CIRCUIT THEORY

Unit 2: Single Phase A.C. Circuits

Diploma / Polytechnic Engineering — Study Notes

Introduction

In electrical engineering, quantities such as voltage and current can either be direct (D.C.), which remain constant with time, or alternating (A.C.), which vary periodically in both magnitude and direction with time. A single phase A.C. circuit is a circuit supplied by a single alternating voltage source, and it forms the basis for understanding domestic and light industrial power supply systems. This chapter explains how alternating voltage is generated, how sinusoidal quantities are represented using phasors, and how the basic circuit elements — resistance (R), inductance (L) and capacitance (C) — respond to an alternating supply.

2.1 Generation of Alternating Voltage

An alternating voltage is a voltage that changes its magnitude and direction (polarity) periodically with respect to time. It is generated by rotating a coil in a uniform magnetic field, or by rotating a magnetic field within a stationary coil, based on the principle of electromagnetic induction (Faraday's law).

Principle of Generation

An alternating voltage is generated by an A.C. generator (alternator) which works on the principle of electromagnetic induction: whenever a conductor moves in a magnetic field such that it cuts the magnetic flux lines, an EMF is induced in the conductor. When a rectangular coil is rotated at a uniform angular velocity (ω) within a uniform magnetic field, the flux linking the coil changes continuously, inducing a sinusoidal EMF in the coil.

Construction (Basic Elements)

- Field system: produces the magnetic field, either by permanent magnets or electromagnets.
- Armature: the rotating (or stationary) coil in which the EMF is induced.
- Slip rings and brushes: used to connect the rotating armature to the external circuit and collect the generated EMF.

Working / Derivation of Sinusoidal Waveform

1. Consider a rectangular coil of N turns rotating with a uniform angular velocity ω (rad/s) in a uniform magnetic field of maximum flux density Bm.
2. At any instant, the flux linking the coil is $\varphi = \varphi m \cos(\omega t)$, where φm is the maximum flux and $\theta = \omega t$ is the angle rotated from the position of maximum flux linkage.
3. According to Faraday's law, the instantaneous EMF induced is $e = -N\,(d\varphi/dt)$, which on differentiation gives $e = N\cdot\omega\cdot\varphi m\cdot\sin(\omega t)$.
4. Substituting $Em = N\cdot\omega\cdot\varphi m$ as the maximum value of the induced EMF, the instantaneous EMF equation becomes $e = Em \sin(\omega t)$, which represents a sinusoidal (sine wave) alternating voltage.

> *Equation of alternating voltage: $e = Em \sin(\omega t)$, where Em = maximum value of EMF, ω = angular velocity = 2πf (rad/s), and f = frequency of rotation in Hz.*

Important Terms Related to Alternating Quantities

Term	Meaning
Instantaneous value	The value of an alternating quantity at any particular instant of time, denoted by small letters e, i, v.
Amplitude (Maximum value)	The maximum value attained by an alternating quantity in one cycle, denoted by Em, Im, Vm.
Cycle	One complete set of positive and negative values of an alternating quantity.
Time period (T)	The time taken to complete one cycle, measured in seconds; $T = 1/f$.
Frequency (f)	The number of cycles completed per second, measured in Hertz (Hz); $f = 1/T$.
Angular frequency (ω)	The rate of change of angle with respect to time, $\omega = 2\pi f$, measured in rad/s.

Applications

- Generation of A.C. power in power plants (thermal, hydro, nuclear) for supply to homes and industries.
- Basis of working of alternators used in automobiles and standby generators.
- Fundamental principle behind transformers, induction motors, and A.C. measuring instruments.

Limitations

- The simple analysis assumes a uniform magnetic field and constant angular velocity, which may not hold exactly in practical machines.
- Sinusoidal generation described here applies mainly to single-phase systems; practical power generation is normally three-phase.

2.2 Phasor Representation of Sinusoidal Quantities

Analysing A.C. circuits directly using instantaneous sinusoidal equations ($e = E_m \sin \omega t$) is mathematically tedious, especially when adding or comparing multiple alternating quantities having different phase angles. To simplify this, sinusoidal quantities are represented by phasors — rotating vectors whose length represents the magnitude and whose angle represents the phase of the alternating quantity.

Concept of a Phasor

A phasor is a straight line of definite length, rotating in the anticlockwise direction at a constant angular velocity (ω), used to represent a sinusoidally varying quantity. The length of the phasor represents the maximum (or RMS) value of the quantity, and its angular position at any instant represents the phase of the quantity at that instant. The projection of the rotating phasor on the vertical axis at any instant gives the instantaneous value of the sinusoidal quantity.

Steps to Represent a Sinusoidal Quantity as a Phasor

1. Draw a reference (horizontal) axis, usually representing the zero-phase position.
2. Draw a line (phasor) of length proportional to the maximum or RMS value of the alternating quantity.
3. Set the angle of the phasor from the reference axis equal to the phase angle of the quantity (positive angle measured anticlockwise for a leading quantity, and clockwise for a lagging quantity).
4. For two or more alternating quantities of the same frequency, draw all phasors from the same origin, with each at its correct phase angle relative to the reference axis, to compare or add them using the parallelogram or triangle law of vector addition.

Phase and Phase Difference

- Phase: the fractional part of a time period or cycle through which the quantity has advanced from a chosen reference (zero) point.
- Phase difference: the angle by which one alternating quantity leads or lags another alternating quantity of the same frequency.
- In-phase quantities: two quantities reaching their maximum and zero values at the same time (phase difference = 0°).
- Quantities in quadrature: phase difference of 90° between two alternating quantities.
- Leading and lagging quantities: a quantity is said to lead another if it reaches its maximum value earlier, and to lag if it reaches its maximum value later.

Mathematical (Complex/Polar) Representation of a Phasor

A phasor can also be expressed mathematically using rectangular form ($A = a + jb$) or polar form ($A = |A|\angle\theta$), where $|A|$ is the magnitude and θ is the phase angle. This mathematical form allows alternating quantities to be added, subtracted, multiplied or divided using the ordinary rules of complex algebra, which greatly simplifies A.C. circuit analysis compared to working with instantaneous sine equations.

Applications

- Used for easy addition and subtraction of alternating voltages and currents having different phase angles.
- Simplifies analysis of series and parallel A.C. circuits using phasor (vector) diagrams.
- Forms the basis of complex number (j-operator) method used in A.C. circuit calculations.
- Used in power calculations (active, reactive, apparent power) through phasor relationships between voltage and current.

Limitations

- Phasor representation is valid only for sinusoidal quantities of the same frequency; it cannot directly represent non-sinusoidal or transient quantities.
- Phasor diagrams become complex and difficult to interpret when many quantities of different magnitudes and phases are involved.

2.3 R, L, C Circuit Elements — Voltage and Current Response

When an alternating voltage is applied to a circuit containing a pure resistor (R), a pure inductor (L) or a pure capacitor (C), each element responds differently in terms of the phase relationship between the applied voltage and the resulting current, and in terms of the opposition it offers to current flow.

2.3.1 Pure Resistive (R) Circuit

When a pure resistor R is connected across an alternating voltage $v = V_m \sin(\omega t)$, the current is given by Ohm's law as $i = v/R = (V_m/R) \sin(\omega t) = I_m \sin(\omega t)$. Since both the voltage and current equations have the same sine function with no phase angle difference, the current is in phase with the applied voltage in a pure resistive circuit.

- Phase relationship: current and voltage reach their zero and maximum values at the same instant (phase difference = 0°).
- Opposition offered: resistance R (in ohms), which is independent of frequency.
- Power: average power $P = V \times I$ (both in RMS values), since voltage and current are in phase; power factor = $\cos(0°) = 1$ (unity power factor).

2.3.2 Pure Inductive (L) Circuit

When a pure inductor L is connected across an alternating voltage $v = V_m \sin(\omega t)$, the current lags behind the applied voltage by 90° ($\pi/2$ radians), and is given by $i = I_m \sin(\omega t - 90°)$. This happens because the inductor opposes any change in current by inducing a back EMF, which delays the build-up of current relative to the voltage.

- Phase relationship: current lags the voltage by 90°.
- Opposition offered: inductive reactance $X_L = \omega L = 2\pi f L$ (in ohms), which increases with frequency.
- Power: average power consumed over a complete cycle is zero, since the positive and negative power values cancel out; power factor = cos(90°) = 0.

2.3.3 Pure Capacitive (C) Circuit

When a pure capacitor C is connected across an alternating voltage $v = V_m \sin(\omega t)$, the current leads the applied voltage by 90° ($\pi/2$ radians), and is given by $i = I_m \sin(\omega t + 90°)$. This happens because the current charging the capacitor is maximum when the voltage starts changing from zero, and becomes zero when the voltage is at its peak (fully charged).

- Phase relationship: current leads the voltage by 90°.
- Opposition offered: capacitive reactance $X_C = 1/(\omega C) = 1/(2\pi f C)$ (in ohms), which decreases with an increase in frequency.
- Power: average power consumed over a complete cycle is zero, similar to the inductive circuit; power factor = cos(90°) = 0.

Comparison of R, L and C Circuit Elements

Parameter	Resistor (R)	Inductor (L)	Capacitor (C)
Phase of current w.r.t. voltage	In phase (0°)	Lags by 90°	Leads by 90°
Opposition offered	Resistance R	Inductive reactance $X_L = \omega L$	Capacitive reactance $X_C = 1/(\omega C)$
Effect of frequency	No effect (independent of f)	X_L increases with frequency	X_C decreases with frequency
Power factor	Unity (1)	Zero (lagging)	Zero (leading)
Average power consumed	$P = V \times I$	Zero	Zero

> *Memory aid — 'CIVIL': In a Capacitor, Current (I) leads Voltage (V); in an inductor (L), Voltage (V) leads Current (I). This helps in quickly recalling the phase relationships for L and C circuits.*

Applications

- Pure resistive behaviour is used in heating elements, incandescent lamps, and resistive loads.
- Inductive behaviour is applied in transformers, chokes, motors and inductive filters.
- Capacitive behaviour is used in power factor correction, filters, and timing/coupling circuits.
- The R, L, C responses together form the basis for analysing series and parallel R-L, R-C and R-L-C A.C. circuits.

Limitations

- Practical resistors, inductors and capacitors are never 'pure'; a practical inductor has some resistance, and a practical capacitor has some leakage, so ideal phase relationships are only approximately true.

- The simple sinusoidal response described applies to linear circuit elements operating in steady state; it does not describe transient (switch-on/switch-off) behaviour.

Multiple Choice Questions (MCQs)

Choose the correct option for each question. The correct answer is shown in bold within the options, and a separate answer key is also given at the end.

Q1. An alternating voltage is generated based on the principle of:

a) Electrostatic induction
b) Electromagnetic induction
c) Thermoelectric effect
d) Photoelectric effect

Q2. The equation of a sinusoidal alternating voltage is given by:

a) e = Em t
b) e = Em sin(ωt)
c) e = Em / t
d) e = Em cos(2t)

Q3. The angular frequency ω is related to frequency f as:

a) $\omega = f$
b) $\omega = 2\pi f$
c) $\omega = f/2\pi$
d) $\omega = \pi f$

Q4. A phasor is used to represent an alternating quantity by:

a) A stationary line only
b) A rotating vector with magnitude and phase angle
c) A single numerical value
d) A DC quantity

Q5. Two alternating quantities are said to be in quadrature when their phase difference is:

a) 0°
b) 45°
c) 90°
d) 180°

Q6. In a pure resistive A.C. circuit, the current and voltage are:

a) In phase with each other
b) Current leads voltage by 90°
c) Current lags voltage by 90°
d) Out of phase by 180°

Q7. In a pure inductive circuit, the current:

a) Leads the voltage by 90°
b) Lags the voltage by 90°
c) Is in phase with voltage
d) Lags the voltage by 180°

Q8. In a pure capacitive circuit, the current:

a) Lags the voltage by 90°
b) Is in phase with voltage
c) Leads the voltage by 90°
d) Leads the voltage by 180°

Q9. The inductive reactance XL of a pure inductor is given by:

a) XL = 1/(ωL)
b) XL = ωL
c) XL = ωC
d) XL = 1/(ωC)

Q10. The power factor of a pure capacitive or pure inductive circuit is:

a) Unity
b) Zero
c) 0.5
d) 0.707

Answer Key

1-b | 2-b | 3-b | 4-b | 5-c | 6-a | 7-b | 8-c | 9-b | 10-b

Broad / Long Answer Questions

1. Explain the principle and working of generation of alternating voltage with a neat diagram, and derive the equation e = Em sin(ωt).
2. Define the following terms related to alternating quantities: instantaneous value, amplitude, cycle, time period and frequency.
3. What is a phasor? Explain how a sinusoidal quantity is represented using a phasor diagram.
4. Explain the terms phase and phase difference. Differentiate between in-phase, leading and lagging quantities with diagrams.
5. Explain the behaviour of a pure resistive circuit connected to an alternating voltage supply, with waveform and phasor diagram.
6. Explain the behaviour of a pure inductive circuit connected to an alternating voltage supply, and derive the expression for current, showing that it lags the voltage by 90°.
7. Explain the behaviour of a pure capacitive circuit connected to an alternating voltage supply, and derive the expression for current, showing that it leads the voltage by 90°.
8. Compare pure R, L and C circuits on the basis of phase relationship, opposition offered, effect of frequency and power factor.
9. Explain the effect of frequency on inductive reactance and capacitive reactance with relevant graphs.
10. Derive the EMF equation of an alternator and explain the significance of maximum value, frequency and angular velocity in the equation.

— End of Notes —