CIRCUIT THEORY

2.4 R-L, R-C, R-L-C Series & Parallel Circuits | 2.5 Resonance, Bandwidth, Q-factor | 2.6 Related Numerical Problems

Diploma / Polytechnic Engineering — Study Notes

2.4 R-L, R-C, R-L-C Series and Parallel A.C. Circuits

Practical A.C. circuits generally contain a combination of resistance (R), inductance (L) and capacitance (C) rather than a single pure element. The combined opposition offered by such circuits to alternating current is called impedance (Z), which depends on both resistance and reactance. This section explains R-L, R-C and R-L-C circuits in series and parallel form, along with impedance, reactance, the impedance triangle, power factor, the three types of power, the power triangle and vector (phasor) diagrams.

2.4.1 R-L Series Circuit

In an R-L series circuit, a resistor R and an inductor L are connected in series across an alternating supply voltage V. Since R and L carry the same current I, the current is taken as the reference phasor. The voltage drop across R ($V_R = IR$) is in phase with the current, while the voltage drop across L ($V_L = IX_L$) leads the current by 90°.

- Applied voltage: $V = \sqrt{(V_R^2 + V_L^2)}$ (obtained by phasor addition, since V_R and V_L are 90° apart).
- Impedance: $Z = \sqrt{(R^2 + X_L^2)}$, where $X_L = \omega L = 2\pi fL$ is the inductive reactance.
- Phase angle: $\varphi = \tan^{-1}(X_L / R)$; the current lags the voltage by angle φ (lagging power factor).
- Current: $I = V / Z$.

2.4.2 R-C Series Circuit

In an R-C series circuit, a resistor R and a capacitor C are connected in series across an alternating supply. The voltage drop across R ($V_R = IR$) is in phase with current, while the voltage drop across C ($V_C = IX_C$) lags the current by 90° (i.e., current leads V_C by 90°).

- Applied voltage: $V = \sqrt{(V_R^2 + V_C^2)}$.
- Impedance: $Z = \sqrt{(R^2 + X_C^2)}$, where $X_C = 1/(\omega C) = 1/(2\pi fC)$ is the capacitive reactance.
- Phase angle: $\varphi = \tan^{-1}(X_C / R)$; the current leads the voltage by angle φ (leading power factor).
- Current: $I = V / Z$.

2.4.3 R-L-C Series Circuit

In an R-L-C series circuit, a resistor R, an inductor L and a capacitor C are connected in series across an alternating supply. The inductive reactance drop (V_L) and capacitive reactance drop (V_C) are 180° out of phase with each other (V_L leads current by 90°, V_C lags current by 90°), so the net reactive drop is their difference, ($V_L - V_C$).

- Applied voltage: $V = \sqrt{[V_R^2 + (V_L - V_C)^2]}$.
- Net reactance: $X = X_L - X_C$ (may be positive, negative or zero, depending on whether X_L or X_C dominates).
- Impedance: $Z = \sqrt{[R^2 + (X_L - X_C)^2]}$.
- Phase angle: $\varphi = \tan^{-1}[(X_L - X_C) / R]$.

- Circuit behaviour: if XL > XC, the circuit is net inductive (current lags voltage); if XC > XL, the circuit is net capacitive (current leads voltage); if XL = XC, the circuit is purely resistive (resonance condition, discussed in Section 2.5).

2.4.4 Parallel R-L, R-C and R-L-C Circuits

In parallel A.C. circuits, all the branches (R, L, C) are connected across the same supply voltage, so voltage is taken as the reference phasor, and the branch currents (IR, IL, IC) are added as phasors to obtain the total current. Admittance (Y = 1/Z), conductance (G = 1/R) and susceptance (B = 1/X) are commonly used for parallel circuit calculations.

- Resistive branch current IR = V/R, in phase with V.
- Inductive branch current IL = V/XL, lagging V by 90°.
- Capacitive branch current IC = V/XC, leading V by 90°.
- Total current (R-L-C parallel): $I = \sqrt{[IR^2 + (IC - IL)^2]}$, obtained by phasor addition of the branch currents.
- Overall power factor: cos φ = IR / I, where φ is the angle between the total current I and the supply voltage V.

2.4.5 Impedance, Reactance and the Impedance Triangle

- Reactance (X): the opposition offered by an inductor or capacitor to alternating current; XL = ωL (inductive), XC = 1/(ωC) (capacitive), net X = XL − XC for a series R-L-C circuit.
- Impedance (Z): the total opposition offered by a circuit to alternating current, combining resistance and reactance; $Z = \sqrt{(R^2 + X^2)}$, measured in ohms.
- Impedance triangle: a right-angled triangle with R as the base, X as the perpendicular (vertical) side, and Z as the hypotenuse; the angle between R and Z is the phase angle φ, where cos φ = R/Z, sin φ = X/Z and tan φ = X/R.

> *Impedance triangle relation: $Z^2 = R^2 + X^2$, cos φ = R/Z (power factor), sin φ = X/Z, tan φ = X/R.*

2.4.6 Power Factor

Power factor (cos φ) is defined as the cosine of the phase angle between the applied voltage and the current in an A.C. circuit. It indicates how effectively the electrical power supplied is converted into useful work, and it also equals the ratio of resistance to impedance (R/Z) or the ratio of active power to apparent power (P/S).

- Unity power factor (cos φ = 1): occurs in a purely resistive circuit, where voltage and current are in phase.
- Lagging power factor: occurs in inductive circuits, where current lags the voltage (φ is positive).
- Leading power factor: occurs in capacitive circuits, where current leads the voltage (φ is negative).
- A low power factor increases the current required for the same power, leading to higher losses and larger conductor size; hence power factor improvement is important in industrial installations.

2.4.7 Active, Reactive and Apparent Power, and the Power Triangle

Power Type	Definition and Formula
Active power (P)	The actual power consumed/converted into useful work (heat, mechanical work, etc.) in a circuit; P = VI cos φ, measured in watts (W).

Power Type	Definition and Formula
Reactive power (Q)	The power that oscillates between the source and the reactive (L or C) elements without being consumed; $Q = VI \sin \varphi$, measured in volt-amperes reactive (VAR).
Apparent power (S)	The total power supplied to the circuit, combining active and reactive power; $S = VI = \sqrt{(P^2 + Q^2)}$, measured in volt-amperes (VA).

The relationship between P, Q and S can be represented by a power triangle, which is a right-angled triangle with active power P as the base (horizontal side), reactive power Q as the perpendicular (vertical) side, and apparent power S as the hypotenuse. The angle between P and S is the same phase angle φ as in the impedance triangle, so $\cos \varphi = P/S$ (power factor), and this triangle is geometrically similar to the impedance triangle.

2.4.8 Vector (Phasor) Diagrams

A vector (phasor) diagram graphically shows the magnitude and phase relationship between the applied voltage, the current and the individual voltage drops (or currents) in an A.C. circuit. For series circuits, current is drawn as the reference phasor, with VR in phase with I, VL leading I by 90°, and VC lagging I by 90°; the applied voltage V is obtained as the phasor sum of these drops. For parallel circuits, voltage is drawn as the reference phasor, and the branch currents IR, IL, IC are drawn relative to it, with the total current I obtained as their phasor sum.

Applications

- Design and analysis of A.C. power distribution networks and industrial electrical installations.
- Selection of capacitors for power factor correction to reduce electricity bills and line losses.
- Design of filter circuits, chokes and industrial motor/transformer windings.
- Sizing of cables, switchgear and protective devices based on apparent power (kVA rating).

Limitations

- The formulas discussed assume sinusoidal, steady-state A.C. supply; they do not directly apply to non-sinusoidal or transient conditions.
- Practical R, L and C elements have some amount of the other properties as well (e.g., a practical inductor has resistance), which slightly changes the ideal calculated results.

2.5 Resonance, Bandwidth, Quality Factor and Voltage Magnification

Resonance in an A.C. circuit is a condition in which the inductive reactance (XL) becomes exactly equal to the capacitive reactance (XC), causing the reactive effects to cancel out. At resonance, the circuit behaves as if it were purely resistive, and this condition is used in tuning circuits, filters and communication receivers.

2.5.1 Series Resonance (R-L-C Series Circuit)

In a series R-L-C circuit, resonance occurs when XL = XC, at a particular frequency called the resonant frequency (fr). At this frequency, the net reactance $X = XL - XC = 0$, so the impedance becomes minimum and equal to R alone, and the current becomes maximum.

- Condition for resonance: XL = XC, i.e., $\omega rL = 1/(\omega rC)$.
- Resonant frequency: $fr = 1 / (2\pi\sqrt{(LC)})$ Hz.
- At resonance: Z = R (minimum impedance), I = V/R (maximum current), and the circuit power factor is unity (current in phase with voltage).

- Below resonant frequency, XC > XL, so the circuit is capacitive (leading power factor); above resonant frequency, XL > XC, so the circuit is inductive (lagging power factor).

2.5.2 Parallel Resonance (R-L-C Parallel Circuit)

In a parallel R-L-C circuit (with a practical coil having resistance R in series with L, in parallel with C), resonance occurs at the frequency where the susceptance of the inductive branch equals the susceptance of the capacitive branch, making the net reactive current zero. At this condition, unlike series resonance, the impedance of the circuit is maximum and the line current drawn from the supply is minimum.

- At parallel resonance, the circuit impedance is maximum (often called the 'dynamic resistance'), so parallel resonant circuits are also called 'rejector circuits' since they reject (offer maximum opposition to) the resonant frequency current from the supply line.
- The resonant frequency for a practical parallel R-L-C circuit is approximately fr = 1/(2π√(LC)), the same basic formula as for series resonance, with a small correction factor when coil resistance is significant.
- At resonance, the circulating current between L and C (within the parallel loop) can be much larger than the current drawn from the supply.

2.5.3 Bandwidth

Bandwidth (BW) is the range of frequencies over which the current (in a series resonant circuit) or the response remains equal to or greater than 70.7% (1/√2) of its maximum (resonant) value. These frequencies are called the lower and upper half-power (cut-off) frequencies, f1 and f2.

- Bandwidth: BW = f2 − f1 (in Hz), where f1 and f2 are the lower and upper half-power frequencies at which power delivered is half of the power at resonance.
- A narrow bandwidth indicates a highly selective circuit (sharper response), while a wide bandwidth indicates a less selective circuit.

2.5.4 Quality Factor (Q-factor)

The quality factor (Q) of a resonant circuit is a measure of its selectivity and sharpness of resonance. It is defined as the ratio of the resonant (centre) frequency to the bandwidth, and also indicates how much the reactive voltage (in series circuits) or reactive current (in parallel circuits) is magnified compared to the applied value at resonance.

- Q-factor (general): Q = fr / BW = fr / (f2 − f1).
- Q-factor (series R-L-C circuit): Q = XL/R = ωrL/R = (1/R)√(L/C) at resonance.
- A higher Q-factor indicates lower resistance relative to reactance, a sharper resonance curve, narrower bandwidth, and greater selectivity — desirable in radio and communication tuning circuits.

2.5.5 Voltage Magnification at Resonance

In a series R-L-C circuit at resonance, although the applied voltage V equals the voltage across R (since Z = R), the voltage across the inductor (VL) and across the capacitor (VC) can individually become much larger than the applied voltage V. This effect is called voltage magnification, and the voltage magnification factor at resonance is equal to the Q-factor of the circuit.

- Voltage magnification factor = VL / V = VC / V = Q (at resonance).
- This magnification can produce very high voltages across L and C in high-Q circuits, which must be considered while selecting the voltage rating of components.

- The same magnification principle applies to current in a parallel resonant circuit, where the circulating current in the L-C loop can be Q times the supply current — this is sometimes called current magnification.

Comparison: Series Resonance vs Parallel Resonance

Parameter	Series Resonance	Parallel Resonance
Impedance at resonance	Minimum (Z = R)	Maximum
Current at resonance	Maximum (I = V/R)	Minimum (line current)
Also known as	Acceptor circuit	Rejector circuit
Magnification effect	Voltage magnification (across L, C)	Current magnification (in L-C loop)

Applications

- Tuning circuits in radio and television receivers to select a particular station/channel frequency.
- Design of band-pass and band-stop filters in communication and signal processing systems.
- Oscillator circuits and impedance matching networks in electronic equipment.
- Induction heating and certain types of power electronic converter circuits.

Limitations

- High-Q resonant circuits, while very selective, can develop dangerously high voltages or currents across L and C, requiring components rated for these magnified values.
- Resonant circuits are sensitive to component tolerances and temperature drift, which can shift the actual resonant frequency from the designed value.
- The simple resonance formulas assume ideal, linear R, L, C elements with negligible stray capacitance/inductance, which is not always true in practical high-frequency circuits.

2.6 Related Numerical Problems

This section presents solved numerical examples based on R-L, R-L-C series circuits, power calculations, and series resonance, following the formulas explained above.

Numerical on R-L Series Circuit

Example 1: Impedance, current and power factor of an R-L series circuit

Given: A coil having resistance R = 8 Ω and inductive reactance XL = 6 Ω is connected across a 230 V, 50 Hz A.C. supply. Find the impedance, current, phase angle and power factor of the circuit.

Solution

Step 1 (Impedance): $Z = \sqrt{R^2 + XL^2} = \sqrt{8^2 + 6^2} = \sqrt{64 + 36} = \sqrt{100} = 10\ \Omega$.

Step 2 (Current): $I = V/Z = 230/10 = 23$ A.

Step 3 (Phase angle): $\varphi = \tan^{-1}(XL/R) = \tan^{-1}(6/8) = \tan^{-1}(0.75) = 36.87°$ (current lags voltage).

Step 4 (Power factor): $\cos \varphi = R/Z = 8/10 = 0.8$ (lagging).

> **Answer: Z = 10 Ω, I = 23 A, φ = 36.87° (lagging), Power factor = 0.8 lagging**

Numerical on R-L-C Series Circuit

Example 2: Impedance and current of an R-L-C series circuit

Given: An R-L-C series circuit has R = 10 Ω, XL = 20 Ω and XC = 8 Ω, connected across a 100 V A.C. supply. Find the net reactance, impedance, current and the nature of the circuit (inductive or capacitive).

Solution

Step 1 (Net reactance): $X = XL - XC = 20 - 8 = 12\ \Omega$ (positive, so net inductive).

Step 2 (Impedance): $Z = \sqrt{(R^2 + X^2)} = \sqrt{(10^2 + 12^2)} = \sqrt{(100 + 144)} = \sqrt{244} \approx 15.62\ \Omega$.

Step 3 (Current): $I = V/Z = 100/15.62 \approx 6.4$ A.

Step 4 (Nature of circuit): Since XL > XC, the net reactance is inductive, so the current lags behind the voltage (lagging power factor).

> **Answer: X = 12 Ω (inductive), Z ≈ 15.62 Ω, I ≈ 6.4 A, circuit is net inductive (lagging power factor)**

Numerical on Power Triangle

Example 3: Active, reactive and apparent power calculation

Given: A single-phase load draws a current of 10 A from a 230 V supply at a lagging power factor of 0.8. Calculate the apparent power, active power and reactive power.

Solution

Step 1 (Apparent power): $S = V \times I = 230 \times 10 = 2300$ VA = 2.3 kVA.

Step 2 (Active power): $P = VI \cos \varphi = 2300 \times 0.8 = 1840$ W = 1.84 kW.

Step 3 (Phase angle): Since $\cos \varphi = 0.8$, $\varphi = \cos^{-1}(0.8) = 36.87°$, so $\sin \varphi = 0.6$.

Step 4 (Reactive power): $Q = VI \sin \varphi = 2300 \times 0.6 = 1380$ VAR = 1.38 kVAR.

> **Answer: Apparent power S = 2.3 kVA, Active power P = 1.84 kW, Reactive power Q = 1.38 kVAR**

Numerical on Series Resonance

Example 4: Resonant frequency, Q-factor and bandwidth of a series R-L-C circuit

Given: A series R-L-C circuit has R = 5 Ω, L = 0.1 H and C = 100 µF. Find (a) the resonant frequency, (b) the Q-factor at resonance, and (c) the bandwidth of the circuit.

Solution

Step 1 (Resonant frequency): $fr = 1/(2\pi\sqrt{(LC)}) = 1/(2\pi\sqrt{(0.1 \times 100 \times 10^{-6})}) = 1/(2\pi\sqrt{(10 \times 10^{-6})}) = 1/(2\pi \times 3.162 \times 10^{-3}) \approx 50.3$ Hz.

Step 2 (Inductive reactance at resonance): $XL = 2\pi\ fr\ L = 2\pi \times 50.3 \times 0.1 \approx 31.6\ \Omega$.

Step 3 (Q-factor): $Q = XL/R = 31.6/5 \approx 6.32$.

Step 4 (Bandwidth): $BW = fr/Q = 50.3/6.32 \approx 7.96$ Hz.

> **Answer: fr ≈ 50.3 Hz, Q ≈ 6.32, Bandwidth ≈ 7.96 Hz**

Numerical on Voltage Magnification

Example 5: Voltage magnification (VL and VC) at resonance

Given: For the series R-L-C circuit of Example 4 (R = 5 Ω, Q ≈ 6.32 at resonance), if the applied voltage at resonance is V = 20 V, find the voltage across the inductor and the capacitor at resonance.

Solution

Step 1 (Recall magnification relation): At resonance, the voltage magnification factor is equal to the Q-factor of the circuit, i.e., VL/V = VC/V = Q.

Step 2 (Calculate VL): VL = Q × V = 6.32 × 20 ≈ 126.4 V.

Step 3 (Calculate VC): Since XL = XC at resonance, VC = VL (equal in magnitude, opposite in phase), so VC ≈ 126.4 V as well.

Answer: VL ≈ 126.4 V and VC ≈ 126.4 V, both much greater than the applied voltage of 20 V, illustrating voltage magnification at resonance

Multiple Choice Questions (MCQs)

Choose the correct option for each question. The correct answer is shown in bold within the options, and a separate answer key is also given at the end.

Q1. In an R-L series circuit, the current with respect to the applied voltage:

a) Leads by 90°
b) Lags by an angle less than 90°
c) Is in phase
d) Lags by 180°

Q2. The impedance of an R-L-C series circuit is given by:

a) Z = R + XL + XC
b) Z = √(R² + (XL − XC)²)
c) Z = R × (XL − XC)
d) Z = (XL − XC) / R

Q3. In a parallel R-L-C circuit, the total current is obtained by:

a) Simple algebraic sum of branch currents
b) Phasor sum of branch currents
c) Subtracting IR from IL
d) Multiplying all branch currents

Q4. The power factor of a circuit is defined as:

a) sin φ
b) tan φ
c) cos φ
d) 1/cos φ

Q5. Active power in an A.C. circuit is given by:

a) P = VI
b) P = VI sin φ
c) P = VI cos φ
d) P = V/I

Q6. Apparent power (S) is related to active power (P) and reactive power (Q) as:

a) S = P + Q
b) S = P − Q
c) S = √(P² + Q²)
d) S = P × Q

Q7. The condition for series resonance in an R-L-C circuit is:

a) R = XL
b) XL = XC
c) R = XC

d) XL = 0

Q8. At series resonance, the impedance of the circuit is:

a) Maximum
b) Minimum, equal to R
c) Equal to zero
d) Equal to XL

Q9. The Q-factor of a series resonant circuit is given by:

a) Q = R/XL
b) Q = XL/R
c) Q = R × XL
d) Q = fr × BW

Q10. At parallel resonance, the line current drawn from the supply is:

a) Maximum
b) Minimum
c) Equal to zero always
d) Independent of frequency

Answer Key

1-b | 2-b | 3-b | 4-c | 5-c | 6-c | 7-b | 8-b | 9-b | 10-b

Broad / Long Answer Questions

1. Derive the expression for impedance and phase angle of an R-L series A.C. circuit, with a neat phasor diagram.
2. Derive the expression for impedance and phase angle of an R-C series A.C. circuit, with a neat phasor diagram.
3. Derive the expression for impedance of an R-L-C series A.C. circuit, and explain the three possible conditions based on the relative values of XL and XC.
4. Explain the working of a parallel R-L-C A.C. circuit and derive the expression for the total current using the phasor method.
5. Define power factor. Explain unity, lagging and leading power factor with suitable examples and phasor diagrams.
6. Define active power, reactive power and apparent power. Draw and explain the power triangle.
7. What is resonance in an A.C. circuit? Derive the expression for the resonant frequency of a series R-L-C circuit.
8. Explain bandwidth and Q-factor of a series resonant circuit, and derive the relation between them.
9. Explain the phenomenon of voltage magnification in a series R-L-C circuit at resonance, showing that the magnification factor equals the Q-factor.
10. Compare series resonance and parallel resonance on the basis of impedance, current, and their common names (acceptor/rejector circuit).

— End of Notes —