Applied Physics 1

Thermal Expansion and Specific Heats of a Gas

1. Expansion of Solids, Liquids and Gases

Thermal expansion refers to the increase in the dimensions (length, area, or volume) of a substance as a result of an increase in its temperature. When a substance is heated, the kinetic energy of its constituent molecules increases, causing them to vibrate or move more vigorously and, on average, occupy a greater amount of space, resulting in an overall expansion of the substance.

The extent of thermal expansion differs considerably among the three common states of matter, mainly because of differences in the strength of intermolecular forces holding their particles together:

State of Matter	Nature of Expansion on Heating	Example
Solids	Expand relatively less; may expand in length (linear), area (surface), or volume (cubical), depending on the shape of the body	A metal rod becomes slightly longer when heated; railway tracks expand in summer
Liquids	Expand more than solids for the same rise in temperature; liquids only exhibit cubical (volume) expansion, since they have no fixed shape	Mercury or alcohol rises in a thermometer tube on heating; water expands slightly on heating (except in the anomalous range near 0–4°C)
Gases	Expand the most for the same rise in temperature, among the three states of matter, since intermolecular forces in gases are very weak	Air inside a balloon expands and the balloon may burst if heated excessively; hot air balloons rise because heated air expands and becomes less dense

In general, gases expand the most for a given rise in temperature, followed by liquids, with solids expanding the least, since the particles in a solid are held together by relatively strong intermolecular forces that restrict their movement, while gas molecules are almost free to move and are only weakly attracted to one another.

2. Coefficients of Linear, Surface and Cubical Expansion of Solids

Since a solid has a definite shape, its thermal expansion can be considered in three distinct ways — as an increase in length, an increase in surface area, or an increase in volume — depending on which dimension of the solid is being considered. Each of these types of expansion is characterised by its own coefficient of expansion:

Coefficient	Symbol	Definition	Formula
Coefficient of Linear Expansion	α	Increase in length per unit original length, per degree rise in temperature	$\alpha = \Delta L / (L\ \Delta\theta)$
Coefficient of Surface (Areal) Expansion	β	Increase in area per unit original area, per degree rise in temperature	$\beta = \Delta A / (A\ \Delta\theta)$
Coefficient of Cubical (Volume) Expansion	γ	Increase in volume per unit original volume, per degree rise in temperature	$\gamma = \Delta V / (V\ \Delta\theta)$

In each of these formulae, $\Delta\theta$ represents the rise in temperature, and L, A, and V represent the original length, area, and volume of the solid respectively, before heating. The units of α, β, and γ are all expressed as "per degree Celsius" ($°C^{-1}$) or "per kelvin" (K^{-1}), since they represent a fractional change per unit rise in temperature.

3. Relation Among the Coefficients of Expansion

For a given solid material, the coefficients of linear, surface, and cubical expansion are not independent of one another; they are related through simple numerical ratios, since surface expansion involves two dimensions of the solid, and cubical expansion involves all three dimensions. These relations are given below:

Relation	Expression
Surface expansion coefficient in terms of linear expansion coefficient	$\beta = 2\alpha$
Cubical expansion coefficient in terms of linear expansion coefficient	$\gamma = 3\alpha$
Combined ratio	$\alpha : \beta : \gamma = 1 : 2 : 3$

These relations show that, for the same material and the same rise in temperature, the coefficient of surface expansion is twice the coefficient of linear expansion, and the coefficient of cubical expansion is three times the coefficient of linear expansion. This is a useful result, since it allows the surface or cubical expansion of a solid to be calculated directly from its coefficient of linear expansion, which is usually the value most commonly tabulated for engineering materials.

Worked Example 1:
A metal rod of length 2 m at 20°C has a coefficient of linear expansion of 1.2×10^{-5} per °C. Calculate its length when heated to 120°C.

Solution: Rise in temperature, $\Delta\theta = 120 - 20 = 100$°C. Increase in length, $\Delta L = \alpha L \Delta\theta = 1.2 \times 10^{-5} \times 2 \times 100 = 2.4 \times 10^{-3}$ m = 2.4 mm. New length = 2 m + 0.0024 m = 2.0024 m.

Worked Example 2:
A metal cube of side 10 cm at 25°C has a coefficient of linear expansion of 2×10^{-5} per °C. Calculate the increase in its volume when heated to 225°C.

Solution: Rise in temperature, $\Delta\theta = 225 - 25 = 200$°C. Original volume, $V = (0.1)^3 = 1 \times 10^{-3}$ m^3. Coefficient of cubical expansion, $\gamma = 3\alpha = 3 \times 2 \times 10^{-5} = 6 \times 10^{-5}$ per °C. Increase in volume, $\Delta V = \gamma V \Delta\theta = 6 \times 10^{-5} \times 1 \times 10^{-3} \times 200 = 1.2 \times 10^{-5}$ m^3.

4. Specific Heats of a Gas: Cp and Cv (Mention Only)

Unlike solids and liquids, a gas can be heated under two distinctly different conditions — at constant volume, or at constant pressure — and the quantity of heat required to raise the temperature of a unit mass of gas by one degree is different in each case. This leads to two distinct specific heats being defined for a gas:

- Specific heat at constant volume (Cv): The quantity of heat required to raise the temperature of a unit mass of gas through one degree, while its volume is kept constant.
- Specific heat at constant pressure (Cp): The quantity of heat required to raise the temperature of a unit mass of gas through one degree, while its pressure is kept constant.

When a gas is heated at constant pressure, it is allowed to expand, and so the heat supplied must not only raise the temperature of the gas (increasing its internal energy) but also provide the additional energy needed for the gas to do external work against the surrounding pressure as it expands. In contrast, when a gas is heated at constant volume, no expansion takes place, so all the heat supplied goes solely towards increasing the internal energy of the gas.

For this reason, the specific heat at constant pressure (Cp) is always greater than the specific heat at constant volume (Cv) for a given gas:

- $C_p > C_v$

This relationship (mention only) reflects the fact that heating a gas at constant pressure requires additional heat energy beyond what is needed at constant volume, since part of that additional heat is converted into external work done by the expanding gas, rather than contributing entirely to a rise in temperature.

5. Multiple Choice Questions (MCQs)

Answer the following questions by selecting the most appropriate option.

1. Thermal expansion refers to the:
A) Decrease in the dimensions of a body on heating
B) Increase in the dimensions of a body on heating
C) Change in the mass of a body on heating
D) Change in the colour of a body on heating

2. Among solids, liquids, and gases, which generally expands the most for the same rise in temperature?
A) Solids
B) Liquids
C) Gases
D) All expand equally

3. Liquids exhibit only which type of thermal expansion?
A) Linear expansion only
B) Surface expansion only
C) Cubical (volume) expansion
D) No expansion at all

4. The coefficient of linear expansion (α) of a solid is defined as the increase in length per unit original length per:
A) Unit area
B) Unit rise in temperature
C) Unit volume
D) Unit time

5. The formula for the coefficient of linear expansion is:
A) $\alpha = \Delta L / (L\, \Delta\theta)$
B) $\alpha = L / (\Delta L\, \Delta\theta)$
C) $\alpha = \Delta L \times L \times \Delta\theta$
D) $\alpha = \Delta\theta / (L\, \Delta L)$

6. The coefficient of surface expansion is related to the coefficient of linear expansion by:

A) $\beta = \alpha$

B) $\beta = 2\alpha$

C) $\beta = 3\alpha$

D) $\beta = \alpha/2$

7. The coefficient of cubical expansion is related to the coefficient of linear expansion by:

A) $\gamma = \alpha$

B) $\gamma = 2\alpha$

C) $\gamma = 3\alpha$

D) $\gamma = \alpha/3$

8. The ratio $\alpha : \beta : \gamma$ for a solid is:

A) 1 : 1 : 1

B) 1 : 2 : 3

C) 3 : 2 : 1

D) 1 : 3 : 2

9. Railway tracks are laid with small gaps between successive rails mainly to allow for:

A) Water drainage

B) Thermal expansion of the rails in summer

C) Ease of installation only

D) Reduction in the weight of rails

10. The symbol Cp represents the specific heat of a gas measured at:

A) Constant volume

B) Constant pressure

C) Constant temperature

D) Constant density

11. The symbol Cv represents the specific heat of a gas measured at:

A) Constant pressure

B) Constant volume

C) Constant temperature

D) Constant mass only

12. For a given gas, the relationship between Cp and Cv is generally such that:

A) Cp is less than Cv

B) Cp is greater than Cv

C) Cp is always equal to Cv

D) Cp and Cv cannot be compared

13. The reason Cp is greater than Cv for a gas is that:

A) Heating at constant volume requires additional work to be done by the gas

B) Heating at constant pressure requires additional heat to also do external work as the gas expands

C) Cv accounts for expansion work while Cp does not

D) There is no physical reason; it is a coincidence

14. Mercury is commonly used in thermometers mainly because it expands:

A) Very irregularly with temperature

B) Fairly uniformly over a wide range of temperature

C) Only at very high temperatures

D) Only when mixed with water

15. A bimetallic strip used in thermostats works on the principle that:

A) All metals expand by exactly the same amount when heated

B) Different metals have different coefficients of linear expansion, causing the strip to bend on heating

C) Metals do not expand at all on heating

D) Only one of the two metals in the strip can conduct heat

Answer Key

1. B) Increase in the dimensions of a body on heating | 2. C) Gases | 3. C) Cubical (volume) expansion | 4. B) Unit rise in temperature | 5. A) $\alpha = \Delta L / (L\ \Delta\theta)$ | 6. B) $\beta = 2\alpha$ | 7. C) $\gamma = 3\alpha$ | 8. B) 1 : 2 : 3 | 9. B) Thermal expansion of the rails in summer | 10. B) Constant pressure | 11. B) Constant volume | 12. B) Cp is greater than Cv | 13. B) Heating at constant pressure requires additional heat to also do external work as the gas expands | 14. B) Fairly uniformly over a wide range of temperature | 15. B) Different metals have different coefficients of linear expansion, causing the strip to bend on heating

6. Broad / Long Answer and Numerical Questions

1. Explain thermal expansion. Compare the extent of expansion in solids, liquids, and gases for the same rise in temperature, giving reasons for the difference.

2. Define the coefficient of linear expansion of a solid, and write its formula.

3. Define the coefficient of surface (areal) expansion of a solid, and write its formula.

4. Define the coefficient of cubical (volume) expansion of a solid, and write its formula.

5. Derive the relationship between the coefficients of linear, surface, and cubical expansion of a solid ($\beta = 2\alpha$ and $\gamma = 3\alpha$).

6. Explain, with two practical examples, why allowance for thermal expansion is an important consideration in engineering design and construction.

7. Explain why liquids exhibit only cubical expansion and not linear or surface expansion.

8. Mention the specific heats of a gas at constant pressure (Cp) and at constant volume (Cv), and state which of the two is greater, giving the reason briefly (mention only, no derivation required).

9. A metal rod of length 2 m at 20°C has a coefficient of linear expansion of 1.2×10^{-5} per °C. Calculate its length when heated to 120°C.

10. A metal cube of side 10 cm at 25°C has a coefficient of linear expansion of 2×10^{-5} per °C. Calculate the increase in its volume when heated to 225°C. (Use $\gamma = 3\alpha$)