CIRCUIT THEORY

3.4 Balanced/Unbalanced Load, Neutral Shift | 3.5 Three Phase Power (Star & Delta) | 3.6 Related Numerical Problems

Diploma / Polytechnic Engineering — Study Notes

3.4 Balanced and Unbalanced Load, Neutral Shift

3.4.1 Balanced Load

A three-phase load is said to be balanced when all three phases (or all three loads connected across the three lines) have exactly equal impedance, i.e., equal magnitude and equal phase angle, so that equal currents flow in each phase and all three currents are displaced from each other by exactly 120°. In a balanced system, all three phase voltages and currents are equal in magnitude, and the neutral current (in a Star system) is zero, since the phasor sum of three equal currents displaced by 120° is zero.

- Examples: three identical heating elements, three identical single-phase motors, or a balanced three-phase induction motor load connected symmetrically across the three phases.
- In a balanced system, calculations can be simplified by analysing only one phase and multiplying the result by three (per-phase analysis).

3.4.2 Unbalanced Load

A three-phase load is said to be unbalanced when the three phase impedances are not equal in magnitude and/or phase angle, so that unequal currents flow in the three phases, or the currents are not displaced from each other by exactly 120°. Unbalanced loads commonly occur when different single-phase loads (of different ratings) are connected across the three phases of a distribution system, such as in residential and commercial buildings.

- Under unbalanced loading, the neutral current (in a 4-wire Star system) is no longer zero; it carries the resultant unbalance current back to the source.
- Unbalanced loading can cause unequal voltage drops in the three phases, resulting in unequal phase voltages, overheating of some equipment, and reduced efficiency.
- In a Delta system (no neutral), unbalanced loading causes unequal line currents but does not affect the phase voltages, since each phase is directly connected across a fixed line voltage.

3.4.3 Neutral Shift in Unbalanced Load

Neutral shift (also called neutral point displacement or neutral voltage shift) occurs in a three-phase, three-wire Star-connected system (i.e., a Star system without a neutral wire, or with a high-impedance/open neutral) when the load is unbalanced. In such a case, the star point of the load no longer remains at zero potential (as it would in a balanced system); instead, it shifts away from the true neutral (centroid) position, causing the three phase (line-to-neutral) voltages across the unbalanced load to become unequal.

Why Neutral Shift Occurs

1. In a balanced Star system, the three equal phase currents (120° apart) sum to zero at the star point, so the star point of the load coincides with the true neutral point (electrically at zero potential) — no shift occurs.
2. In an unbalanced Star system without a neutral wire, the currents in the three unequal load impedances must still sum to zero at the load's star point (by Kirchhoff's Current Law), because there is no separate neutral conductor to carry any unbalance current back to the source.

3. To satisfy this condition with unequal impedances, the potential of the load's star point must shift away from the supply's true neutral point, adjusting itself until the resulting three phase currents balance out to zero at that point.
4. As a result, the voltage across each unbalanced phase load (measured from the line terminal to the shifted star point) becomes different from the ideal phase voltage ($V_L/\sqrt{3}$); some phases experience higher voltage and others lower voltage than normal.

Effects of Neutral Shift

- Unequal voltages appear across the different phase loads — some loads may be subjected to over-voltage (risk of damage or reduced life), while others are subjected to under-voltage (poor/insufficient performance).
- In severe cases of unbalance, the neutral point can shift so far that one phase voltage becomes very high (close to the full line voltage) while another becomes very low, causing serious equipment damage, particularly for voltage-sensitive loads like lamps and electronic equipment.
- The line currents and voltages become unequal, increasing losses and reducing overall system efficiency.

Remedy for Neutral Shift

- Provide a solid, low-impedance neutral wire (three-phase, four-wire system) connecting the load's star point directly to the supply's neutral point; this fixes the star point at zero potential and carries the unbalance current, preventing neutral shift.
- Ensure the neutral conductor is properly earthed and never fitted with a fuse or switch that could open the neutral connection under unbalanced loading.
- Distribute single-phase loads as evenly as possible across the three phases to minimise the degree of unbalance.

> *Key point: Neutral shift occurs only when a Star-connected system is unbalanced AND has no solid neutral connection. A properly connected, low-impedance neutral wire prevents neutral shift by holding the star point fixed at zero (neutral) potential.*

Applications and Limitations

Type	Details
Applications	Understanding balanced/unbalanced loads and neutral shift is essential for the safe design of three-phase, four-wire distribution systems supplying mixed single-phase and three-phase loads in residential, commercial and industrial installations.
Limitations	The simple star-point-shift analysis assumes purely resistive or known impedance loads and steady-state balanced supply voltages; sudden or dynamically changing loads require more detailed time-domain or simulation-based analysis.

3.5 Three Phase Power — Active, Reactive and Apparent Power in Star and Delta System

The total power in a three-phase system is the sum of the power consumed in each of the three phases. For a balanced three-phase system, since all three phases carry equal current at the same power factor, the total power is simply three times the power per phase. Interestingly, when expressed in terms of line quantities (V_L and I_L), the total power formula turns out to be exactly the same for both Star and Delta connections.

3.5.1 Power in Star-Connected System (Derivation)

1. In a balanced Star connection, power per phase = $Vph \times Iph \times \cos\varphi$, where φ is the phase angle between the phase voltage and phase current.
2. Total active power for three phases: $P = 3 \times Vph \times Iph \times \cos\varphi$.
3. In a Star connection, the relations are $VL = \sqrt{3} \times Vph$ (so $Vph = VL/\sqrt{3}$) and $IL = Iph$.
4. Substituting these into the power formula: $P = 3 \times (VL/\sqrt{3}) \times IL \times \cos\varphi = \sqrt{3} \times VL \times IL \times \cos\varphi$.

3.5.2 Power in Delta-Connected System (Derivation)

1. In a balanced Delta connection, power per phase = $Vph \times Iph \times \cos\varphi$, where φ is the phase angle between the phase voltage and phase current (same definition as before).
2. Total active power for three phases: $P = 3 \times Vph \times Iph \times \cos\varphi$.
3. In a Delta connection, the relations are $VL = Vph$ and $IL = \sqrt{3} \times Iph$ (so $Iph = IL/\sqrt{3}$).
4. Substituting these into the power formula: $P = 3 \times VL \times (IL/\sqrt{3}) \times \cos\varphi = \sqrt{3} \times VL \times IL \times \cos\varphi$.

> *Important result: For a balanced three-phase load, whether connected in Star or Delta, the total active power expressed in terms of line quantities is the same: $P = \sqrt{3} \times VL \times IL \times \cos\varphi$. This is why line voltage and line current (which can be directly measured without knowing the internal connection) are normally used for three-phase power calculations.*

3.5.3 Reactive Power and Apparent Power

Power Type	Formula (Balanced 3-Phase, Star or Delta)
Active power (P)	The actual power consumed/converted into useful work; for a balanced three-phase system, $P = \sqrt{3} \times VL \times IL \times \cos\varphi$ (also $P = 3 \times Vph \times Iph \times \cos\varphi$), measured in watts (W).
Reactive power (Q)	The power exchanged between the source and the reactive (inductive/capacitive) elements without being consumed; for a balanced three-phase system, $Q = \sqrt{3} \times VL \times IL \times \sin\varphi$ (also $Q = 3 \times Vph \times Iph \times \sin\varphi$), measured in VAR.
Apparent power (S)	The total power supplied to the three-phase circuit; $S = \sqrt{3} \times VL \times IL$ (also $S = 3 \times Vph \times Iph$), measured in volt-amperes (VA); also $S = \sqrt{P^2 + Q^2}$.

Comparison of Star and Delta Power Formulas

Quantity	Star Connection	Delta Connection
Phase voltage / current relation	$Vph = VL/\sqrt{3}$, $Iph = IL$	$Vph = VL$, $Iph = IL/\sqrt{3}$
Active power (per-phase form)	$P = 3\ Vph\ Iph \cos\varphi$	$P = 3\ Vph\ Iph \cos\varphi$
Active power (line form)	$P = \sqrt{3}\ VL\ IL \cos\varphi$	$P = \sqrt{3}\ VL\ IL \cos\varphi$
Reactive power (line form)	$Q = \sqrt{3}\ VL\ IL \sin\varphi$	$Q = \sqrt{3}\ VL\ IL \sin\varphi$
Apparent power (line form)	$S = \sqrt{3}\ VL\ IL$	$S = \sqrt{3}\ VL\ IL$

Applications

- Power billing and energy metering in three-phase industrial and commercial installations.
- Sizing of three-phase transformers, generators, cables and switchgear based on kVA (apparent power) rating.
- Power factor correction calculations using active and reactive power values in three-phase systems.

- Load analysis and power flow studies in three-phase distribution and transmission networks.

Limitations

- The formulas $P = \sqrt{3}\ V_L I_L \cos\varphi$, $Q = \sqrt{3}\ V_L I_L \sin\varphi$, and $S = \sqrt{3}\ V_L I_L$ are valid only for balanced three-phase loads; unbalanced loads require separate phase-wise (per-phase) power calculation and summation.
- These formulas assume a purely sinusoidal supply; the presence of harmonics in practical systems requires additional harmonic power analysis.

3.6 Related Numerical Problems

This section presents solved numerical examples based on the relationship between phase and line quantities, and three-phase power calculations in Star and Delta connected systems.

Numerical on Star-Connected System

Example 1: Phase voltage, phase current and power of a balanced Star-connected load

Given: A balanced Star-connected load is supplied from a 400 V, three-phase supply. Each phase of the load has an impedance that draws a phase current of 10 A at a lagging power factor of 0.8. Find the phase voltage, line current, and the total active power consumed.

Solution

Step 1 (Phase voltage): Since $V_L = \sqrt{3} \times V_{ph}$, $V_{ph} = V_L/\sqrt{3} = 400/1.732 \approx 231$ V.

Step 2 (Line current): In a Star connection, $I_L = I_{ph}$, so $I_L = 10$ A.

Step 3 (Total active power): $P = \sqrt{3} \times V_L \times I_L \times \cos\varphi = 1.732 \times 400 \times 10 \times 0.8 \approx 5545.6$ W ≈ 5.55 kW.

Answer: Phase voltage $V_{ph} \approx 231$ V, Line current $I_L = 10$ A, Total active power $P \approx 5.55$ kW

Numerical on Delta-Connected System

Example 2: Phase current, line current and power of a balanced Delta-connected load

Given: A balanced Delta-connected load is supplied from a 400 V, three-phase supply. Each phase of the load draws a phase current of 15 A at a lagging power factor of 0.9. Find the phase voltage, line current, and the total active power consumed.

Solution

Step 1 (Phase voltage): In a Delta connection, $V_L = V_{ph}$, so $V_{ph} = 400$ V.

Step 2 (Line current): Since $I_L = \sqrt{3} \times I_{ph}$, $I_L = 1.732 \times 15 \approx 25.98$ A.

Step 3 (Total active power): $P = \sqrt{3} \times V_L \times I_L \times \cos\varphi = 1.732 \times 400 \times 25.98 \times 0.9 \approx 16{,}200$ W ≈ 16.2 kW. (Alternatively, using per-phase form: $P = 3 \times V_{ph} \times I_{ph} \times \cos\varphi = 3 \times 400 \times 15 \times 0.9 = 16{,}200$ W, which matches.)

Answer: Phase voltage $V_{ph} = 400$ V, Line current $I_L \approx 25.98$ A, Total active power $P \approx 16.2$ kW

Numerical on Total Power from Line Quantities

Example 3: Active, reactive and apparent power from line voltage and line current

Given: A balanced three-phase load draws a line current of 20 A from a 415 V, three-phase supply at a lagging power factor of 0.85. Calculate the apparent power, active power and reactive power.

Solution

Step 1 (Apparent power): $S = \sqrt{3} \times VL \times IL = 1.732 \times 415 \times 20 \approx 14{,}378$ VA ≈ 14.38 kVA.

Step 2 (Active power): $P = \sqrt{3} \times VL \times IL \times \cos\varphi = 14{,}378 \times 0.85 \approx 12{,}221$ W ≈ 12.22 kW.

Step 3 (Reactive power): Since $\cos\varphi = 0.85$, $\varphi = \cos^{-1}(0.85) = 31.79°$, so $\sin\varphi \approx 0.527$; $Q = \sqrt{3} \times VL \times IL \times \sin\varphi = 14{,}378 \times 0.527 \approx 7577$ VAR ≈ 7.58 kVAR.

> **Answer: Apparent power S ≈ 14.38 kVA, Active power P ≈ 12.22 kW, Reactive power Q ≈ 7.58 kVAR**

Numerical on Comparing Star and Delta for the Same Load

Example 4: Comparing line current when the same three-phase load is connected in Star vs Delta

Given: Three identical impedances, each of 10 Ω resistance, are connected first in Star and then in Delta across a 400 V, three-phase supply. Compare the phase current, line current and total power drawn in the two cases (assume purely resistive load, so $\cos\varphi = 1$).

Solution

Step 1 (Star connection — phase voltage): $Vph = VL/\sqrt{3} = 400/1.732 \approx 231$ V. Phase current (= line current, since Star): $Iph = Vph/R = 231/10 \approx 23.1$ A, so IL(star) = 23.1 A.

Step 2 (Star connection — power): $P(star) = \sqrt{3} \times VL \times IL \times \cos\varphi = 1.732 \times 400 \times 23.1 \times 1 \approx 16{,}000$ W ≈ 16 kW.

Step 3 (Delta connection — phase voltage): $Vph = VL = 400$ V. Phase current: $Iph = Vph/R = 400/10 = 40$ A. Line current: $IL(delta) = \sqrt{3} \times Iph = 1.732 \times 40 \approx 69.28$ A.

Step 4 (Delta connection — power): $P(delta) = \sqrt{3} \times VL \times IL \times \cos\varphi = 1.732 \times 400 \times 69.28 \times 1 \approx 48{,}000$ W = 48 kW.

Step 5 (Comparison): P(delta) / P(star) = 48/16 = 3. This confirms the standard result that for the same three impedances and the same supply voltage, the power drawn in Delta connection is exactly three times the power drawn in Star connection (and the line current in Delta is three times the line current in Star).

> **Answer: IL(star) ≈ 23.1 A, P(star) ≈ 16 kW; IL(delta) ≈ 69.28 A, P(delta) ≈ 48 kW — Delta draws 3 times the power of Star for the same impedances and supply voltage**

Multiple Choice Questions (MCQs)

Choose the correct option for each question. The correct answer is shown in bold within the options, and a separate answer key is also given at the end.

Q1. A three-phase load is said to be balanced when:

- **a) All three phase impedances are equal in magnitude and phase angle**
- b) Only two phases have equal impedance
- c) The neutral wire is removed
- d) The load is connected only in Delta

Q2. In a balanced Star-connected system, the neutral current is:

- a) Maximum
- b) Equal to line current
- **c) Zero**
- d) Equal to phase current

Q3. Neutral shift occurs in a Star-connected system when:

a) The load is balanced and neutral wire is present
b) The load is unbalanced and there is no solid neutral connection
c) The load is balanced and Delta connected
d) The supply frequency changes

Q4. The main remedy to prevent neutral shift in an unbalanced Star system is to:

a) Remove the neutral wire
b) Provide a solid, low-impedance neutral connection
c) Increase the load unbalance
d) Use only two phases

Q5. The formula for total active power in a balanced three-phase system (in terms of line quantities) is:

a) P = VL IL cos φ
b) P = √3 VL IL cos φ
c) P = 3 VL IL cos φ
d) P = VL IL / √3

Q6. For a balanced three-phase load, the active power formula in terms of line quantities is the same for:

a) Star connection only
b) Delta connection only
c) Both Star and Delta connections
d) Neither Star nor Delta

Q7. The reactive power in a balanced three-phase system is given by:

a) Q = √3 VL IL cos φ
b) Q = √3 VL IL sin φ
c) Q = VL IL
d) Q = √3 VL IL

Q8. The apparent power in a balanced three-phase system is given by:

a) S = √3 VL IL
b) S = VL IL cos φ
c) S = √3 VL IL cos φ
d) S = VL IL sin φ

Q9. If identical impedances are connected first in Star and then in Delta across the same supply, the power drawn in Delta compared to Star is:

a) Same as Star
b) Half of Star
c) Three times Star
d) One-third of Star

Q10. In an unbalanced Delta-connected load (no neutral involved), which of the following is generally affected?

a) Phase voltages become unequal
b) Line currents become unequal, but phase voltages remain fixed by the line voltage
c) Neutral current increases
d) Frequency changes

Answer Key

1-a | 2-c | 3-b | 4-b | 5-b | 6-c | 7-b | 8-a | 9-c | 10-b

Broad / Long Answer Questions

1. Explain the concept of balanced and unbalanced load in a three-phase system with suitable examples.

2. What is neutral shift? Explain why neutral shift occurs in an unbalanced Star-connected system without a solid neutral connection.
3. Explain the effects of neutral shift on the phase voltages of an unbalanced Star-connected load, and state the remedy to avoid it.
4. Derive the expression for total active power in a balanced Star-connected three-phase system in terms of line voltage and line current.
5. Derive the expression for total active power in a balanced Delta-connected three-phase system in terms of line voltage and line current.
6. Show that the total power formula $P = \sqrt{3}$ VL IL $\cos \varphi$ is the same for both Star and Delta connected balanced three-phase loads.
7. Define active power, reactive power and apparent power for a balanced three-phase system, and write their formulas in terms of line quantities.
8. Explain why a three-phase, four-wire distribution system is preferred over a three-wire system for supplying mixed single-phase and three-phase loads.
9. Three identical impedances are connected in Star and then in Delta across the same three-phase supply. Show, with derivation, that the power drawn in Delta is three times the power drawn in Star.
10. Explain, with a phasor/vector approach, how unequal phase currents in an unbalanced Star load without a neutral wire cause the star point to shift from the true neutral position.

— End of Notes —