

MATHEMATICS 1

Chapter: Binomial Theorem

Short Answer Questions (2 Marks) with Solutions: Set 2

Part 1: Expansions

Q1. Expand $(a - 2b)^3$.

Solution:

1. Coefficients 1, 3, 3, 1 with second term $-2b$.
2. $a^3 + 3a^2(-2b) + 3a(4b^2) + (-8b^3)$

Ans. $a^3 - 6a^2b + 12ab^2 - 8b^3$.

Q2. Expand $(2x + 1/x)^4$.

Solution:

1. $(2x)^4 + 4(2x)^3(1/x) + 6(2x)^2(1/x)^2 + 4(2x)(1/x)^3 + (1/x)^4$
2. $= 16x^4 + 32x^2 + 24 + 8/x^2 + 1/x^4$

Ans. $16x^4 + 32x^2 + 24 + 8/x^2 + 1/x^4$.

Q3. Expand $(1 + x/2)^5$.

Solution:

1. Coefficients 1, 5, 10, 10, 5, 1 with second term $x/2$.
2. $1 + 5(x/2) + 10(x^2/4) + 10(x^3/8) + 5(x^4/16) + x^5/32$

Ans. $1 + 5x/2 + 5x^2/2 + 5x^3/4 + 5x^4/16 + x^5/32$.

Q4. Simplify $(x + \sqrt{x^2 - 1})^3 + (x - \sqrt{x^2 - 1})^3$.

Solution:

1. $(a + b)^3 + (a - b)^3 = 2[a^3 + 3ab^2]$, with $a = x$ and $b^2 = x^2 - 1$.
2. $= 2[x^3 + 3x(x^2 - 1)] = 2[4x^3 - 3x]$

Ans. $8x^3 - 6x$.

Part 2: Terms and Coefficients

Q5. Find the 3rd term of $(2x - 3)^6$.

Solution: $T_3 = T_{2+1} = {}^6C_2 (2x)^4 (-3)^2 = 15 \times 16x^4 \times 9$

Ans. $T_3 = 2160 x^4$.

Q6. Find the 7th term of $(x - y)^8$.

Solution: $T_7 = T_{6+1} = {}^8C_6 x^2 (-y)^6 = 28 x^2 y^6$

Ans. $T_7 = 28 x^2 y^6$.

Q7. Find the middle term of $(2/x + x/2)^6$.

Solution:

1. $n = 6$ is even, so the middle term is T_4 .
2. $T_4 = {}^6C_3 (2/x)^3 (x/2)^3 = 20 \times (8/x^3) \times (x^3/8) = 20$

Ans. Middle term = 20.

Q8. Find the middle terms of $(1 - x)^7$.

Solution:

1. $n = 7$ is odd, so the middle terms are T_4 and T_5 .
2. $T_4 = {}^7C_3 (-x)^3 = -35x^3$; $T_5 = {}^7C_4 (-x)^4 = 35x^4$

Ans. $-35x^3$ and $35x^4$.

Q9. Find the 2nd term from the end in the expansion of $(a + b)^{12}$.

Solution:

1. The r -th term from the end is the $(n - r + 2)$ -th term from the beginning: $(12 - 2 + 2) = 12$ th term.
2. $T_{12} = {}^{12}C_{11} a b^{11} = 12ab^{11}$

Ans. $12ab^{11}$.

Q10. The coefficients of the 2nd and 3rd terms in the expansion of $(1 + x)^n$ are 8 and 28 respectively. Find n .

Solution:

1. Coefficient of 2nd term = ${}^nC_1 = n = 8$.
2. Check with the 3rd term: ${}^8C_2 = 28$ ✓

Ans. $n = 8$.

Q11. Find the coefficient of x^4 in the expansion of $(1 + 2x)^6$.

Solution: Coefficient of $x^4 = {}^6C_4 \times 2^4 = 15 \times 16$

Ans. Coefficient = 240.

Q12. Find the coefficient of x^6 in the expansion of $(x^2 - 1/x)^6$.

Solution:

1. $T_{r+1} = {}^6C_r (x^2)^{6-r} (-1/x)^r = {}^6C_r (-1)^r x^{12-3r}$.
2. Put $12 - 3r = 6$, so $r = 2$. Coefficient = ${}^6C_2 (-1)^2$

Ans. Coefficient = 15.

Q13. Find the term independent of x in $(2x - 1/x)^4$.

Solution:

1. $T_{r+1} = {}^4C_r (2x)^{4-r} (-1/x)^r = {}^4C_r 2^{4-r} (-1)^r x^{4-2r}$.
2. Put $4 - 2r = 0$, so $r = 2$. Term = ${}^4C_2 \times 2^2 \times 1 = 6 \times 4$

Ans. Term independent of $x = 24$.

Part 3: Binomial Coefficients and Properties

Q14. Evaluate $^{12}C_4$.

Solution: $^{12}C_4 = (12 \times 11 \times 10 \times 9)/(4 \times 3 \times 2 \times 1) = 11880/24$

Ans. 495.

Q15. If ${}^nC_3 = {}^nC_5$, find n and hence find nC_2 .

Solution:

1. Since $3 \neq 5$, $n = 3 + 5 = 8$.
2. ${}^8C_2 = (8 \times 7)/2 = 28$

Ans. $n = 8$ and ${}^nC_2 = 28$.

Q16. If ${}^nC_4 : {}^nC_2 = 5 : 2$, find n .

Solution:

1. ${}^nC_4/{}^nC_2 = [n(n-1)(n-2)(n-3)/24] \times [2/(n(n-1))] = (n-2)(n-3)/12$.
2. Given $(n-2)(n-3)/12 = 5/2$, so $(n-2)(n-3) = 30$, so $n^2 - 5n - 24 = 0$.
3. $(n-8)(n+3) = 0$. Since n is positive, $n = 8$.

Ans. $n = 8$.

Q17. Prove that $r \cdot {}^nC_r = n \cdot {}^{n-1}C_{r-1}$.

Solution:

1. LHS $= r \times n!/[r!(n-r)!] = n!/[r!(n-r)!]$
2. $= n \times (n-1)!/[r!(n-1-(r-1))!] = n \cdot {}^{n-1}C_{r-1}$

Ans. Hence proved.

Q18. Show that ${}^nC_r / {}^nC_{r-1} = (n-r+1)/r$.

Solution:

1. ${}^nC_r/{}^nC_{r-1} = \{n!/[r!(n-r)!]\} \times \{(r-1)!(n-r+1)!/n!\}$
2. Since $r! = r(r-1)!$ and $(n-r+1)! = (n-r+1)(n-r)!$, the factors cancel to leave $(n-r+1)/r$.

Ans. Hence proved.

Q19. Prove that ${}^nC_r = {}^nC_{n-r}$.

Solution: ${}^nC_{n-r} = n!/[r!(n-r)!(n-(n-r))!] = n!/[r!(n-r)!] = {}^nC_r$

Ans. Hence proved.

Q20. Prove that $C_0 + C_1 + C_2 + \dots + C_n = 2^n$.

Solution:

1. $(1+x)^n = C_0 + C_1x + C_2x^2 + \dots + C_nx^n$.
2. Put $x = 1$: $(1+1)^n = C_0 + C_1 + C_2 + \dots + C_n$, so the sum $= 2^n$.

Ans. Hence proved.

Q21. Find the value of $C_1 + C_3 + C_5 + C_7$ for $n = 8$.

Solution: The sum of the odd-position coefficients is $2^{n-1} = 2^7$

Ans. 128.

Q22. Find the sum of the coefficients in the expansion of $(3x + 2y)^3$.

Solution: Put $x = 1$ and $y = 1$: $(3 + 2)^3 = 5^3$

Ans. 125.

Part 4: Numerical Problems

Q23. Evaluate $(98)^3$ using the binomial theorem.

Solution:

1. $(100 - 2)^3 = 100^3 - 3(100)^2(2) + 3(100)(2)^2 - 2^3$
2. $= 1000000 - 60000 + 1200 - 8$

Ans. 941192.

Q24. Evaluate $(101)^4$ using the binomial theorem.

Solution:

1. $(100 + 1)^4 = 100^4 + 4(100)^3 + 6(100)^2 + 4(100) + 1$
2. $= 100000000 + 4000000 + 60000 + 400 + 1$

Ans. 104060401.

Q25. Find the value of $(1.03)^3$ correct to 4 decimal places.

Solution:

1. $(1 + 0.03)^3 = 1 + 3(0.03) + 3(0.03)^2 + (0.03)^3$
2. $= 1 + 0.09 + 0.0027 + 0.000027 = 1.092727$

Ans. 1.0927.

Q26. Find the exact value of $(1.1)^5$.

Solution:

1. $(1 + 0.1)^5 = 1 + 5(0.1) + 10(0.01) + 10(0.001) + 5(0.0001) + 0.00001$
2. $= 1 + 0.5 + 0.1 + 0.01 + 0.0005 + 0.00001$

Ans. 1.61051.

Q27. Evaluate $(\sqrt{2} + 1)^3 - (\sqrt{2} - 1)^3$.

Solution:

1. $(a + b)^3 - (a - b)^3 = 2[3a^2b + b^3]$, with $a = \sqrt{2}$, $b = 1$.
2. $= 2[3(2)(1) + 1] = 2 \times 7$

Ans. 14.

Part 5: Negative and Fractional Index

Q28. Write the first three terms of $(1 + x)^{-3}$.

Solution: $1 + (-3)x + [(-3)(-4)/2!]x^2 = 1 - 3x + 6x^2$

Ans. $1 - 3x + 6x^2 - \dots$

Q29. Expand $(1 - 2x)^{1/2}$ up to x^2 .

Solution:

1. Use $(1 + y)^{1/2} = 1 + y/2 - y^2/8 + \dots$ with $y = -2x$.

2. $= 1 + (-2x)/2 - (4x^2)/8$

Ans. $1 - x - x^2/2 - \dots$

Q30. Write the first three terms of $(1 - x)^{-1/2}$.

Solution: Coefficient of x : $(-1/2)(-1) = 1/2$. Coefficient of x^2 : $[(-1/2)(-3/2)/2!] \times (-1)^2 = 3/8$.

Ans. $1 + x/2 + 3x^2/8 + \dots$

Q31. Write the general term in the expansion of $(1 - x)^{-2}$.

Solution: $(1 - x)^{-2} = 1 + 2x + 3x^2 + 4x^3 + \dots$ The coefficient of x^r is one more than r .

Ans. $T_{r+1} = (r + 1) x^r$.

Q32. Find the coefficient of x^3 in the expansion of $(1 - x)^{-4}$.

Solution:

1. Coefficient of x^r in $(1 - x)^{-n}$ is ${}^{n+r-1}C_r$.

2. For $n = 4$, $r = 3$: 6C_3

Ans. Coefficient = 20.

Q33. For what values of x is the expansion of $(5 - 2x)^{-1}$ valid?

Solution: $(5 - 2x)^{-1} = (1/5)(1 - 2x/5)^{-1}$. The series is valid when $|2x/5| < 1$.

Ans. $|x| < 5/2$.

Q34. Expand $(1 + x)/(1 - x)$ up to x^3 .

Solution: $(1 + x)(1 + x + x^2 + x^3 + \dots) = (1 + x + x^2 + x^3 + \dots) + (x + x^2 + x^3 + \dots)$

Ans. $1 + 2x + 2x^2 + 2x^3 + \dots$

Q35. Using $(1 + x)^n \approx 1 + nx$, find the approximate value of $(1.03)^{1/3}$.

Solution: $(1 + 0.03)^{1/3} \approx 1 + (1/3)(0.03) = 1 + 0.01$

Ans. ≈ 1.01 .

Q36. Using $(1 + x)^n \approx 1 + nx$, find the approximate value of $(1.05)^{-2}$.

Solution: $(1 + 0.05)^{-2} \approx 1 + (-2)(0.05) = 1 - 0.10$

Ans. ≈ 0.90 .