Applied Physics 1

Hydrodynamics — Fluid Flow, Continuity and Bernoulli's Theorem

1. Fluid Motion

Hydrodynamics is the branch of physics that deals with the study of fluids in motion, along with the forces acting on them and the effects these forces produce. It forms an essential part of applied physics and engineering, since the flow of liquids and gases plays a central role in numerous natural processes and engineering systems, including water supply networks, aircraft design, blood circulation in the human body, and the flow of fluids in pipes and channels.

The motion of a fluid can be broadly classified as steady or unsteady, depending on whether the velocity of the fluid at any given point remains constant with time or varies with time, and further as streamline (laminar) or turbulent, depending on the pattern in which fluid particles move, as discussed below.

2. Streamline Flow and Turbulent Flow

Based on the nature of the motion of fluid particles, the flow of a fluid is classified into two main types:

Type of Flow	Description
Streamline (Laminar) Flow	Flow in which each particle of the fluid follows a smooth, well-defined path, called a streamline, and the velocity at any given point remains constant with time; different streamlines never cross each other.
Turbulent Flow	Flow in which the motion of fluid particles becomes irregular and chaotic, with velocity at a given point fluctuating randomly with time, and the formation of eddies and swirls within the fluid.

Streamline flow generally occurs at low velocities and in fluids of higher viscosity, while turbulent flow tends to occur at higher velocities, in fluids of lower viscosity, or when the fluid encounters obstacles or sharp changes in the shape of its container or flow path. The transition from streamline to turbulent flow, and the conditions under which it occurs, can be predicted using Reynold's number.

3. Reynold's Number

Reynold's number (Re) is a dimensionless quantity used to predict whether the flow of a fluid through a given pipe or channel will be streamline or turbulent. It represents the ratio of the inertial forces to the viscous forces acting within the flowing fluid, and depends on the velocity of flow, the density and viscosity of the fluid, and a characteristic length (such as the diameter of the pipe through which the fluid flows).

Since Reynold's number combines all the physical factors that influence the nature of flow into a single dimensionless number, it provides a convenient and reliable criterion for predicting the type of flow that will occur under given conditions. The approximate ranges of Reynold's number associated with different types of flow (for flow through a cylindrical pipe) are given below:

Range of Reynold's Number (Re)	**Nature of Flow**
Re < 2000 (approximately)	Streamline (laminar) flow
2000 < Re < 3000 (approximately)	Unstable / transitional flow
Re > 3000 (approximately)	Turbulent flow

A low Reynold's number indicates that viscous forces dominate over inertial forces, favouring smooth, orderly streamline flow, while a high Reynold's number indicates that inertial forces dominate, favouring the onset of irregular, turbulent flow.

4. Equation of Continuity

The equation of continuity expresses the principle of conservation of mass for a fluid in steady flow through a pipe or channel of varying cross-sectional area. For an incompressible fluid (a fluid whose density does not change) flowing steadily through a pipe, the mass of fluid entering any section of the pipe per unit time must be equal to the mass of fluid leaving any other section per unit time, since the fluid does not accumulate or diminish within the pipe.

If a_1 and v_1 are the cross-sectional area and velocity of the fluid at one section of the pipe, and a_2 and v_2 are the corresponding values at another section, the equation of continuity is expressed as:

- $a_1 v_1 = a_2 v_2$

This equation shows that, for an incompressible fluid in steady flow, the product of the cross-sectional area and the velocity of flow remains constant along the length of the pipe.

Consequently, wherever the cross-sectional area of the pipe decreases, the velocity of the fluid must increase correspondingly, and wherever the area increases, the velocity must decrease, so that the volume flow rate ($a \times v$) remains the same throughout the pipe.

Worked Example 1:
Water flows through a horizontal pipe of cross-sectional area 4 cm^2 with a velocity of 3 m/s. If the pipe narrows to a cross-sectional area of 2 cm^2, find the velocity of water in the narrower section.

Solution: Using the equation of continuity, $a_1v_1 = a_2v_2$: $4 \times 3 = 2 \times v_2$, so $v_2 = 12/2 = 6$ m/s.

5. Bernoulli's Theorem (Formula and Numericals)

Bernoulli's theorem is based on the principle of conservation of energy applied to a fluid in streamline flow. It states that, for an ideal fluid (incompressible and non-viscous) undergoing steady, streamline flow, the sum of the pressure energy, kinetic energy, and potential energy per unit volume at any point along a streamline remains constant.

The formula for Bernoulli's theorem is expressed as:

- $P + \frac{1}{2} \rho v^2 + \rho g h = \text{constant}$

where P is the pressure at a point, ρ is the density of the fluid, v is the velocity of flow at that point, g is the acceleration due to gravity, and h is the height of that point above a chosen reference level. Applying this equation to two points (1 and 2) along the same streamline gives:

- $P_1 + \frac{1}{2} \rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2} \rho v_2^2 + \rho g h_2$

This equation shows that, along a streamline, an increase in the velocity of a fluid is accompanied by a decrease in its pressure (and vice versa), provided the height remains the same — this important consequence is widely used in explaining a range of practical phenomena, discussed further below.

Worked Example 2:
Water flows through a horizontal pipe of cross-sectional area 5 cm^2 at a velocity of 2 m/s, at a pressure of 2×10^5 Pa. The pipe narrows to a cross-sectional area of 2 cm^2 at the same height. Find the velocity and the pressure of water in the narrower section. (Take density of water $\rho = 1000$ kg/m^3)

Solution: Using the equation of continuity, $a_1v_1 = a_2v_2$: $5 \times 2 = 2 \times v_2$, so $v_2 = 10/2 = 5$ m/s. Since the pipe is horizontal, $h_1 = h_2$, so Bernoulli's equation reduces to $P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$. Substituting values: $2 \times 10^5 + \frac{1}{2} \times 1000 \times 2^2 = P_2 + \frac{1}{2} \times 1000 \times 5^2$. This gives $2 \times 10^5 +$

2000 = P_2 + 12500, so P_2 = 202000 − 12500 = 189500 Pa (approximately 1.895×10^5 Pa). This confirms that as the velocity of water increases in the narrower section, its pressure decreases, in accordance with Bernoulli's theorem.

6. Applications of Bernoulli's Theorem (Name Only)

Bernoulli's theorem finds application in the working principle of a number of devices and natural phenomena, some of which are listed below (names only):

- Venturimeter (used to measure the rate of flow of a fluid through a pipe)
- Lift on an aircraft wing (aerofoil)
- Working of an atomiser / sprayer
- Blowing off of roofs during a storm
- Bunsen burner
- Blood flow measurement in the human circulatory system

7. Multiple Choice Questions (MCQs)

Answer the following questions by selecting the most appropriate option.

1. Hydrodynamics is the branch of physics that deals with the study of:
A) Fluids at rest
B) Fluids in motion
C) Solids under stress
D) Gases at high temperature only

2. A path followed by a fluid particle in streamline flow is called a:
A) Turbulent line
B) Streamline
C) Flow line
D) Reynold's line

3. In streamline (laminar) flow, two streamlines:
A) Always intersect
B) Never intersect
C) Are always parallel to the ground
D) Exist only in gases

4. Turbulent flow is characterised by:
A) Smooth, orderly motion of fluid particles
B) Irregular, chaotic motion with eddies and swirls

C) Zero velocity throughout the fluid
D) Constant velocity at every point at all times

5. Reynold's number is a dimensionless quantity used to predict:

A) The density of a fluid
B) Whether the flow of a fluid will be streamline or turbulent
C) The viscosity of a fluid directly
D) The pressure at a point in a fluid

6. A low value of Reynold's number generally indicates:

A) Turbulent flow
B) Streamline (laminar) flow
C) No flow at all
D) Maximum turbulence

7. A high value of Reynold's number generally indicates:

A) Streamline flow
B) Turbulent flow
C) Zero viscosity
D) Zero velocity

8. The equation of continuity for an incompressible fluid in steady flow through a pipe of varying cross-section is based on the conservation of:

A) Energy
B) Mass
C) Momentum only
D) Charge

9. The equation of continuity is expressed as:

A) $A_1v_1 = A_2v_2$
B) $A_1 + v_1 = A_2 + v_2$
C) $A_1/v_1 = A_2/v_2$
D) $A_1v_1^2 = A_2v_2^2$

10. According to the equation of continuity, if the cross-sectional area of a pipe decreases, the velocity of the fluid flowing through it:

A) Decreases
B) Increases
C) Remains constant
D) Becomes zero

11. Bernoulli's theorem is based on the principle of conservation of:

A) Mass only
B) Energy
C) Momentum only

D) Charge

12. Bernoulli's equation for a fluid in streamline flow is given by:

A) $P + \frac{1}{2}\rho v^2 + \rho gh$ = constant

B) $P - \frac{1}{2}\rho v^2 - \rho gh$ = constant

C) $P \times \frac{1}{2}\rho v^2 \times \rho gh$ = constant

D) $P/\rho v^2$ = constant

13. In Bernoulli's equation, the term $\frac{1}{2}\rho v^2$ represents:

A) Pressure energy per unit volume

B) Kinetic energy per unit volume

C) Potential energy per unit volume

D) Total energy per unit mass

14. Which of the following is an application of Bernoulli's theorem?

A) Venturimeter

B) Screw gauge

C) Vernier callipers

D) Spring balance

15. The lift generated on an aircraft wing is explained on the basis of:

A) Newton's third law only

B) Bernoulli's theorem

C) Stoke's law

D) Pascal's law

Answer Key

1. B) Fluids in motion | 2. B) Streamline | 3. B) Never intersect | 4. B) Irregular, chaotic motion with eddies and swirls | 5. B) Whether the flow of a fluid will be streamline or turbulent | 6. B) Streamline (laminar) flow | 7. B) Turbulent flow | 8. B) Mass | 9. A) $A_1v_1 = A_2v_2$ | 10. B) Increases | 11. B) Energy | 12. A) $P + \frac{1}{2}\rho v^2 + \rho gh$ = constant | 13. B) Kinetic energy per unit volume | 14. A) Venturimeter | 15. B) Bernoulli's theorem

8. Broad / Long Answer and Numerical Questions

1. Define hydrodynamics. Distinguish between streamline (laminar) flow and turbulent flow, with a suitable example of each.

2. Define Reynold's number and explain its significance in predicting the nature of fluid flow in a pipe.

3. Explain, with approximate ranges of Reynold's number, how the nature of flow changes from streamline to turbulent.

4. State the equation of continuity for the flow of an incompressible fluid through a pipe of varying cross-section, and explain the principle on which it is based.

5. Explain, using the equation of continuity, why the speed of a fluid increases as it flows through a narrower section of a pipe.

6. State Bernoulli's theorem and write its equation, explaining the physical significance of each term.

7. Mention any four applications of Bernoulli's theorem (names only, no explanation required).

8. Explain briefly how Bernoulli's theorem accounts for the lift generated on the wing of an aircraft in flight.

9. Water flows through a horizontal pipe of cross-sectional area 4 cm^2 with a velocity of 3 m/s. If the pipe narrows to a cross-sectional area of 2 cm^2, find the velocity of water in the narrower section, using the equation of continuity.

10. Water flows through a pipe of cross-sectional area 5 cm^2 at a velocity of 2 m/s, at a pressure of 2×10^5 Pa. The pipe then narrows to a cross-sectional area of 2 cm^2 at the same height. Using the equation of continuity and Bernoulli's theorem, find the velocity and the pressure of water in the narrower section. (Take density of water ρ = 1000 kg/m^3)