Engineering Graphics

Geometrical Construction

2.1(a) Regular Polygons, Ellipse, Parabola and Hyperbola

Regular Polygons

A regular polygon is a closed plane figure having all sides equal in length and all interior angles equal in measure, such as an equilateral triangle, a square, a regular pentagon, or a regular hexagon. Regular polygons are most conveniently constructed with the help of a circumscribing circle (a circle passing through all the vertices of the polygon).

The general method of construction is as follows: a circle of the required size is first drawn, and its circumference is then divided into the same number of equal parts as the number of sides required in the polygon (for example, five equal parts for a pentagon, or six equal parts for a hexagon). The resulting division points, when joined in sequence by straight lines, form the required regular polygon.

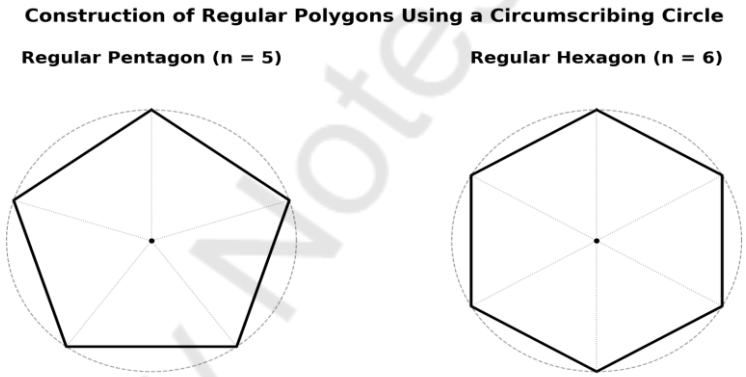

Figure 1: Construction of regular polygons (pentagon and hexagon) using a circumscribing circle

Ellipse, Parabola and Hyperbola (Conic Sections)

An ellipse, a parabola, and a hyperbola are collectively known as conic sections, since each of these curves can be obtained by cutting a right circular cone with a plane at different angles of inclination relative to the axis of the cone. Each curve also has an equivalent geometric definition based on the ratio of the distance of a moving point from a fixed point (the focus) to its distance from a fixed straight line (the directrix), known as the eccentricity of the curve.

Curve	General Method of Construction (Concept)
Regular Polygon	A circumscribing (or inscribing) circle is drawn first; the circle is then divided into the required number of equal parts (equal to the number of

	sides), and the division points are joined in order by straight lines to form the polygon.
Ellipse	Commonly constructed by the concentric circle method (using two concentric circles of radii equal to the semi-major and semi-minor axes) or the arcs of circles (foci) method, plotting a series of points that satisfy the defining property of the ellipse, and joining them with a smooth curve.
Parabola	Commonly constructed by the rectangle method or the offset method, using the given base and height (or the focus and directrix), plotting a series of points and joining them with a smooth curve.
Hyperbola	Commonly constructed using the given asymptotes and a point on the curve (or the focus-directrix property), plotting a series of points that satisfy the defining property of the hyperbola, and joining them with a smooth curve.

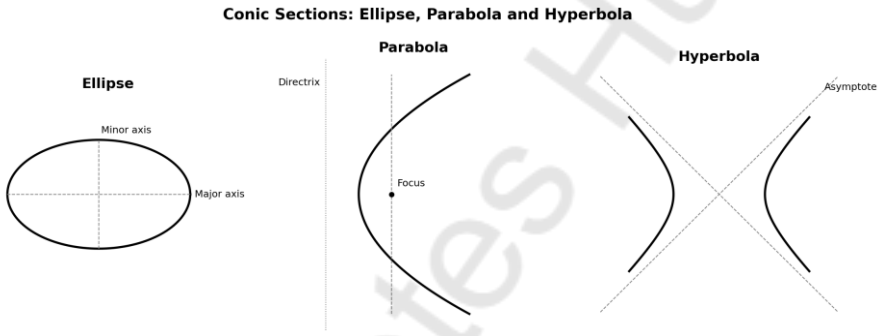

Figure 2: The three conic sections — ellipse, parabola, and hyperbola

An ellipse is a closed curve with an eccentricity less than 1, possessing two axes of symmetry — a longer major axis and a shorter minor axis. A parabola is an open curve with an eccentricity exactly equal to 1, possessing a single axis of symmetry, a focus, and a directrix. A hyperbola is an open curve with an eccentricity greater than 1, consisting of two separate branches, and is characterised by two straight-line asymptotes that the curve approaches indefinitely but never actually touches.

2.1(b) Curve Through Given Points, Cycloid, and Involutes

Curve Passing Through a Given Number of Points

In many practical drawing and graphical situations (such as plotting the results of an experiment, or drawing an irregular outline), a series of individual points is first established, and a smooth, continuous curve must then be drawn passing through all of these points in sequence. Such a curve is generally constructed using a French curve (a drafting instrument with a variety of curved edges) or freehand, ensuring that the resulting curve passes smoothly through every given point, without sharp, unnatural corners or kinks, and follows the general trend suggested by the sequence of points.

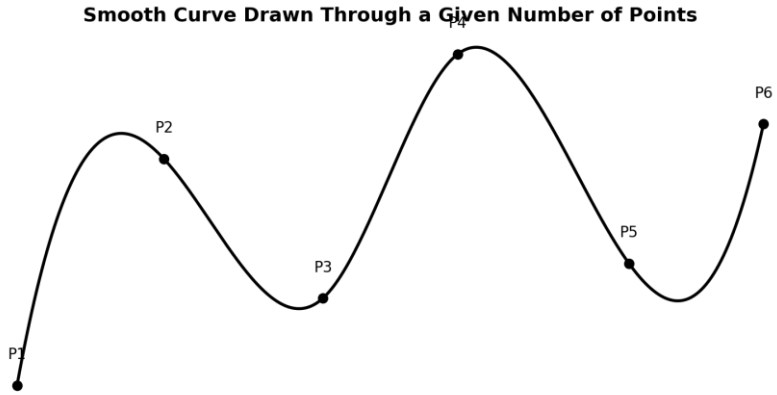

Figure 3: A smooth curve drawn through a given series of points

Cycloidal and Involute Curves

Cycloidal curves and involutes are important classes of curves generated by the rolling or unwinding motion of one geometric figure relative to another, and have significant practical applications in engineering, particularly in the design of gear teeth profiles and cam mechanisms.

Curve	Definition
Cycloid	The curve traced by a point on the circumference of a circle as the circle rolls, without slipping, along a straight line (called the directing line).
Involute of a Circle	The curve traced by the free end of a taut string as it is gradually unwound from around a circle, the string always remaining tangential to the circle.
Involute of a Polygon	The curve traced by the free end of a taut string as it is gradually unwound from around the perimeter of a regular polygon, formed of a series of circular arcs, each centred at a successive corner of the polygon, with the radius increasing by one side length at each corner.

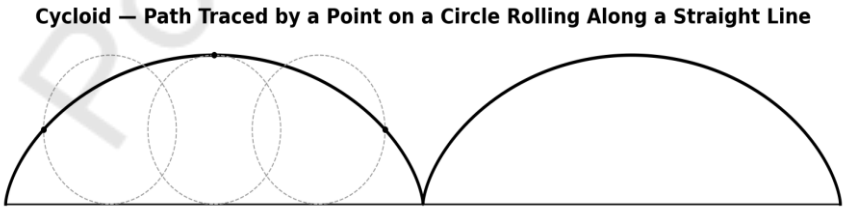

Figure 4: Cycloid — generated by a circle rolling along a straight line

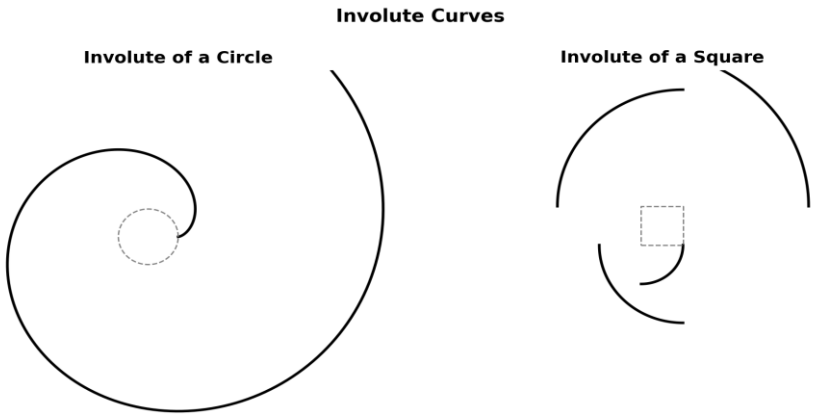

Figure 5: Involute of a circle and involute of a square

As shown in Figure 5, the involute of a circle spirals continuously outward from the circle, always remaining tangential to it, tracing a smooth curve of continuously increasing radius of curvature. The involute of a polygon, in contrast, is constructed piecewise, as a sequence of circular arcs, each centred on a successive corner of the polygon, with the radius of each arc increasing by exactly one side length as the string unwinds around each corner in turn.

Both the involute of a circle and cycloidal curves (including the cycloid and its related epicycloid and hypocycloid curves, generated by a circle rolling on the outside or inside of another circle rather than a straight line) are of considerable practical importance in engineering, most notably in the design of the involute and cycloidal tooth profiles used in gears, where the specific geometric properties of these curves ensure smooth, efficient, and accurately predictable transmission of motion between meshing gear teeth.

3. Multiple Choice Questions (MCQs)

Answer the following questions by selecting the most appropriate option.

1. A regular polygon is generally constructed by first drawing:

A) A straight base line only
B) A circumscribing or inscribing circle, divided into equal parts
C) A random freehand curve
D) Two perpendicular axes only

2. An ellipse is a conic section obtained by cutting a cone with a plane:

A) Parallel to the base of the cone
B) Inclined to the axis, cutting all the generators (but not parallel to the base or any generator)
C) Parallel to one of the generators of the cone
D) Passing through the apex only

3. A parabola is a conic section obtained by cutting a cone with a plane:

A) Parallel to the base of the cone

B) Parallel to one of the generators (slant sides) of the cone

C) Passing through the apex only

D) Perpendicular to the axis of the cone

4. A hyperbola is a conic section obtained by cutting a cone with a plane:

A) Parallel to the base of the cone

B) Parallel to the axis of the cone (or at an angle greater than the semi-angle of the cone)

C) Perpendicular to a generator

D) Through the apex parallel to the base

5. The longer axis of an ellipse is called its:

A) Minor axis

B) Major axis

C) Conjugate axis

D) Transverse axis

6. A hyperbola is characterised by having two straight lines, which the curve approaches but never touches, called:

A) Directrices

B) Asymptotes

C) Tangents

D) Normals

7. A curve passing through a given number of points, where the points do not lie on a simple known geometric curve, is generally drawn using:

A) A compass only, without any curve-fitting

B) A smooth freehand or French-curve-guided line connecting the points in sequence

C) Only straight lines joining the points

D) Random, unconnected marks

8. A cycloid is defined as the curve traced by a point on:

A) A stationary circle

B) The circumference of a circle rolling without slipping along a straight line

C) A straight line moving along a circle

D) The diameter of a fixed circle

9. The involute of a circle is defined as the path traced by:

A) A point fixed at the centre of the circle

B) The free end of a taut string as it is unwound from the circle

C) A point moving along the diameter

D) A point on a straight line only

10. The involute of a polygon is constructed using a series of:

A) Straight lines of equal length only

B) Circular arcs of successively increasing radius, each centred at a corner of the polygon

C) Parabolic segments

D) Concentric circles of equal radius

11. In the involute of a polygon with side length 's', the radius of each successive arc increases by:

A) Half the side length

B) One side length, 's'

C) Twice the side length

D) A random, unrelated amount

12. Which of the following curves is generated by rolling motion of a circle along a straight line?

A) Involute

B) Cycloid

C) Ellipse

D) Hyperbola

13. Practical engineering applications of the involute curve include the design of:

A) Railway tracks

B) Gear tooth profiles

C) Aircraft wings

D) Window frames

14. A smooth curve through several given points, such as in plotting graphical data, should generally:

A) Have sharp corners at every given point

B) Pass through all the given points with a smooth, continuously curving line

C) Ignore some of the given points

D) Consist only of straight line segments

15. The number of equal parts into which a circumscribing circle is divided to construct a regular polygon is equal to:

A) Twice the number of sides of the polygon

B) The number of sides of the polygon

C) Half the number of sides of the polygon

D) A fixed number, always six

Answer Key

1. B) A circumscribing or inscribing circle, divided into equal parts | 2. B) Inclined to the axis, cutting all the generators (but not parallel to the base or any generator) | 3. B) Parallel to one of the generators (slant sides) of the cone | 4. B) Parallel to the axis of the

cone (or at an angle greater than the semi-angle of the cone) | 5. B) Major axis | 6. B) Asymptotes | 7. B) A smooth freehand or French-curve-guided line connecting the points in sequence | 8. B) The circumference of a circle rolling without slipping along a straight line | 9. B) The free end of a taut string as it is unwound from the circle | 10. B) Circular arcs of successively increasing radius, each centred at a corner of the polygon | 11. B) One side length, 's' | 12. B) Cycloid | 13. B) Gear tooth profiles | 14. B) Pass through all the given points with a smooth, continuously curving line | 15. B) The number of sides of the polygon

4. Broad / Long Answer and Practical Questions

1. Explain the general method used to construct a regular polygon using a circumscribing circle, illustrating your answer with reference to a regular hexagon.

2. Define an ellipse, and explain, in general terms, the concentric circle method used to construct it.

3. Define a parabola, and explain, in general terms, the rectangle method used to construct it.

4. Define a hyperbola, and explain the significance of its asymptotes in relation to the shape of the curve.

5. Explain the method used to draw a smooth curve passing through a given number of points, and discuss why a smooth, continuous curve is generally preferred over straight line segments in such constructions.

6. Define a cycloid, and explain the geometric principle by which it is generated.

7. Define the involute of a circle, and explain the geometric principle by which it is generated.

8. Define the involute of a polygon, and explain how the construction differs from the involute of a circle.

9. Draw (freehand or with instruments) a regular pentagon of side 30 mm, using the circumscribing circle method, and label the construction clearly.

10. Draw (freehand or with instruments) an involute of a circle of diameter 40 mm, and explain each step of your construction.