum load-bearing capacity of the material.
5. Breaking (fracture) point: The point at which the material actually fractures or breaks under the applied stress.

The stress-strain curve is significant in engineering for several reasons:

- It allows engineers to determine important material properties such as the modulus of elasticity, yield strength, and ultimate tensile strength, which are essential for the safe design of structures and machine components.
- It helps distinguish between ductile materials (which show a large plastic region before breaking, such as mild steel) and brittle materials (which break shortly after the elastic limit, with little plastic deformation, such as cast iron or glass).
- The area under the stress-strain curve, up to a given strain, represents the energy stored per unit volume of the material (strain energy density) during deformation, which is important in applications involving impact loading and energy absorption.

- It enables engineers to select appropriate materials for specific applications, ensuring that components operate safely within the elastic region under expected working loads, avoiding permanent deformation or failure.

7. Multiple Choice Questions (MCQs)

Answer the following questions by selecting the most appropriate option.

1. Elasticity is the property of a material by virtue of which it:

A) Permanently changes shape under load
B) Regains its original shape and size after the removal of a deforming force
C) Breaks easily under small loads
D) Cannot be deformed at all

2. Stress is defined as:

A) Force per unit volume
B) Restoring force per unit area
C) Change in length per unit length
D) Force per unit length

3. The SI unit of stress is:

A) Newton
B) Pascal (N/m^2)
C) Joule
D) No unit (dimensionless)

4. Strain is defined as:

A) The force applied per unit area
B) The ratio of change in dimension to the original dimension
C) The restoring force per unit volume
D) The energy stored per unit volume

5. Strain is a:

A) Dimensionless quantity with no units
B) Quantity measured in newtons
C) Quantity measured in pascals
D) Vector quantity with direction

6. Which type of strain is produced by a force applied tangentially to a surface?

A) Longitudinal strain
B) Volumetric strain
C) Shearing strain
D) No strain is produced

7. Young's modulus is defined as the ratio of:

A) Volumetric stress to volumetric strain
B) Longitudinal stress to longitudinal strain
C) Shearing stress to shearing strain
D) Force to area

8. Bulk modulus relates to which type of stress and strain?

A) Longitudinal
B) Shearing
C) Volumetric
D) None of these

9. The modulus of rigidity relates:

A) Longitudinal stress to longitudinal strain
B) Shearing stress to shearing strain
C) Volumetric stress to volumetric strain
D) Force to velocity

10. Hooke's law states that, within the elastic limit:

A) Stress is inversely proportional to strain
B) Stress is directly proportional to strain
C) Stress and strain are unrelated
D) Strain is always zero

11. The constant of proportionality in Hooke's law is called:

A) Elastic limit
B) Modulus of elasticity
C) Breaking stress
D) Yield point

12. On a stress-strain curve, the region where Hooke's law is obeyed is:

A) The curved region beyond the elastic limit
B) The straight-line (linear) region near the origin
C) The region after the breaking point
D) The entire curve

13. The point on the stress-strain curve beyond which the material does not return to its original shape after removal of stress is called:

A) The origin
B) The elastic limit
C) The breaking point
D) The proportional constant

14. The maximum stress a material can withstand before it breaks is called:

A) Yield stress
B) Ultimate (breaking) stress

C) Proportional stress
D) Elastic stress

15. The area under the stress-strain curve up to a given strain represents:
A) The modulus of elasticity
B) The energy stored per unit volume in the material
C) The breaking stress
D) The applied force

Answer Key

1. B) Regains its original shape and size after the removal of a deforming force | 2. B) Restoring force per unit area | 3. B) Pascal (N/m^2) | 4. B) The ratio of change in dimension to the original dimension | 5. A) Dimensionless quantity with no units | 6. C) Shearing strain | 7. B) Longitudinal stress to longitudinal strain | 8. C) Volumetric | 9. B) Shearing stress to shearing strain | 10. B) Stress is directly proportional to strain | 11. B) Modulus of elasticity | 12. B) The straight-line (linear) region near the origin | 13. B) The elastic limit | 14. B) Ultimate (breaking) stress | 15. B) The energy stored per unit volume in the material

8. Broad / Long Answer and Numerical Questions

1. Define elasticity, stress, and strain. State the SI unit of stress and explain why strain has no unit.

2. Explain the three types of stress — longitudinal, shearing, and volumetric — with suitable examples of each.

3. Explain the three types of strain — longitudinal, shearing, and volumetric — corresponding to the three types of stress.

4. State Hooke's law. Explain the significance of the elastic limit in relation to this law.

5. Define Young's modulus, bulk modulus, and modulus of rigidity, giving the formula for each.

6. Explain, with the help of a labelled stress-strain curve, the significance of the proportional limit, elastic limit, yield point, and breaking point.

7. Explain the physical significance of the area under a stress-strain curve.

8. Distinguish between elastic and plastic deformation of a material, referring to the stress-strain curve in your explanation.

9. A wire of length 2 m and cross-sectional area 1×10^{-6} m^2 is stretched by 0.5 mm under a load. If the Young's modulus of the material is 2×10^{11} N/m^2, calculate the tensile stress, tensile strain, and the force applied.

10. A metal cube of volume 0.01 m^3 is subjected to a pressure that reduces its volume by 2×10^{-5} m^3. If the bulk modulus of the material is 1.6×10^{10} N/m^2, calculate the volumetric strain and the pressure (volumetric stress) applied.