CIRCUIT THEORY

Unit 6: Two Port Network

Diploma / Polytechnic Engineering — Study Notes

Introduction

A two-port network is an electrical network having two pairs of terminals — one pair designated as the input port (Port 1) and the other as the output port (Port 2) — through which current enters and leaves the network. Two-port networks are widely used to represent devices such as transmission lines, amplifiers, filters, transformers and attenuators, since their external behaviour can be completely described by a set of four parameters relating the port voltages and currents, without needing to know the internal circuit details. This chapter covers the Z (open-circuit impedance) and Y (short-circuit admittance) parameters, the ABCD (transmission) parameters, and the relationships between them.

For any two-port network, the port variables are defined as: V1 and I1 (voltage and current at the input port, Port 1), and V2 and I2 (voltage and current at the output port, Port 2). By convention, I1 and I2 are taken as entering the network at their respective ports.

6.1 Open Circuit Impedance (Z) and Short Circuit Admittance (Y) Parameters

6.1.1 Open Circuit Impedance (Z) Parameters

The Z-parameters (impedance parameters) express the two port voltages (V1, V2) in terms of the two port currents (I1, I2). They are called 'open-circuit' parameters because each parameter is measured/calculated by keeping one of the ports open-circuited (i.e., the current at that port is made zero).

The defining equations of the Z-parameters are:

- $V1 = Z11 \cdot I1 + Z12 \cdot I2$
- $V2 = Z21 \cdot I1 + Z22 \cdot I2$

Determination of Z-Parameters (Open Circuit Test)

1. Z11 (open-circuit input impedance): With Port 2 open-circuited ($I2 = 0$), apply a voltage/current at Port 1 and find $Z11 = V1/I1$ (with $I2 = 0$). It represents the driving-point impedance seen at Port 1 with Port 2 open.
2. Z21 (open-circuit forward transfer impedance): With Port 2 open-circuited ($I2 = 0$), find $Z21 = V2/I1$ (with $I2 = 0$). It represents how much voltage appears at Port 2 for a given current injected at Port 1.
3. Z22 (open-circuit output impedance): With Port 1 open-circuited ($I1 = 0$), apply a voltage/current at Port 2 and find $Z22 = V2/I2$ (with $I1 = 0$). It represents the driving-point impedance seen at Port 2 with Port 1 open.
4. Z12 (open-circuit reverse transfer impedance): With Port 1 open-circuited ($I1 = 0$), find $Z12 = V1/I2$ (with $I1 = 0$). It represents how much voltage appears at Port 1 for a given current injected at Port 2.

> *For a reciprocal (passive, bilateral) network, $Z12 = Z21$. For a symmetrical network, additionally $Z11 = Z22$.*

6.1.2 Short Circuit Admittance (Y) Parameters

The Y-parameters (admittance parameters) express the two port currents (I1, I2) in terms of the two port voltages (V1, V2). They are called 'short-circuit' parameters because each parameter is measured/calculated by keeping one of the ports short-circuited (i.e., the voltage at that port is made zero).

The defining equations of the Y-parameters are:

- I1 = Y11·V1 + Y12·V2
- I2 = Y21·V1 + Y22·V2

Determination of Y-Parameters (Short Circuit Test)

1. Y11 (short-circuit input admittance): With Port 2 short-circuited (V2 = 0), find Y11 = I1/V1 (with V2 = 0). It represents the driving-point admittance seen at Port 1 with Port 2 shorted.
2. Y21 (short-circuit forward transfer admittance): With Port 2 short-circuited (V2 = 0), find Y21 = I2/V1 (with V2 = 0). It represents how much current flows at Port 2 for a given voltage applied at Port 1.
3. Y22 (short-circuit output admittance): With Port 1 short-circuited (V1 = 0), find Y22 = I2/V2 (with V1 = 0). It represents the driving-point admittance seen at Port 2 with Port 1 shorted.
4. Y12 (short-circuit reverse transfer admittance): With Port 1 short-circuited (V1 = 0), find Y12 = I1/V2 (with V1 = 0). It represents how much current flows at Port 1 for a given voltage applied at Port 2.

> *For a reciprocal (passive, bilateral) network, Y12 = Y21. Also, the Y-parameter matrix is the inverse of the Z-parameter matrix (in matrix form), i.e., $[Y] = [Z]^{-1}$.*

Applications

- Z-parameters are convenient for networks connected in series (series-series connection of two two-port networks), since impedances add directly.
- Y-parameters are convenient for networks connected in parallel (parallel-parallel connection of two two-port networks), since admittances add directly.
- Used in the modelling and analysis of transmission lines, filters, transistor amplifier circuits (at a given operating point/frequency), and impedance-matching networks.

Limitations

- Some networks do not have a defined set of Z-parameters (e.g., if the open-circuit condition leads to an indeterminate or infinite impedance) or Y-parameters (if the short-circuit condition is not physically realisable or leads to infinite current); in such cases, a different parameter set (like ABCD or hybrid parameters) must be used.
- Z and Y parameters (as commonly used) apply to linear, time-invariant two-port networks at a fixed frequency; non-linear or time-varying networks require different analysis techniques.

6.2 Transmission (ABCD) Parameters and Their Inter-Relations

The Transmission parameters, also called ABCD parameters or general circuit parameters, express the input port quantities (V1, I1) in terms of the output port quantities (V2, I2). They are particularly useful for analysing networks connected in cascade (one network's output feeding directly into the next network's input), such as transmission lines and cascaded filter/amplifier stages, because the overall ABCD matrix of cascaded networks is simply the matrix product of the individual ABCD matrices.

The defining equations of the ABCD parameters are:

- V1 = A·V2 + B·(−I2)
- I1 = C·V2 + D·(−I2)

Note the sign convention: I2 is taken as (−I2), i.e., the current leaving Port 2 into the load, rather than entering Port 2, which is the standard convention used for transmission parameters (since the output port normally feeds a load).

Determination of ABCD Parameters

1. A (open-circuit reverse voltage ratio): With Port 2 open-circuited (I2 = 0), A = V1/V2 (with I2 = 0). It is a dimensionless ratio representing the reverse voltage gain from output to input under open-circuit conditions.
2. B (short-circuit reverse transfer impedance): With Port 2 short-circuited (V2 = 0), B = −V1/I2 (with V2 = 0). It has the dimension of impedance (ohms).
3. C (open-circuit reverse transfer admittance): With Port 2 open-circuited (I2 = 0), C = I1/V2 (with I2 = 0). It has the dimension of admittance (siemens/mho).
4. D (short-circuit reverse current ratio): With Port 2 short-circuited (V2 = 0), D = −I1/I2 (with V2 = 0). It is a dimensionless ratio representing the reverse current gain from output to input under short-circuit conditions.

> *For a reciprocal (passive, bilateral) network, the ABCD parameters satisfy the condition AD − BC = 1. For a symmetrical network, additionally A = D.*

Cascade Connection Property

The most important property of ABCD parameters is that, if two (or more) two-port networks are connected in cascade (the output of the first network is the input of the second network), the overall ABCD matrix of the combined network is obtained simply by multiplying the individual ABCD matrices of each network in the same order as they are cascaded. This makes ABCD parameters especially convenient for analysing long transmission lines or multi-stage networks built from repeated identical sections.

6.2.1 Inter-Relations Between Z, Y and ABCD Parameters

Since the Z, Y and ABCD parameters all describe the same two-port network, they are mathematically related to each other, and any one set can be derived from another set using standard conversion formulas.

Relation	Conversion Formulas
ABCD in terms of Z-parameters	A = Z11/Z21, B = (Z11·Z22 − Z12·Z21)/Z21, C = 1/Z21, D = Z22/Z21.
Z-parameters in terms of ABCD	Z11 = A/C, Z12 = (A·D − B·C)/C, Z21 = 1/C, Z22 = D/C.
ABCD in terms of Y-parameters	A = −Y22/Y21, B = −1/Y21, C = −(Y11·Y22 − Y12·Y21)/Y21, D = −Y11/Y21.
Y-parameters in terms of ABCD	Y11 = D/B, Y12 = (B·C − A·D)/B, Y21 = −1/B, Y22 = A/B.
Y-parameters in terms of Z-parameters	Y11 = Z22/ΔZ, Y12 = −Z12/ΔZ, Y21 = −Z21/ΔZ, Y22 = Z11/ΔZ, where ΔZ = Z11·Z22 − Z12·Z21 (determinant of the Z-matrix).

> *A quick way to remember these relations is that the [Y] matrix is the matrix inverse of the [Z] matrix, and the ABCD matrix can be obtained from either the Z or Y matrix using the standard conversion table (as commonly provided in circuit theory textbooks/reference tables).*

Applications

- ABCD parameters are extensively used in transmission line analysis, to relate sending-end voltage/current to receiving-end voltage/current.
- Used in cascade analysis of filter sections, attenuator networks, and multi-stage amplifier circuits.

- Conversion formulas allow engineers to freely move between Z, Y and ABCD representations depending on which is most convenient for a given problem (e.g., Z for series connections, Y for parallel connections, ABCD for cascade connections).

Limitations

- ABCD parameters (as defined here) apply directly to networks with a well-defined input and output port and a specific direction of signal flow (input to output); using them for networks without a clear cascade structure adds unnecessary complexity.
- Conversion between parameter sets fails (division by zero) when the reference parameter (e.g., Z21, Y21, B) used in the denominator of a conversion formula happens to be zero for a particular network, requiring the use of an alternative parameter set instead.

Related Numerical Problems

This section presents simple solved numerical examples on finding Z-parameters, Y-parameters and ABCD parameters of basic two-port networks, and converting between parameter sets.

Numerical on Z-Parameters

Example 1: Z-parameters of a simple T-network

Given: A symmetrical T-network has a series arm of 5 Ω at the input, a series arm of 5 Ω at the output, and a shunt arm of 10 Ω between them (connected to the common/reference terminal). Find the Z-parameters (Z11, Z12, Z21, Z22) of this network.

Solution

Step 1 (Z11 — Port 2 open): With Port 2 open (I2 = 0), the impedance seen from Port 1 is the input series arm (5 Ω) in series with the shunt arm (10 Ω), since no current flows in the output series arm. Z11 = 5 + 10 = 15 Ω.

Step 2 (Z22 — Port 1 open): By symmetry of the T-network (both series arms are 5 Ω), Z22 = 5 + 10 = 15 Ω.

Step 3 (Z21 — Port 2 open, find V2/I1): With Port 2 open, all of I1 flows through the shunt arm (10 Ω) only (since the output series arm carries no current when Port 2 is open), so V2 (measured across the open Port 2 through the output series arm, which carries no current) equals the voltage across the shunt arm: V2 = I1 × 10. Hence Z21 = V2/I1 = 10 Ω.

Step 4 (Z12 — by reciprocity): Since this is a passive, reciprocal network, Z12 = Z21 = 10 Ω.

Answer: Z11 = 15 Ω, Z22 = 15 Ω, Z12 = Z21 = 10 Ω

Numerical on Y-Parameters

Example 2: Y-parameters of a simple Pi (π) network

Given: A symmetrical π-network has a shunt admittance of 0.1 S at the input terminal, a shunt admittance of 0.1 S at the output terminal, and a series admittance of 0.2 S connecting the two. Find the Y-parameters of this network.

Solution

Step 1 (Y11 — Port 2 shorted): With Port 2 shorted (V2 = 0), the admittance seen at Port 1 is the input shunt admittance (0.1 S) in parallel with the series admittance (0.2 S), since the series arm now connects directly to the shorted (zero-voltage) output. Y11 = 0.1 + 0.2 = 0.3 S.

Step 2 (Y22 — Port 1 shorted): By symmetry of the π-network (both shunt admittances are 0.1 S), Y22 = 0.1 + 0.2 = 0.3 S.

Step 3 (Y21 — Port 2 shorted, find I2/V1): With Port 2 shorted, the current flowing into the short from the series arm is I2 = −V1 × 0.2 (negative sign because the defined current direction I2 enters Port 2, but current actually flows out of Port 2 into the short from the network side). So Y21 = I2/V1 = −0.2 S.

Step 4 (Y12 — by reciprocity): Since this is a passive, reciprocal network, Y12 = Y21 = −0.2 S.

> **Answer: Y11 = 0.3 S, Y22 = 0.3 S, Y12 = Y21 = −0.2 S**

Numerical on ABCD Parameters from Z-Parameters

Example 3: Finding ABCD parameters from known Z-parameters

Given: For the T-network of Example 1 (Z11 = 15 Ω, Z22 = 15 Ω, Z12 = Z21 = 10 Ω), find the ABCD (transmission) parameters.

Solution

Step 1 (Parameter A): A = Z11/Z21 = 15/10 = 1.5.

Step 2 (Parameter B): B = (Z11·Z22 − Z12·Z21)/Z21 = (15×15 − 10×10)/10 = (225 − 100)/10 = 125/10 = 12.5 Ω.

Step 3 (Parameter C): C = 1/Z21 = 1/10 = 0.1 S.

Step 4 (Parameter D): D = Z22/Z21 = 15/10 = 1.5.

Step 5 (Verify reciprocity condition): AD − BC = (1.5×1.5) − (12.5×0.1) = 2.25 − 1.25 = 1, confirming the network is reciprocal, as expected for a passive resistive T-network.

> **Answer: A = 1.5, B = 12.5 Ω, C = 0.1 S, D = 1.5 (and AD − BC = 1, confirming reciprocity)**

Numerical on ABCD Parameters — Series Impedance Network

Example 4: ABCD parameters of a simple series impedance

Given: A two-port network consists of a single series impedance Z = 20 Ω connected directly between the input and output ports (no shunt element). Find the ABCD parameters of this network.

Solution

Step 1 (Parameter A): With Port 2 open (I2 = 0), no current flows through the series impedance, so V1 = V2 (no voltage drop). Hence A = V1/V2 = 1.

Step 2 (Parameter C): With Port 2 open (I2 = 0), since no current flows anywhere in this simple series network, I1 = 0 for any finite V2. Hence C = I1/V2 = 0 S.

Step 3 (Parameter B): With Port 2 shorted (V2 = 0), the entire input voltage V1 appears across the series impedance Z, driving current I1 = −I2 (current simply passes through). So V1 = I1 × Z = (−I2) × Z, giving B = −V1/I2 = Z = 20 Ω.

Step 4 (Parameter D): With Port 2 shorted (V2 = 0), the same current flows through the series element, so I1 = −I2 directly (undivided), giving D = −I1/I2 = 1.

> **Answer: A = 1, B = 20 Ω (= Z), C = 0 S, D = 1 — the standard ABCD matrix for a simple series impedance**

Multiple Choice Questions (MCQs)

Choose the correct option for each question. The correct answer is shown in bold within the options, and a separate answer key is also given at the end.

Q1. In a two-port network, the Z-parameters express:

a) Currents in terms of voltages
b) Voltages in terms of currents
c) Input quantities in terms of output quantities
d) Power in terms of impedance only

Q2. Z-parameters are called 'open-circuit' parameters because they are found by:

a) Short-circuiting one port
b) Open-circuiting one port
c) Grounding both ports
d) Connecting a matched load

Q3. Y-parameters are called 'short-circuit' parameters because they are found by:

a) Open-circuiting one port
b) Short-circuiting one port
c) Removing the source
d) Reversing polarity

Q4. For a reciprocal two-port network, the Z-parameters satisfy:

a) $Z11 = Z22$
b) $Z12 = Z21$
c) $Z12 = -Z21$
d) $Z11 = Z12$

Q5. The relationship between the Y-matrix and Z-matrix of a two-port network is:

a) $[Y] = [Z]$
b) $[Y] = [Z]^{-1}$
c) $[Y] = -[Z]$
d) $[Y] = [Z]^2$

Q6. ABCD (transmission) parameters express:

a) Output quantities in terms of input quantities
b) Input quantities (V1, I1) in terms of output quantities (V2, −I2)
c) Currents only, without voltages
d) Power gain only

Q7. For a reciprocal two-port network, the ABCD parameters satisfy the condition:

a) $A = D$ only
b) $AD - BC = 1$
c) $AD + BC = 0$
d) $A + D = 1$

Q8. ABCD parameters are especially convenient for analysing networks connected in:

a) Series only
b) Parallel only
c) Cascade (one network feeding the next)
d) Star only

Q9. Parameter B in the ABCD parameter set has the unit of:

a) Siemens (mho)
b) Ohms (impedance)

c) Dimensionless ratio
d) Henries

Q10. Z21 in terms of ABCD parameters is given by:

a) Z21 = 1/C
b) Z21 = C
c) Z21 = A/C
d) Z21 = D/C

Answer Key

1-b | 2-b | 3-b | 4-b | 5-b | 6-b | 7-b | 8-c | 9-b | 10-a

Broad / Long Answer Questions

1. Define a two-port network and explain the significance of the port variables V1, I1, V2 and I2.
2. Define the Z-parameters of a two-port network and explain how each parameter (Z11, Z12, Z21, Z22) is determined using the open-circuit test.
3. Define the Y-parameters of a two-port network and explain how each parameter (Y11, Y12, Y21, Y22) is determined using the short-circuit test.
4. Define the ABCD (transmission) parameters of a two-port network, and explain how each parameter (A, B, C, D) is determined.
5. Explain the significance of the cascade connection property of ABCD parameters, and why it makes them useful for transmission line and multi-stage network analysis.
6. State the conditions for reciprocity and symmetry in terms of the Z-parameters, Y-parameters and ABCD parameters of a two-port network.
7. Derive the relationship between the ABCD parameters and the Z-parameters of a two-port network.
8. Derive the relationship between the Y-parameters and the Z-parameters of a two-port network.
9. Find the Z-parameters of a symmetrical T-network having series arms of Za, Zc and a shunt arm of Zb, and express Z11, Z12, Z21 and Z22 in terms of Za, Zb and Zc.
10. Explain the practical applications of Z, Y and ABCD parameters in the analysis of electrical networks such as transmission lines, filters and amplifier circuits.

— End of Notes —