CIRCUIT THEORY

Unit 4: Transient Analysis

Diploma / Polytechnic Engineering — Study Notes

4.1 Introduction

Whenever there is a sudden change in an electrical circuit — such as switching a supply ON or OFF, or suddenly changing a circuit parameter — the circuit does not reach its new steady-state condition instantaneously. Instead, the currents and voltages pass through a short-lived, changing condition called the transient state before settling down to the new steady-state (final) value. The branch of circuit analysis that studies this temporary behaviour is called Transient Analysis.

Transient State and Steady State

Term	Meaning
Transient state	The temporary condition that exists in a circuit for a short time immediately after a sudden change (such as switching), during which currents and voltages change with time before settling to their final value.
Steady state	The condition reached by the circuit after the transient has died out, in which currents and voltages either remain constant (for a D.C. supply) or vary in a fixed repetitive sinusoidal pattern (for an A.C. supply).

Transients occur mainly because energy-storing elements — inductors (which store energy in a magnetic field) and capacitors (which store energy in an electric field) — cannot change their stored energy, and hence their current (for an inductor) or voltage (for a capacitor), instantaneously. Any sudden attempt to change these quantities results in a gradual, exponential transition governed by a first-order (or higher-order) differential equation.

Causes of Transients

- Switching operations — closing or opening a switch to connect or disconnect a source, load or circuit element.
- Sudden changes in supply voltage, such as a fault or a lightning surge on a power line.
- Sudden change in circuit parameters, such as a short circuit or the connection/disconnection of an additional load.

Important Basic Rules

- The current through an inductor cannot change instantaneously (it must be continuous), because an inductor opposes any sudden change in current by inducing a back EMF.
- The voltage across a capacitor cannot change instantaneously (it must be continuous), because a sudden change in voltage would require an infinite charging current.
- In a purely resistive circuit, there is no transient period since a resistor has no energy-storing property — the current changes instantaneously with the voltage.

Applications

- Design of protection systems (fuses, circuit breakers, surge arresters) that must withstand or safely interrupt transient currents and voltages.

- Design of timing and control circuits, such as flash circuits, timer circuits, and pulse-shaping networks, based on RC/RL charging and discharging behaviour.
- Analysis of switching transients in power systems, transformers and motor starting circuits.

Limitations

- The simple exponential (first-order) transient analysis discussed in this chapter is applicable mainly to simple single-energy-storage-element (R-L or R-C) circuits; circuits with both L and C together require second-order differential equation analysis.
- Practical component behaviour (non-linearity, parasitic elements) can cause deviations from the ideal exponential response predicted by simple transient theory.

4.2 Simple R-L Circuit Supplied from a D.C. Voltage Source

Consider a series circuit consisting of a resistor R and an inductor L connected in series with a switch S, supplied by a D.C. voltage source of value V. When the switch is closed at time $t = 0$, the current in the circuit does not instantly rise to its final (steady-state) value V/R; instead, it grows gradually in an exponential manner because the inductor opposes the sudden change in current.

Growth of Current (Switch Closed — Charging/Growth Transient)

Derivation

1. Applying Kirchhoff's Voltage Law to the R-L series circuit after the switch is closed: $V = iR + L(di/dt)$, where i is the instantaneous current.
2. Rearranging this first-order linear differential equation: $L(di/dt) + iR = V$.
3. Solving this differential equation (using the standard method for first-order linear differential equations, with initial condition $i = 0$ at $t = 0$, since inductor current cannot change instantaneously) gives the instantaneous current as: $i(t) = (V/R)\,[1 - e^{(-Rt/L)}]$.
4. The term V/R represents the final steady-state current ($I_m = V/R$), which the circuit approaches as $t \to \infty$. The term $e^{(-Rt/L)}$ represents the decaying part that dies out with time.

> *Growth equation for current in an R-L circuit: $i(t) = I_m\,[1 - e^{(-Rt/L)}]$, where $I_m = V/R$ is the final (steady-state) current.*

Voltage Across R and L During Growth

- Voltage across resistor: $V_R(t) = i \times R = V\,[1 - e^{(-Rt/L)}]$, which grows from 0 and approaches V as t increases (same exponential shape as the current).
- Voltage across inductor: $V_L(t) = V - V_R(t) = V \cdot e^{(-Rt/L)}$, which starts at its maximum value V at $t = 0$ (since the inductor opposes the sudden current change) and decays exponentially to zero as t increases.

Nature of the Growth Curve

The current i(t) starts at zero at the instant of switching ($t = 0$) and rises exponentially, approaching but never mathematically reaching the final steady value $I_m = V/R$; practically, it is considered to reach the steady value after about 5 time constants. The graph of i(t) versus t is an exponentially rising curve, while $V_L(t)$ versus t is an exponentially falling curve, and $V_R(t)$ versus t has the same shape as i(t).

Applications

- Analysis of inrush current when energising inductive loads such as transformers, chokes, relay coils and motor windings.

- Design of protective circuits (freewheeling diodes, snubber circuits) to handle the energy released by inductive elements during switching.
- Understanding starting transients in D.C. motor and generator field windings.

Limitations

- The derivation assumes an ideal inductor (no internal resistance beyond the series R, no core saturation) and an ideal constant D.C. source; practical inductors and sources introduce deviations from the ideal exponential response.
- The analysis applies to a simple series R-L circuit; more complex networks with multiple energy-storage elements require more advanced (higher-order) differential equation techniques.

4.3 Simple R-C Circuit Supplied from a D.C. Voltage Source

Consider a series circuit consisting of a resistor R and a capacitor C connected in series with a switch S, supplied by a D.C. voltage source of value V. When the switch is closed at $t = 0$ (with the capacitor initially uncharged), the capacitor voltage does not rise instantly to V; instead, it charges gradually in an exponential manner, since a sudden change in capacitor voltage would require an infinite charging current.

Charging of Capacitor (Switch Closed — Growth Transient)

Derivation

1. Applying Kirchhoff's Voltage Law to the R-C series circuit after the switch is closed: $V = iR + v_C$, where v_C is the instantaneous capacitor voltage and $i = C(dv_C/dt)$.
2. Substituting for i: $V = RC(dv_C/dt) + v_C$, which is a first-order linear differential equation in v_C.
3. Solving this differential equation with the initial condition $v_C = 0$ at $t = 0$ (uncharged capacitor) gives the instantaneous capacitor voltage as: $v_C(t) = V [1 - e^{-t/RC}]$.
4. The charging current can be found from $i = C(dv_C/dt)$, which gives: $i(t) = (V/R) e^{-t/RC}$.

> *Charging equations for an R-C circuit: $v_C(t) = V [1 - e^{-t/RC}]$, and charging current $i(t) = (V/R) e^{-t/RC} = I_m e^{-t/RC}$, where $I_m = V/R$ is the maximum (initial) charging current.*

Nature of the Charging Curves

- Capacitor voltage $v_C(t)$: starts at zero at $t = 0$ and rises exponentially, approaching V as t increases; practically considered fully charged after about 5 time constants.
- Charging current $i(t)$: starts at its maximum value $I_m = V/R$ at $t = 0$ (since the uncharged capacitor initially behaves like a short circuit) and decays exponentially to zero as the capacitor becomes fully charged.
- Voltage across resistor $V_R(t) = i \times R = V \cdot e^{-t/RC}$, which has the same exponentially decaying shape as the charging current.

Discharging of Capacitor (Through Resistor R)

If a fully charged capacitor (charged to voltage V) is disconnected from the source and discharged through the same resistor R, the capacitor voltage decays exponentially from V to zero, following: $v_C(t) = V \cdot e^{-t/RC}$, and the discharge current follows $i(t) = -(V/R) e^{-t/RC}$, where the negative sign indicates the current now flows in the opposite direction (out of the capacitor) compared to the charging current.

Applications

- Timing circuits, such as those used in timers, flash photography circuits, and pulse generators, which rely on the predictable exponential charging/discharging of a capacitor.

- Filter and coupling circuits in electronic and communication equipment.
- Energy storage and smoothing circuits in D.C. power supplies (filter capacitors after rectification).

Limitations

- The derivation assumes an ideal capacitor (no leakage resistance, no equivalent series resistance beyond R) and an ideal constant D.C. source.
- Practical capacitors have some leakage current and equivalent series resistance/inductance, which slightly modify the ideal exponential charging/discharging behaviour, especially at high frequencies or over long time periods.

4.4 Time Constant

The time constant (denoted by the Greek letter τ, tau) of a circuit is a measure of how quickly the transient response of the circuit changes; it is defined as the time required for the current (in an R-L circuit) or the voltage (in an R-C circuit) to change by a factor of $(1 - 1/e)$, i.e., to reach approximately 63.2% of its total change from the initial to the final value, starting from the instant of switching.

Time Constant of R-L Circuit

- Time constant: $\tau = L/R$ (measured in seconds), where L is the inductance in henries and R is the resistance in ohms.
- At $t = \tau$, the current has risen to about 63.2% of its final value: $i(\tau) = Im\,(1 - e^{-1}) \approx 0.632\ Im$.
- A larger L/R ratio gives a longer time constant, meaning the current takes longer to reach its steady-state value; a smaller L/R ratio means the current rises (or decays) more quickly.

Time Constant of R-C Circuit

- Time constant: $\tau = RC$ (measured in seconds), where R is the resistance in ohms and C is the capacitance in farads.
- At $t = \tau$, the capacitor voltage has risen to about 63.2% of its final value: $vC(\tau) = V\,(1 - e^{-1}) \approx 0.632\ V$.
- A larger RC value gives a longer time constant, meaning the capacitor takes longer to charge or discharge; a smaller RC value means faster charging/discharging.

Significance of the Time Constant

1. The time constant indicates the speed of the transient response — a smaller time constant means the circuit reaches steady state more quickly, and a larger time constant means it takes longer.
2. Theoretically, the transient response takes infinite time to reach exactly the final steady-state value, since it follows an exponential curve; however, in practice, the transient is considered to have effectively died out after about 5 time constants (5τ), at which point the response has reached more than 99% of its final value.
3. The time constant is used to compare the response speed of different R-L or R-C circuits and to design circuits (like timers and filters) for a specific desired response time.

Percentage Completion of Transient at Multiples of Time Constant

Time elapsed	1τ	2τ	3τ	4τ	5τ
% of final value reached	63.2%	86.5%	95.0%	98.2%	99.3%

> *Rule of thumb: A circuit's transient is considered to have practically ended, and the circuit is considered to have reached steady state, after approximately 5 time constants (5τ).*

Applications

- Selecting R and L (or R and C) values to design circuits with a desired charging/discharging speed, such as timer circuits and pulse-shaping networks.
- Estimating the settling time of protective devices and switching circuits after a disturbance.
- Comparing the transient response speed of different electrical and electronic circuits.

Limitations

- The time constant concept, as derived here, applies directly only to simple first-order (single R-L or single R-C) circuits; higher-order circuits (with both L and C) require more complex analysis involving multiple time constants or damped oscillatory behaviour.
- Component tolerances and non-ideal behaviour (temperature effects, parasitic elements) can cause the actual time constant to differ slightly from the calculated theoretical value.

4.5 Related Numerical Problems

This section presents solved numerical examples based on R-L and R-C transient circuits and time constant calculations, following the formulas explained above.

Numerical on R-L Circuit — Time Constant and Current Growth

Example 1: Time constant and current at a given instant in an R-L circuit

Given: A series R-L circuit with $R = 50\ \Omega$ and $L = 2$ H is connected to a 100 V D.C. supply through a switch. Find (a) the time constant of the circuit, (b) the final steady-state current, and (c) the current 0.02 s after the switch is closed.

Solution

Step 1 (Time constant): $\tau = L/R = 2/50 = 0.04$ s.

Step 2 (Final steady-state current): $I_m = V/R = 100/50 = 2$ A.

Step 3 (Current at $t = 0.02$ s): $i(t) = I_m [1 - e^{(-t/\tau)}] = 2 \times [1 - e^{(-0.02/0.04)}] = 2 \times [1 - e^{(-0.5)}]$.

Step 4 (Evaluate): $e^{(-0.5)} \approx 0.6065$, so $i(0.02) = 2 \times (1 - 0.6065) = 2 \times 0.3935 \approx 0.787$ A.

Answer: $\tau = 0.04$ s, $I_m = 2$ A, $i(0.02\text{ s}) \approx 0.787$ A

Numerical on R-L Circuit — Time to Reach a Given Fraction of Final Current

Example 2: Time required for current to reach 90% of its final value

Given: For the same R-L circuit as Example 1 ($R = 50\ \Omega$, $L = 2$ H, $\tau = 0.04$ s), find the time taken for the current to reach 90% of its final steady-state value.

Solution

Step 1 (Set up the equation): $i(t) = I_m [1 - e^{(-t/\tau)}] = 0.9\, I_m$, so $1 - e^{(-t/\tau)} = 0.9$, which gives $e^{(-t/\tau)} = 0.1$.

Step 2 (Take natural log of both sides): $-t/\tau = \ln(0.1) = -2.303$, so $t = 2.303 \times \tau$.

Step 3 (Substitute $\tau = 0.04$ s): $t = 2.303 \times 0.04 \approx 0.0921$ s.

Answer: $t \approx 0.0921$ s (about 92.1 ms) for the current to reach 90% of its final value

Numerical on R-C Circuit — Charging

Example 3: Time constant and capacitor voltage during charging

Given: A series R-C circuit with $R = 1000\ \Omega$ and $C = 50\ \mu F$ is connected to a 20 V D.C. supply through a switch, with the capacitor initially uncharged. Find (a) the time constant, and (b) the capacitor voltage 0.03 s after the switch is closed.

Solution

Step 1 (Time constant): $\tau = RC = 1000 \times 50 \times 10^{-6} = 0.05$ s.

Step 2 (Capacitor voltage at t = 0.03 s): $v_C(t) = V\,[1 - e^{(-t/\tau)}] = 20 \times [1 - e^{(-0.03/0.05)}] = 20 \times [1 - e^{(-0.6)}]$.

Step 3 (Evaluate): $e^{(-0.6)} \approx 0.5488$, so $v_C(0.03) = 20 \times (1 - 0.5488) = 20 \times 0.4512 \approx 9.02$ V.

> **Answer: $\tau = 0.05$ s, $v_C(0.03\text{ s}) \approx 9.02$ V**

Numerical on R-C Circuit — Discharging

Example 4: Capacitor discharge through a resistor

Given: A capacitor of $100\ \mu F$, initially charged to 50 V, is discharged through a resistor of $2000\ \Omega$. Find (a) the time constant, and (b) the capacitor voltage 0.1 s after the discharge begins.

Solution

Step 1 (Time constant): $\tau = RC = 2000 \times 100 \times 10^{-6} = 0.2$ s.

Step 2 (Discharge equation): $v_C(t) = V \cdot e^{(-t/\tau)}$, where V = 50 V is the initial voltage.

Step 3 (Evaluate at t = 0.1 s): $v_C(0.1) = 50 \times e^{(-0.1/0.2)} = 50 \times e^{(-0.5)} \approx 50 \times 0.6065 \approx 30.3$ V.

> **Answer: $\tau = 0.2$ s, $v_C(0.1\text{ s}) \approx 30.3$ V**

Numerical on Approximate Time for Steady State

Example 5: Time to reach practical steady state (5 time constants)

Given: For the R-C circuit of Example 3 ($\tau = 0.05$ s), estimate the time after which the capacitor can be considered to have reached its final (fully charged) steady-state value.

Solution

Step 1 (Recall the rule): A circuit is considered to have practically reached steady state after about 5 time constants (5τ), at which point the response has reached more than 99% of its final value.

Step 2 (Calculate): $t(\text{steady state}) \approx 5 \times \tau = 5 \times 0.05 = 0.25$ s.

> **Answer: The capacitor can be considered practically fully charged after approximately 0.25 s (5 time constants)**

Multiple Choice Questions (MCQs)

Choose the correct option for each question. The correct answer is shown in bold within the options, and a separate answer key is also given at the end.

Q1. A transient state in a circuit occurs due to:

a) Steady, unchanging supply conditions
b) A sudden change such as switching
c) Purely resistive circuits only
d) Constant frequency operation

Q2. In an inductor, which quantity cannot change instantaneously?

a) Voltage
b) Current
c) Resistance
d) Power

Q3. In a capacitor, which quantity cannot change instantaneously?

a) Current
b) Voltage
c) Resistance
d) Charge only, not voltage

Q4. The equation for current growth in a series R-L circuit connected to a D.C. supply V is:

a) $i(t) = (V/R)\, e^{(-Rt/L)}$
b) $i(t) = (V/R)\, [1 - e^{(-Rt/L)}]$
c) $i(t) = V \cdot e^{(-t/RC)}$
d) $i(t) = (V/R)(Rt/L)$

Q5. In a charging R-C circuit, the capacitor voltage equation is:

a) $v_C(t) = V\, e^{(-t/RC)}$
b) $v_C(t) = V\, [1 - e^{(-t/RC)}]$
c) $v_C(t) = (V/R)[1 - e^{(-t/RC)}]$
d) $v_C(t) = V\, (t/RC)$

Q6. The time constant of a series R-L circuit is given by:

a) $\tau = RC$
b) $\tau = L/R$
c) $\tau = R/L$
d) $\tau = 1/(RC)$

Q7. The time constant of a series R-C circuit is given by:

a) $\tau = L/R$
b) $\tau = RC$
c) $\tau = R/C$
d) $\tau = 1/(RL)$

Q8. At one time constant ($t = \tau$), the response of a charging R-L or R-C circuit reaches approximately:

a) 37%
b) 50%
c) 63.2%
d) 99%

Q9. A transient is generally considered to have practically died out after:

a) 1 time constant
b) 2 time constants
c) 5 time constants
d) 10 time constants

Q10. In a purely resistive circuit, the transient period is:

a) Very long
b) Equal to L/R
c) Absent (current changes instantaneously)
d) Equal to RC

Answer Key

1-b | 2-b | 3-b | 4-b | 5-b | 6-b | 7-b | 8-c | 9-c | 10-c

Broad / Long Answer Questions

1. Explain the terms transient state and steady state with suitable examples. What are the main causes of transients in electrical circuits?
2. Why can the current through an inductor and the voltage across a capacitor not change instantaneously? Explain with reasoning.
3. Derive the expression for the growth of current in a series R-L circuit connected to a D.C. voltage source, starting from the KVL equation.
4. Explain, with waveforms, the variation of current, voltage across R, and voltage across L during the growth transient in a series R-L circuit.
5. Derive the expression for the charging voltage across a capacitor in a series R-C circuit connected to a D.C. source, starting from the KVL equation.
6. Explain, with waveforms, the variation of capacitor voltage and charging current during the charging of a capacitor in a series R-C circuit.
7. Explain the discharging of a capacitor through a resistor, and derive the expressions for capacitor voltage and discharge current.
8. Define time constant for R-L and R-C circuits. Explain its physical significance and derive the percentage of final value reached at $t = \tau$.
9. Explain why a circuit is considered to have reached practical steady state after about 5 time constants, with the help of a table or graph showing percentage completion at 1τ, 2τ, 3τ, 4τ and 5τ.
10. A series R-L circuit with given values of R and L is connected to a D.C. supply. Explain the complete procedure to calculate the time constant, the final current, and the current at any given instant of time.

— End of Notes —