

MATHEMATICS 1

Chapter: Binomial Theorem

Short Answer Questions (2 Marks) with Solutions

Part 1: Expansions

Q1. Expand $(x + y)^5$ using Pascal's triangle.

Solution: The row of Pascal's triangle for $n = 5$ is 1, 5, 10, 10, 5, 1. Powers of x decrease and powers of y increase.

Ans. $x^5 + 5x^4y + 10x^3y^2 + 10x^2y^3 + 5xy^4 + y^5$.

Q2. Expand $(1 - 3x)^4$.

Solution:

1. Coefficients 1, 4, 6, 4, 1 with second term $-3x$.

2. $1 + 4(-3x) + 6(9x^2) + 4(-27x^3) + 81x^4$

Ans. $1 - 12x + 54x^2 - 108x^3 + 81x^4$.

Q3. Expand $(x^2 + 1/x)^3$.

Solution:

1. $(x^2)^3 + 3(x^2)^2(1/x) + 3(x^2)(1/x)^2 + (1/x)^3$

2. $= x^6 + 3x^3 + 3 + x^{-3}$

Ans. $x^6 + 3x^3 + 3 + 1/x^3$.

Part 2: Terms and Coefficients

Q4. Find the 4th term of $(x + 2y)^7$.

Solution: $T_4 = T_{3+1} = {}^7C_3 x^4 (2y)^3 = 35 \times 8 x^4 y^3$

Ans. $T_4 = 280 x^4 y^3$.

Q5. Find the 5th term of $(1 - x/2)^6$.

Solution: $T_5 = {}^6C_4 (1)^2 (-x/2)^4 = 15 \times x^4/16$

Ans. $T_5 = (15/16) x^4$.

Q6. Find the middle term of $(1 + x)^8$.

Solution: $n = 8$ is even, so the middle term is $T_5: {}^8C_4 x^4$.

Ans. Middle term $= 70x^4$.

Q7. Find the middle terms of $(a + b)^5$.

Solution:

1. $n = 5$ is odd, so the middle terms are T_3 and T_4 .

2. $T_3 = {}^5C_2 a^3 b^2 = 10a^3 b^2$; $T_4 = {}^5C_3 a^2 b^3 = 10a^2 b^3$

Ans. $10a^3 b^2$ and $10a^2 b^3$.

Q8. Find the 3rd term from the end in the expansion of $(a + b)^9$.

Solution:

1. The r -th term from the end is the $(n - r + 2)$ -th term from the beginning: $(9 - 3 + 2) = 8$ th term.
2. $T_8 = {}^9C_7 a^2 b^7 = 36a^2b^7$

Ans. $36a^2b^7$.

Q9. The expansion of $(2x + 3)^n$ has 15 terms. Find n .

Solution: Number of terms = $n + 1 = 15$

Ans. $n = 14$.

Q10. Find the coefficient of x^3 in the expansion of $(1 + x)^7$.

Solution: Coefficient of x^r in $(1 + x)^n$ is nC_r , so it is ${}^7C_3 = (7 \times 6 \times 5)/(3 \times 2 \times 1)$

Ans. Coefficient = 35.

Q11. Find the coefficient of x^4 in the expansion of $(2 - x)^6$.

Solution:

1. $T_{r+1} = {}^6C_r 2^{6-r} (-x)^r$. For x^4 , $r = 4$.
2. Coefficient = ${}^6C_4 \times 2^2 \times (-1)^4 = 15 \times 4$

Ans. Coefficient = 60.

Q12. Find the term independent of x in $(x + 1/x)^6$.

Solution:

1. $T_{r+1} = {}^6C_r x^{6-r} (1/x)^r = {}^6C_r x^{6-2r}$.
2. Put $6 - 2r = 0$, so $r = 3$. Term = 6C_3

Ans. Term independent of $x = 20$.

Q13. Find the greatest binomial coefficient in the expansion of $(1 + x)^9$.

Solution:

1. $n = 9$ is odd, so the greatest coefficients are the two middle ones: 9C_4 and 9C_5 , which are equal.
2. ${}^9C_4 = (9 \times 8 \times 7 \times 6)/(4 \times 3 \times 2 \times 1) = 126$

Ans. Greatest binomial coefficient = 126.

Part 3: Binomial Coefficients and Properties

Q14. Find the value of ${}^{10}C_3 + {}^{10}C_4$.

Solution: Use ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$: ${}^{10}C_3 + {}^{10}C_4 = {}^{11}C_4 = (11 \times 10 \times 9 \times 8)/(4 \times 3 \times 2 \times 1)$

Ans. 330.

Q15. If ${}^nC_2 = 45$, find n .

Solution:

1. $n(n - 1)/2 = 45$, so $n^2 - n - 90 = 0$.
2. $(n - 10)(n + 9) = 0$. Since n is positive, $n = 10$.

Ans. $n = 10$.

Q16. If ${}^{15}C_{3r} = {}^{15}C_{r+3}$, find r .

Solution:

1. If ${}^nC_x = {}^nC_y$, then $x = y$ or $x + y = n$.
2. Case 1: $3r = r + 3$ gives $r = 3/2$, which is not an integer, so it is rejected.
3. Case 2: $3r + (r + 3) = 15$ gives $4r = 12$, so $r = 3$.
4. Check: ${}^{15}C_9 = {}^{15}C_6$ ✓

Ans. $r = 3$.

Q17. Prove that ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$.

Solution:

1. LHS = $n!/[r!(n-r)!] + n!/[(r-1)!(n-r+1)!]$
2. Take $n!/[(r-1)!(n-r+1)!]$ common: $= n!/[(r-1)!(n-r+1)!] \times [(n-r+1) + r]$
3. $= n!(n+1)/[r!(n-r+1)!] = (n+1)!/[r!(n+1-r)!] = {}^{n+1}C_r$

Ans. Hence proved.

Q18. Find the sum of all the binomial coefficients in the expansion of $(a + b)^8$.

Solution: Put $a = b = 1$: the sum of coefficients $= 2^8$

Ans. 256.

Q19. Find the value of $C_0 + C_2 + C_4 + \dots + C_{10}$ for $n = 10$.

Solution: The sum of the even-position coefficients is $2^{n-1} = 2^9$

Ans. 512.

Q20. Find the sum of the coefficients in the expansion of $(1 + 2x)^5$.

Solution: Put $x = 1$: $(1 + 2)^5 = 3^5$

Ans. 243.

Part 4: Numerical Problems

Q21. Evaluate $(102)^3$ using the binomial theorem.

Solution:

1. $(100 + 2)^3 = 100^3 + 3(100)^2(2) + 3(100)(2)^2 + 2^3$
2. $= 1000000 + 60000 + 1200 + 8$

Ans. 1061208.

Q22. Find the value of $(1.02)^4$ correct to 4 decimal places.

Solution:

1. $(1 + 0.02)^4 = 1 + 4(0.02) + 6(0.02)^2 + 4(0.02)^3 + \dots$
2. $= 1 + 0.08 + 0.0024 + 0.00032 + \dots = 1.082432\dots$

Ans. 1.0824.

Q23. Evaluate $(\sqrt{3} + 1)^4 + (\sqrt{3} - 1)^4$.

Solution:

1. $(a + b)^4 + (a - b)^4 = 2[a^4 + 6a^2b^2 + b^4]$, with $a = \sqrt{3}$, $b = 1$.

2. $= 2[9 + 6(3) + 1] = 2 \times 28$

Ans. 56.

Q24. Evaluate $(2 + \sqrt{3})^3 + (2 - \sqrt{3})^3$.

Solution:

1. $(a + b)^3 + (a - b)^3 = 2[a^3 + 3ab^2]$, with $a = 2$, $b = \sqrt{3}$.

2. $= 2[8 + 3(2)(3)] = 2 \times 26$

Ans. 52.

Part 5: Negative and Fractional Index

Q25. Expand $(1 - x)^{-1}$ up to the term in x^4 .

Solution: Using $(1 + y)^{-1} = 1 - y + y^2 - y^3 + y^4 - \dots$ with $y = -x$, all signs become positive.

Ans. $1 + x + x^2 + x^3 + x^4 + \dots$

Q26. Expand $(1 + x)^{1/2}$ up to x^2 .

Solution: $n = 1/2$. Coefficient of $x = 1/2$. Coefficient of $x^2 = (1/2)(-1/2)/2! = -1/8$.

Ans. $1 + x/2 - x^2/8 + \dots$

Q27. Write the first three terms of $(1 - x)^{-3}$.

Solution: $1 + (-3)(-x) + [(-3)(-4)/2!](x)^2 = 1 + 3x + 6x^2$

Ans. $1 + 3x + 6x^2 + \dots$

Q28. Write the first three terms of $(1 + 2x)^{-1}$.

Solution: Use $(1 + y)^{-1} = 1 - y + y^2 - \dots$ with $y = 2x$.

Ans. $1 - 2x + 4x^2 - \dots$

Q29. For what values of x is the expansion of $(2 - 3x)^{-1}$ valid?

Solution: $(2 - 3x)^{-1} = (1/2)(1 - 3x/2)^{-1}$. The series is valid when $|3x/2| < 1$.

Ans. $|x| < 2/3$.

Q30. Write the number of terms in the expansion of $(1 + x)^n$ when n is a negative integer or a fraction. State the condition for validity.

Solution: For negative or fractional n , the factors n , $(n - 1)$, $(n - 2)$, ... never become zero, so the terms never stop.

Ans. The expansion has infinitely many terms and is valid only for $|x| < 1$.

Q31. Using $(1 + x)^n \approx 1 + nx$, find the approximate value of $(1.002)^{10}$.

Solution: $(1 + 0.002)^{10} \approx 1 + 10(0.002) = 1 + 0.02$

Ans. ≈ 1.02 .

Q32. Using $(1 + x)^n \approx 1 + nx$, find the approximate value of $(0.98)^{1/2}$.

Solution: $(1 - 0.02)^{1/2} \approx 1 + (1/2)(-0.02) = 1 - 0.01$

Ans. ≈ 0.99 .

Poly Notes Hub