Applied Physics 1

Measurements and Errors in Measurement

1. Measurements — Need and Importance

Measurement is the process of comparing an unknown physical quantity with a known standard quantity of the same kind, called a unit, in order to express its magnitude numerically. Physics is fundamentally an experimental science, and every physical law or theory must ultimately be verified through measurement.

The need for measurement in science and engineering arises from the following reasons:

- It allows physical quantities to be expressed precisely and communicated unambiguously between people.
- It forms the basis for testing and verifying scientific laws and theories through experiments.
- It is essential for the design, manufacture, and quality control of engineering components and machines.
- It enables comparison of results obtained by different observers, at different times and places.
- It provides the numerical data required for calculations, analysis, and decision-making in technical work.

Without accurate measurement, neither scientific progress nor reliable engineering practice would be possible, since every design, construction, and experiment depends on quantities that have been measured correctly.

2. Measuring Instruments and Least Count

A measuring instrument is a device used to determine the magnitude of a physical quantity by comparing it with a standard unit. Different instruments are used depending on the quantity being measured and the degree of accuracy required.

The least count of an instrument is the smallest value of a physical quantity that can be measured accurately using that instrument. It represents the smallest division on the measuring scale of the instrument, and every measurement made with that instrument is uncertain by at least this amount.

Some commonly used instruments, the quantities they measure, and their typical least counts are given below:

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Instrument	Quantity Measured	Least Count
Metre Scale	Length	1 mm (0.1 cm)
Vernier Callipers	Length (small objects)	0.01 cm (0.1 mm)
Screw Gauge	Length (very small / diameter)	0.001 cm (0.01 mm)
Stopwatch	Time	0.1 s or 0.01 s
Spring Balance	Force / Weight	Depends on scale marking
Ammeter / Voltmeter	Current / Voltage	Depends on scale marking

A smaller least count indicates a more precise instrument. For example, a screw gauge, with a least count of 0.001 cm, is far more precise than a metre scale, which has a least count of only 0.1 cm, and is therefore preferred for measuring very small lengths such as the diameter of a thin wire.

3. Types of Measurement

Measurements in physics can be broadly classified into two types, depending on how the value of the physical quantity is obtained.

3.1 Direct Measurement

In direct measurement, the physical quantity is measured directly using a suitable instrument, without the need for any calculation involving other quantities. The reading obtained from the instrument itself gives the value of the required quantity.

Examples of direct measurement:

- Measuring the length of a rod using a metre scale.
- Measuring the mass of an object using a physical balance.
- Measuring time using a stopwatch.
- Measuring temperature using a thermometer.

3.2 Indirect Measurement

In indirect measurement, the required physical quantity cannot be measured directly. Instead, it is calculated using a formula that relates it to other quantities which can themselves be measured directly.

Examples of indirect measurement:

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- Finding the area of a rectangular sheet by measuring its length and breadth and multiplying them.
- Finding the density of a solid by measuring its mass and volume separately and using density = mass / volume.
- Finding the speed of a moving object by measuring distance travelled and time taken, and using speed = distance / time.
- Finding the acceleration due to gravity using the time period and length of a simple pendulum.

4. Errors in Measurement

No measurement, however carefully made, can be perfectly exact. The difference between the measured value of a quantity and its true value is called the error in measurement. Errors arise due to limitations of the measuring instrument, the observer, and the conditions under which the measurement is made.

Errors in measurement are broadly classified into two types: systematic errors and random errors.

Type of Error	Cause	Example
Systematic Error	Arises from a definite cause; follows a fixed pattern; can often be corrected	Zero error in an instrument, faulty calibration, personal bias of observer
Random Error	Arises from unpredictable, uncontrollable variations during measurement	Small fluctuations in repeated readings due to environmental changes

4.1 Systematic Errors

Systematic errors are errors that occur according to a definite rule or pattern and tend to be either consistently positive or consistently negative in successive measurements. Because they follow a predictable pattern, systematic errors can often be identified and corrected.

Common causes of systematic errors include:

- Instrumental errors — arising from faulty design, zero error, or poor calibration of the instrument.

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- Imperfections in experimental technique — such as heat losses in calorimetry experiments.
- Personal errors — arising from an individual observer's bias, such as habitually reading a scale from an angle (parallax error).
- Errors due to external conditions — such as changes in temperature, pressure, or humidity affecting the instrument or the quantity being measured.

Systematic errors can be minimised by using properly calibrated instruments, applying correction factors, improving experimental technique, and taking careful, unbiased readings.

4.2 Random Errors

Random errors are errors that occur irregularly and unpredictably, without following any definite pattern, due to uncontrollable and often unknown variations during the measurement process. Unlike systematic errors, random errors can be positive or negative in an unpredictable manner from one reading to the next.

Random errors typically arise from minor variations in experimental conditions, small fluctuations in the observer's judgement, or slight disturbances in the environment during the experiment.

Since random errors are unpredictable, they cannot be entirely eliminated, but their effect can be reduced by taking a large number of readings and calculating the mean value, since random errors tend to cancel out when averaged over many observations.

5. Absolute Error and Relative Error

5.1 Absolute Error

If $a_1, a_2, a_3, \ldots, a_n$ are the values obtained in n measurements of a quantity, the arithmetic mean a_m is taken as the best possible (true) value of the quantity. The absolute error in each measurement is the magnitude of the difference between the mean value and the individual measured value:

- $\Delta a_i = |a_m - a_i|$

The final absolute error in the experiment is usually taken as the mean of the magnitudes of the individual absolute errors:

- $\Delta a_{mean} = (\Delta a_1 + \Delta a_2 + \ldots + \Delta a_n) / n$

The result of the measurement is then reported as $a = a_m \pm \Delta a_{mean}$, indicating that the true value lies somewhere between $(a_m - \Delta a_{mean})$ and $(a_m + \Delta a_{mean})$.

5.2 Relative Error and Percentage Error

The relative error, also called the fractional error, is the ratio of the mean absolute error to the mean value of the quantity measured:

- Relative error = $\Delta a_{mean} / a_m$

When the relative error is expressed as a percentage, it is called the percentage error:

- Percentage error = $(\Delta a_{mean} / a_m) \times 100\%$

Relative and percentage errors are particularly useful because they express the error in relation to the size of the measured quantity, allowing meaningful comparison of the accuracy of measurements made on quantities of very different magnitudes.

6. Error Propagation

In most experiments, the final result is calculated using a formula that combines several independently measured quantities, each having its own error. Error propagation refers to the method by which the errors in these individual measured quantities combine to determine the overall error in the final calculated result.

6.1 Error in Sum or Difference

If $Z = A + B$ or $Z = A - B$, where A and B are measured with absolute errors ΔA and ΔB respectively, the maximum absolute error in Z is the sum of the individual absolute errors:

- $\Delta Z = \Delta A + \Delta B$

This holds true for both addition and subtraction, since in the worst case the errors in A and B can both act in the same direction to increase the total error in Z.

6.2 Error in Product or Quotient

If $Z = A \times B$ or $Z = A / B$, the maximum relative (fractional) error in Z is the sum of the relative errors in A and B:

- $\Delta Z / Z = \Delta A / A + \Delta B / B$

6.3 Error in a Quantity Raised to a Power

If $Z = A^n / B^m$, where n and m are powers, the relative error in Z is given by:

- $\Delta Z / Z = n(\Delta A / A) + m(\Delta B / B)$

This rule shows that quantities raised to higher powers contribute proportionally more to the overall error, and therefore quantities appearing with large powers in a formula must be measured with special care and precision.

7. Error Estimation and Significant Figures

Error estimation refers to the process of determining, as accurately as possible, the uncertainty associated with a measured or calculated quantity, so that the reliability of an experimental result can be properly assessed and reported.

Closely related to error estimation is the concept of significant figures, which are the digits in a measured or calculated quantity that are known with certainty, plus one additional digit that is estimated or uncertain. The number of significant figures in a result reflects the precision of the measurement used to obtain it.

Rules for determining the number of significant figures:

1. All non-zero digits are significant. For example, 4527 has four significant figures.
2. Zeros between two non-zero digits are significant. For example, 2005 has four significant figures.
3. Leading zeros (zeros to the left of the first non-zero digit) are not significant. For example, 0.0042 has two significant figures.
4. Trailing zeros after a decimal point are significant. For example, 3.500 has four significant figures.
5. Trailing zeros in a number without a decimal point are generally not considered significant unless stated otherwise, or unless the number is expressed in scientific notation.
6. In scientific notation, all digits in the coefficient are significant. For example, 4.20×10^3 has three significant figures.

Significant figures are important because the result of a calculation cannot be more precise than the least precise measurement used in obtaining it. Therefore, when adding, subtracting, multiplying, or dividing measured quantities, the final result must be rounded off to the appropriate number of significant figures, ensuring that reported measurements accurately reflect the true precision of the experiment rather than implying a false sense of accuracy.

8. Multiple Choice Questions (MCQs)

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Answer the following questions by selecting the most appropriate option.

1. The smallest measurement that can be accurately measured by an instrument is called its:

A) Accuracy
B) Precision
C) Least count
D) Range

2. The least count of a vernier callipers is usually:

A) 0.1 cm
B) 0.01 cm
C) 0.001 cm
D) 1 cm

3. Measuring the length of a table directly with a metre scale is an example of:

A) Indirect measurement
B) Direct measurement
C) Systematic error
D) Random error

4. Finding the area of a rectangular sheet by measuring length and breadth and multiplying them is an example of:

A) Direct measurement
B) Indirect measurement
C) Absolute error
D) Relative error

5. A zero error in a screw gauge is an example of:

A) Random error
B) Systematic error
C) Relative error
D) Percentage error

6. Errors caused by unpredictable fluctuations during repeated measurements are called:

A) Systematic errors
B) Random errors
C) Instrumental errors
D) Gross errors

7. The absolute error in a measurement is defined as:

A) The ratio of error to the true value
B) The magnitude of the difference between the true value and measured value
C) The error expressed as a percentage
D) The least count of the instrument

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8. Relative error is also known as:

A) Absolute error
B) Fractional error
C) Systematic error
D) Gross error

9. Percentage error is obtained by:

A) Multiplying relative error by 100
B) Dividing absolute error by 100
C) Adding absolute and relative errors
D) Multiplying absolute error by the mean value

10. When two quantities are added or subtracted, the resultant absolute error is:

A) The product of individual errors
B) The sum of individual absolute errors
C) The difference of individual errors
D) Always zero

11. When two quantities are multiplied or divided, the resultant relative error is:

A) The sum of individual relative errors
B) The product of individual relative errors
C) The difference of individual relative errors
D) Always constant

12. The number of significant figures in the measurement 0.00420 is:

A) 2
B) 3
C) 5
D) 6

13. Which of the following is NOT a type of systematic error?

A) Instrumental error
B) Personal error
C) Random fluctuation error
D) Environmental error

14. Which instrument is used to measure very small lengths such as the diameter of a wire?

A) Metre scale
B) Vernier callipers
C) Screw gauge
D) Measuring tape

15. Repeated measurements are taken and averaged mainly to minimise:

A) Systematic errors

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B) Random errors
C) Gross errors
D) Instrumental errors

Answer Key

1. C) Least count | 2. B) 0.01 cm | 3. B) Direct measurement | 4. B) Indirect measurement | 5. B) Systematic error | 6. B) Random errors | 7. B) The magnitude of the difference between the true value and measured value | 8. B) Fractional error | 9. A) Multiplying relative error by 100 | 10. B) The sum of individual absolute errors | 11. A) The sum of individual relative errors | 12. B) 3 | 13. C) Random fluctuation error | 14. C) Screw gauge | 15. B) Random errors

9. Broad / Long Answer Questions

1. Explain the need for measurement in physics and engineering, giving suitable examples from daily and technical life.

2. What is meant by the least count of an instrument? Explain the least count of a vernier callipers and a screw gauge with examples.

3. Distinguish between direct and indirect measurement, giving two examples of each.

4. Explain systematic errors in measurement. Describe their possible causes and the methods used to minimise them.

5. Explain random errors in measurement. How do they differ from systematic errors, and how can their effect be reduced?

6. Define absolute error, relative error, and percentage error. Explain each with a suitable numerical example.

7. State the rules for error propagation in the sum and difference of two measured quantities, with a derivation.

8. State the rules for error propagation in the product and quotient of two measured quantities, with a derivation.

9. What are significant figures? Explain the rules used for determining the number of significant figures in a measured quantity.

10. Explain the concept of error estimation in an experiment and discuss why reporting a measurement with the correct number of significant figures is important.

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