CIRCUIT THEORY

Unit 3: Three Phase A.C. Circuits

Diploma / Polytechnic Engineering — Study Notes

Introduction

A three-phase A.C. system consists of three alternating voltages of the same magnitude and frequency, but displaced from one another by 120 electrical degrees. Three-phase systems are the standard method of generating, transmitting and distributing electrical power because they are more efficient, require less conductor material for the same power transfer, and produce a smoother, more constant torque in A.C. motors compared to single-phase systems. This chapter explains the phasor and complex representation of a three-phase supply, phase sequence and polarity, and the two standard three-phase connections — Star and Delta — along with the relationship between their phase and line quantities.

3.1 Phasor and Complex Representation of Three Phase Supply

A balanced three-phase supply is generated by three identical coils placed 120° apart (in space) on the stator of an alternator, so that the EMFs induced in them are equal in magnitude and frequency, but displaced from each other by 120° in time (electrical degrees). These three voltages are usually named R (Red), Y (Yellow) and B (Blue), or A, B and C phases.

Instantaneous (Time-Domain) Equations

If eR is taken as the reference phase, the instantaneous EMF equations of the three phases can be written as follows, each displaced from the previous one by 120°:

- $e_R = E_m \sin(\omega t)$
- $e_Y = E_m \sin(\omega t - 120°)$
- $e_B = E_m \sin(\omega t - 240°) = E_m \sin(\omega t + 120°)$

These three sinusoidal equations show that the Yellow phase lags the Red phase by 120°, and the Blue phase lags the Yellow phase by 120° (or equivalently, leads the Red phase by 120°). Since the three EMFs are equal in magnitude and equally displaced in phase, their algebraic (phasor) sum is zero at every instant for a balanced system.

Phasor Representation

In the phasor diagram, the three voltage phasors ER, EY and EB are drawn from a common origin, each having equal length (representing equal magnitude) and separated from each other by 120°, rotating together in the anticlockwise direction at angular velocity ω. This graphical representation makes it easy to visualise the phase relationship between the three voltages and to add or subtract them using the laws of vector addition.

Complex (Symbolic) Representation Using the Operator 'a'

For mathematical convenience, three-phase quantities are often represented using complex numbers with the help of an operator called 'a', which represents a rotation of 120° in the anticlockwise direction without changing the magnitude. It is defined as $a = 1\angle 120° = -0.5 + j0.866$. Using this operator, if $E_R = E_R\angle 0°$ is taken as reference, the other two phase voltages can be written as:

- $E_R = E_R\angle 0°$
- $E_Y = a^2 E_R = E_R\angle -120°$ (or $E_R\angle 240°$)
- $E_B = a E_Y = E_R\angle 120°$
- Properties of the operator 'a': $a = 1\angle 120°$, $a^2 = 1\angle 240° = 1\angle -120°$, and $a^3 = 1\angle 360° = 1$ (i.e., $a^3 = 1$).
- Sum of the three unit operators: $1 + a + a^2 = 0$, which reflects the fact that the sum of three balanced three-phase quantities is zero.

> *Key relation: $1 + a + a^2 = 0$, and $a^3 = 1$. This identity is frequently used to simplify three-phase calculations expressed in complex (symbolic) form.*

Applications

- Simplifies analysis and calculation of balanced and unbalanced three-phase circuits.
- Forms the basis of symmetrical component analysis used in power system fault studies.
- Used in the design of three-phase generators, motors, and transmission/distribution systems.

Limitations

- The simple 120° phasor and 'a' operator relations apply directly only to balanced three-phase systems; unbalanced systems require more advanced techniques (e.g., symmetrical components).
- Complex/phasor representation assumes sinusoidal, steady-state quantities of the same frequency.

3.2 Phase Sequence and Polarity

Phase Sequence

Phase sequence (or phase order) is the order in which the three voltages of a three-phase supply reach their maximum (positive peak) values. It is an important property because it determines the direction of rotation of three-phase induction motors and affects the correct operation of three-phase equipment.

- Positive sequence (RYB or ABC): the phase voltages reach their maximum values in the order R, then Y, then B (i.e., R leads Y, and Y leads B, each by 120°). This is the normal/standard sequence used in most power systems.
- Negative sequence (RBY or ACB): the phase voltages reach their maximum values in the reverse order, R, then B, then Y. This sequence is obtained by interchanging any two of the three supply lines.

The phase sequence of a supply can be determined practically using a phase sequence indicator instrument, or by observing the direction of rotation of a small three-phase induction motor connected to the supply.

Importance of Phase Sequence

- Determines the direction of rotation of three-phase induction motors; reversing any two supply lines reverses the phase sequence and hence reverses the direction of motor rotation.
- Incorrect phase sequence in parallel operation of alternators or transformers can cause severe short-circuit currents and equipment damage.
- Important in the correct connection of three-phase measuring instruments, protection relays and synchronising equipment.

Polarity

Polarity in a three-phase system refers to the relative instantaneous direction (or sense) of the induced EMF in each phase winding, i.e., which terminal of a winding is at a higher potential at a given instant compared to the other terminal. Correct polarity of windings must be maintained while connecting the three phases in Star or

Delta, and while connecting transformer windings, so that the voltages add up correctly instead of cancelling or creating a short circuit.

- Windings are usually marked with polarity/terminal markings (such as start and finish ends, e.g., A1-A2, B1-B2, C1-C2) to indicate correct connection.
- Wrong polarity connection in a Star or Delta system can result in unequal (unbalanced) line voltages, or in a Delta connection, can create a short-circuit path within the closed loop of windings.
- In transformers, correct polarity (additive or subtractive) must be ensured before connecting windings in parallel or in three-phase banks, to avoid circulating currents and equipment damage.

Applications

- Ensuring correct direction of rotation of three-phase motors in industrial applications (pumps, fans, conveyors, etc.).
- Safe and correct paralleling of alternators and transformers in power stations and substations.
- Correct functioning of three-phase energy meters, protection relays and synchronising equipment.

Limitations

- Phase sequence indicators and polarity tests add extra steps to installation and commissioning, which are sometimes overlooked, leading to equipment faults.
- Incorrect phase sequence or polarity may not always be immediately visible and can cause damage only when the equipment is switched on or loaded.

3.3 Types of Three-Phase Connections — Star and Delta

The three windings (or loads) of a three-phase system can be interconnected in two standard ways: Star (Y) connection and Delta (Δ) connection. The choice of connection affects the relationship between phase and line voltages/currents, and is selected based on the application (generation, transmission, distribution, or type of load).

3.3.1 Definitions — Phase and Line Quantities

Term	Meaning
Phase voltage (Vph)	The voltage across a single winding or phase of the source/load.
Phase current (Iph)	The current flowing through a single winding or phase of the source/load.
Line voltage (VL)	The voltage measured between any two line (outer) conductors of the three-phase supply.
Line current (IL)	The current flowing through any one line (outer) conductor connecting the source to the load.

3.3.2 Star (Y) Connection

In a Star connection, similar ends (either all 'start' ends or all 'finish' ends) of the three windings are joined together at a common point called the neutral (or star) point, and the other three ends are brought out as the three line terminals (R, Y, B). A fourth wire can be taken out from the neutral point to form a three-phase, four-wire system, commonly used for distribution where both line and phase (line-to-neutral) voltages are required.

Relationship Between Line and Phase Quantities in Star Connection (Derivation)

1. In a Star connection, each line conductor is connected to only one phase winding in series; therefore, the same current flows through the line conductor and the phase winding connected to it. Hence, Line current I_L = Phase current I_{ph}.
2. To find the relationship between V_L and V_{ph}, consider the line voltage V_{RY} between lines R and Y, which is the phasor difference of the phase voltages V_R and V_Y, i.e., $V_{RY} = V_R - V_Y$.
3. Since V_R and V_Y are equal in magnitude (V_{ph}) and displaced by 120° from each other, their phasor difference is found using the parallelogram law: $V_{RY} = \sqrt{V_R^2 + V_Y^2 - 2 \cdot V_R \cdot V_Y \cdot \cos 120°}$.
4. Substituting $V_R = V_Y = V_{ph}$ and $\cos 120° = -0.5$: $V_{RY} = \sqrt{V_{ph}^2 + V_{ph}^2 + V_{ph}^2} = \sqrt{3 \cdot V_{ph}^2} = \sqrt{3} \cdot V_{ph}$.
5. By symmetry, the same relation holds for the other two line voltages (V_{YB} and V_{BR}); therefore, in a balanced Star connection, Line voltage $V_L = \sqrt{3} \times$ Phase voltage V_{ph}.

> *Star connection relations: $V_L = \sqrt{3} \times V_{ph}$, and $I_L = I_{ph}$.*

3.3.3 Delta (Δ) Connection

In a Delta connection, the windings are connected end-to-end to form a closed triangular loop, i.e., the finish end of one winding is connected to the start end of the next winding, and so on, with the three junction points brought out as the three line terminals (R, Y, B). There is no neutral point in a Delta connection, so it is normally used as a three-phase, three-wire system.

Relationship Between Line and Phase Quantities in Delta Connection (Derivation)

1. In a Delta connection, each phase winding is connected directly between two line terminals; therefore, the voltage across each phase winding is the same as the voltage between the corresponding two lines. Hence, Line voltage V_L = Phase voltage V_{ph}.
2. To find the relationship between I_L and I_{ph}, consider line current I_R flowing in line R, which is the phasor difference of the two phase currents meeting at that junction, I_{RY} and I_{BR}, i.e., $I_R = I_{RY} - I_{BR}$.
3. Since I_{RY} and I_{BR} are equal in magnitude (I_{ph}) and displaced by 120° from each other, their phasor difference is found using the parallelogram law: $I_R = \sqrt{I_{RY}^2 + I_{BR}^2 - 2 \cdot I_{RY} \cdot I_{BR} \cdot \cos 120°}$.
4. Substituting $I_{RY} = I_{BR} = I_{ph}$ and $\cos 120° = -0.5$: $I_R = \sqrt{I_{ph}^2 + I_{ph}^2 + I_{ph}^2} = \sqrt{3 \cdot I_{ph}^2} = \sqrt{3} \cdot I_{ph}$.
5. By symmetry, the same relation holds for the other two line currents; therefore, in a balanced Delta connection, Line current $I_L = \sqrt{3} \times$ Phase current I_{ph}.

> *Delta connection relations: $V_L = V_{ph}$, and $I_L = \sqrt{3} \times I_{ph}$.*

Comparison of Star and Delta Connections

Parameter	Star (Y) Connection	Delta (Δ) Connection
Line voltage vs phase voltage	$V_L = \sqrt{3} \times V_{ph}$	$V_L = V_{ph}$
Line current vs phase current	$I_L = I_{ph}$	$I_L = \sqrt{3} \times I_{ph}$
Neutral point	Available (4-wire system possible)	Not available (3-wire system)
Insulation requirement	Lower (each winding sees $V_{ph} = V_L/\sqrt{3}$)	Higher (each winding sees full V_L)
Common use	Power generation, transmission, and distribution (with neutral)	Delta-connected transformer windings, motor windings, industrial loads

Applications

- Star connection is used in alternators and distribution transformers where a neutral is required for single-phase loads and for reducing insulation levels at high voltages.

- Delta connection is used in transformer windings (especially on the high-current side), in industrial motor windings, and where a neutral connection is not needed.
- Star-Delta starters use both connections to reduce the starting current of large three-phase induction motors.
- The choice between Star and Delta affects power system design, protection scheme design and fault current calculations.

Limitations

- The simple $V_L = \sqrt{3}\, V_{ph}$ and $I_L = \sqrt{3}\, I_{ph}$ relations are valid only for balanced three-phase systems with equal phase quantities and 120° displacement; unbalanced systems require separate phase-wise analysis.
- Delta connection does not provide a neutral point, so it cannot supply single-phase loads that require a neutral reference directly.
- Star connection without a properly connected neutral can lead to unequal (distorted) phase voltages under unbalanced loading conditions.

Multiple Choice Questions (MCQs)

Choose the correct option for each question. The correct answer is shown in bold within the options, and a separate answer key is also given at the end.

Q1. In a balanced three-phase system, the three phase voltages are displaced from each other by:

a) 60°
b) 90°
c) 120°
d) 180°

Q2. The operator 'a' used in three-phase complex representation corresponds to a rotation of:

a) 90°
b) 120°
c) 180°
d) 240°

Q3. The value of $(1 + a + a^2)$ for the three-phase operator 'a' is:

a) 1
b) 3
c) 0
d) a^3

Q4. Positive phase sequence is also written as:

a) RYB
b) RBY
c) YRB
d) BYR

Q5. Reversing any two supply lines of a three-phase supply changes the:

a) Frequency
b) Phase sequence
c) Voltage magnitude
d) Power factor only

Q6. In a Star-connected system, the relationship between line current and phase current is:

a) $I_L = \sqrt{3}\, I_{ph}$
b) $I_L = I_{ph}$

c) IL = Iph / √3
d) IL = 3 Iph

Q7. In a Star-connected system, the relationship between line voltage and phase voltage is:

a) VL = Vph
b) VL = √3 Vph
c) VL = Vph / √3
d) VL = 3 Vph

Q8. In a Delta-connected system, the relationship between line voltage and phase voltage is:

a) VL = √3 Vph
b) VL = Vph
c) VL = Vph / √3
d) VL = 3 Vph

Q9. In a Delta-connected system, the relationship between line current and phase current is:

a) IL = Iph
b) IL = √3 Iph
c) IL = Iph / √3
d) IL = 3 Iph

Q10. A three-phase, four-wire system with a neutral conductor is normally obtained from a:

a) Delta connection
b) Star connection
c) Open-delta connection
d) Series connection

Answer Key

1-c | 2-b | 3-c | 4-a | 5-b | 6-b | 7-b | 8-b | 9-b | 10-b

Broad / Long Answer Questions

1. Explain the phasor representation of a balanced three-phase supply and write the instantaneous EMF equations of the three phases.
2. Explain the complex (symbolic) representation of three-phase quantities using the operator 'a', and state its important properties.
3. What is phase sequence? Explain positive and negative phase sequence with a suitable diagram, and state its importance.
4. Explain the term 'polarity' in a three-phase system, and discuss its importance while connecting windings in Star or Delta.
5. With a neat diagram, explain the Star connection of a three-phase system.
6. With a neat diagram, explain the Delta connection of a three-phase system.
7. Derive the relationship between line voltage and phase voltage in a balanced Star-connected three-phase system.
8. Derive the relationship between line current and phase current in a balanced Delta-connected three-phase system.

9. Compare Star and Delta connections on the basis of line/phase relationships, neutral availability, insulation requirement and typical applications.
10. Explain how the phase sequence of a three-phase supply affects the direction of rotation of a three-phase induction motor.

— End of Notes —