Applied Physics 1

Force, Momentum and Conservation of Linear Momentum

1. Force

Force is defined as an external agency, a push or a pull, which changes or tends to change the state of rest or uniform motion of a body, or which changes its direction of motion or shape. Force is a vector quantity, having both magnitude and direction, and its SI unit is the newton (N).

According to Newton's second law of motion, force is directly related to the rate of change of momentum of a body. Mathematically:

- $F = dp/dt$

where p is the momentum of the body. For a body of constant mass m moving with velocity v, since $p = mv$, this reduces to the familiar form $F = ma$, where a is the acceleration produced in the body. This shows that force and momentum are closely linked concepts, with force being the physical cause of any change in a body's momentum.

2. Momentum

Momentum (also called linear momentum) is defined as the product of the mass of a body and its velocity. It is a measure of the quantity of motion contained in a moving body and is denoted by p:

- $p = m v$

Momentum is a vector quantity, having the same direction as the velocity of the body. Its SI unit is kilogram-metre per second (kg·m/s), which is dimensionally equivalent to newton-second (N·s).

Key characteristics of momentum:

- A heavier body moving at the same velocity as a lighter body has greater momentum.
- A body at rest has zero momentum, since its velocity is zero.
- Momentum depends on both the mass and the velocity of the body, and a change in either changes the momentum.
- Momentum plays a central role in collisions, explosions, and propulsion problems, where mass is often exchanged or redistributed between interacting bodies.

3. Conservation of Linear Momentum

The law of conservation of linear momentum states that, in the absence of any external force, the total linear momentum of a system of bodies remains constant. In other words, the total momentum of an isolated system before an event, such as a collision or an explosion, is equal to the total momentum of the system after the event.

This law can be derived directly from Newton's third law of motion. Consider two bodies A and B interacting with each other, exerting equal and opposite forces on one another during the interaction (Newton's third law: action = −reaction). Since these are internal forces, and no external force acts on the system, the force on A due to B is exactly balanced by the force on B due to A at every instant. Integrating this over the duration of the interaction shows that the change in momentum of A is equal and opposite to the change in momentum of B, so the total momentum of the system (A + B) remains unchanged.

This can be summarised as:

- Total momentum before interaction = Total momentum after interaction (provided no external force acts)

The law of conservation of linear momentum is one of the most fundamental principles in physics and holds true in all types of interactions, including collisions (elastic and inelastic) and explosions, regardless of the internal forces involved.

4. Applications of Conservation of Linear Momentum

4.1 Recoil of a Gun

When a bullet is fired from a gun, the gun and the bullet form an isolated system before firing, with zero total momentum since both are initially at rest. According to the law of conservation of linear momentum, since no external horizontal force acts on the system during firing, the total momentum immediately after firing must also be zero.

If a bullet of mass m is fired with velocity v, and the gun of mass M recoils with velocity V in the opposite direction, then applying conservation of momentum:

- Total momentum before firing = Total momentum after firing
- $0 = mv + M(-V)$
- $MV = mv$
- $V = mv / M$

This shows that the gun recoils (moves backward) with a velocity inversely proportional to its own mass. Since the mass of the gun M is usually much greater than the mass of the bullet m, the recoil velocity V of the gun is correspondingly much smaller than the velocity v of the bullet, which is why the shooter experiences a noticeable but manageable backward jerk rather than being thrown backward violently.

Worked Example:
A bullet of mass 20 g (0.02 kg) is fired from a gun of mass 4 kg with a velocity of 300 m/s. Find the recoil velocity of the gun.

Solution: Using MV = mv, we get V = mv / M = (0.02 × 300) / 4 = 6 / 4 = 1.5 m/s. The gun therefore recoils with a velocity of 1.5 m/s in the direction opposite to the motion of the bullet.

4.2 Rocket Propulsion (Concept Only)

The forward motion of a rocket is another important application of the law of conservation of linear momentum. A rocket carries its own fuel, which is burnt in a combustion chamber to produce a large volume of hot exhaust gases. These gases are ejected at very high velocity through a nozzle at the rear of the rocket.

Before the gases are ejected, the rocket and the fuel together are at rest (or moving with some initial velocity), and the total momentum of the system has a certain value. As the exhaust gases are expelled backward at high speed, they carry a certain backward momentum with them. Since no external force acts on the rocket-fuel system (ignoring gravity and air resistance for the basic concept), the total momentum of the system must remain conserved. Therefore, to balance the backward momentum carried by the ejected gases, the rocket itself must acquire an equal and opposite forward momentum, causing it to accelerate forward.

This is essentially the same principle as the recoil of a gun, except that in a rocket the process is continuous, with gas being ejected steadily as long as fuel is burnt, resulting in a continuously increasing forward velocity of the rocket as its mass decreases. This is a conceptual application of momentum conservation and does not require detailed derivation at this level.

5. Impulse and Impulsive Force

5.1 Impulse

Impulse is defined as the product of the force acting on a body and the time for which it acts. It is a measure of the total effect of a force applied over a short interval of time and is equal to the total change in momentum produced in the body:

- Impulse $J = F \times \Delta t = \Delta p = m v_2 - m v_1$

Impulse is a vector quantity, with the same direction as the applied force, and its SI unit is newton-second (N·s), which is dimensionally equivalent to the unit of momentum, kg·m/s.

5.2 Impulsive Force

An impulsive force is a very large force that acts on a body for an extremely short interval of time, producing a finite and measurable change in the momentum of the body. Although the force itself may be difficult to measure directly because it varies rapidly during the interaction, its overall effect can be conveniently expressed and calculated in terms of the impulse it produces.

Examples of impulsive forces in everyday life:

- The force exerted by a bat on a ball during a hit in cricket or baseball.
- The force experienced during a hammer striking a nail.
- The force during a collision between two vehicles.
- The force exerted by the ground on a person jumping down from a height and landing.

5.3 Practical Significance of Impulse

Since impulse ($J = F \times \Delta t$) for a given change in momentum is constant, increasing the time of contact (Δt) during an impact reduces the average force (F) experienced, and decreasing the time of contact increases the average force. This principle is widely applied in daily life and engineering design:

- A cricketer draws their hands backward while catching a fast-moving ball, increasing the time of impact and thereby reducing the force on the hands.
- Cushions, mattresses, and padded flooring are used to increase the time of impact during a fall, reducing the force of injury.
- Vehicles are fitted with airbags and crumple zones, which increase the time over which the body decelerates during a collision, reducing the impulsive force on passengers.
- A person jumping from a height bends their knees on landing, increasing the time taken to come to rest and reducing the impact force on the legs.

Worked Example:

A force of 50 N acts on a body for 0.2 s. Find the impulse produced and the change in momentum of the body.

Solution: Impulse $J = F \times \Delta t = 50 \times 0.2 = 10$ N·s. Since impulse equals the change in momentum, the change in momentum of the body is also 10 kg·m/s.

6. Multiple Choice Questions (MCQs)

Answer the following questions by selecting the most appropriate option.

1. The SI unit of momentum is:

A) N
B) kg·m/s
C) kg·m/s^2
D) N·s^{-1}

2. Newton's second law of motion relates force to the rate of change of:

A) Velocity
B) Momentum
C) Displacement
D) Acceleration only

3. Impulse is equal to the change in:

A) Mass
B) Momentum
C) Kinetic energy
D) Velocity only

4. The SI unit of impulse is:

A) N·s
B) N/s
C) kg·m
D) J·s

5. The law of conservation of linear momentum applies to a system when:

A) Internal forces alone act
B) No external force acts on the system
C) Only gravity acts
D) The system is accelerating

6. In the recoil of a gun, the momentum of the gun and the bullet:

A) Are unrelated
B) Are equal in magnitude and opposite in direction
C) Are equal in magnitude and same in direction
D) Both become zero

7. A rocket moves forward mainly due to:

A) Air pushing against it

B) The conservation of angular momentum

C) The conservation of linear momentum as exhaust gases are ejected backward

D) Friction with the atmosphere

8. An impulsive force is characterised by:

A) A small force acting for a long time

B) A large force acting for a very short time

C) A constant force acting indefinitely

D) Zero force acting instantaneously

9. Which of the following is an example of an impulsive force?

A) Gravity acting on a falling stone

B) A bat striking a ball

C) Tension in a slowly stretched string

D) Friction on a sliding block

10. Momentum is a quantity that is:

A) Scalar

B) Vector

C) Dimensionless

D) Always positive

11. According to Newton's second law, F =

A) m/v

B) dp/dt

C) mv^2

D) p/t^2

12. A gun of mass M fires a bullet of mass m with velocity v. The recoil velocity of the gun is given by:

A) V = mv/M

B) V = Mv/m

C) V = mv × M

D) V = (m+M)v

13. In an elastic collision, which of the following is conserved?

A) Only momentum

B) Only kinetic energy

C) Both momentum and kinetic energy

D) Neither momentum nor kinetic energy

14. The dimensional formula of impulse is the same as that of:

A) Force

B) Momentum
C) Energy
D) Power

15. A cricketer moves his hands backward while catching a fast ball mainly to:
A) Increase the force of impact
B) Decrease the time of impact and increase force
C) Increase the time of impact and reduce the force
D) Have no effect on the force

Answer Key

1. B) kg·m/s | 2. B) Momentum | 3. B) Momentum | 4. A) N·s | 5. B) No external force acts on the system | 6. B) Are equal in magnitude and opposite in direction | 7. C) The conservation of linear momentum as exhaust gases are ejected backward | 8. B) A large force acting for a very short time | 9. B) A bat striking a ball | 10. B) Vector | 11. B) dp/dt | 12. A) V = mv/M | 13. C) Both momentum and kinetic energy | 14. B) Momentum | 15. C) Increase the time of impact and reduce the force

7. Broad / Long Answer and Numerical Questions

1. Define force and momentum. Derive the relation between force and rate of change of momentum from Newton's second law.

2. State the law of conservation of linear momentum. Derive it using Newton's third law of motion for a two-body system.

3. Explain the recoil of a gun on the basis of the law of conservation of linear momentum, and derive an expression for the recoil velocity of the gun.

4. Explain the working principle of a rocket on the basis of the law of conservation of linear momentum (concept only, no derivation required).

5. Define impulse and impulsive force. Derive the relation between impulse and change in momentum.

6. Explain, with suitable examples, how the concept of impulsive force is applied in daily life to reduce injury or damage during sudden impacts.

7. A bullet of mass 20 g is fired from a gun of mass 4 kg with a velocity of 300 m/s. Calculate the recoil velocity of the gun.

8. Two bodies of masses 3 kg and 5 kg moving in opposite directions with velocities 4 m/s and 2 m/s respectively collide and stick together. Find their common velocity after collision using conservation of momentum.

9. A force of 50 N acts on a body for 0.2 s. Calculate the impulse produced and the change in momentum of the body.

10. Explain why a glass tumbler dropped on a cushioned surface is less likely to break than when dropped on a hard concrete floor, using the concept of impulse and impulsive force.