

MATHEMATICS 1

Chapter: Complex Numbers

Short Answer Questions with Solutions | All Types of Short Questions

This document contains the most important short answer (2 marks) questions of the chapter Complex Numbers, arranged type-wise, with brief solution steps and the final answer. Keep the quick formulae below in mind while solving. Each question can be answered in 3 to 5 lines in the exam.

Quick formulae:

- $i^2 = -1$, $i^3 = -i$, $i^4 = 1$; for i^n , use the remainder of $n \div 4$.
- For $z = a + ib$: $\bar{z} = a - ib$, $|z| = \sqrt{a^2 + b^2}$, $z\bar{z} = |z|^2$.
- Polar form: $z = r(\cos\theta + i \sin\theta)$; Euler form: $z = re^{i\theta}$. Find the quadrant before writing θ .
- De Moivre: $(\cos\theta + i \sin\theta)^n = \cos n\theta + i \sin n\theta$.
- n th roots: $r^{1/n}[\cos((\theta + 2k\pi)/n) + i \sin((\theta + 2k\pi)/n)]$, $k = 0, 1, \dots, n - 1$.
- Cube roots of unity: $1, \omega, \omega^2$; $\omega^3 = 1$; $1 + \omega + \omega^2 = 0$.

Type 1: Basics, Definitions and Powers of i

Q1. Define a complex number. Write the real and imaginary parts of $3 - 5i$. [2 Marks]

Solution:

1. A complex number is a number of the form $z = a + ib$, where a and b are real numbers and $i = \sqrt{-1}$.
2. For $z = 3 - 5i$, comparing with $a + ib$ gives $a = 3$ and $b = -5$.

Answer: $\text{Re}(z) = 3$, $\text{Im}(z) = -5$

Q2. Evaluate $i^{49} + i^{50} + i^{51} + i^{52}$. [2 Marks]

Solution:

1. Powers of i repeat after every 4 terms: $i^{49} = i$, $i^{50} = -1$, $i^{51} = -i$, $i^{52} = 1$.
2. Sum $= i - 1 - i + 1 = 0$.

Answer: 0

Q3. Find the values of i^{-9} and i^{-22} . [2 Marks]

Solution:

1. $i^{-9} = 1/i^9$. Since $9 = 4 \times 2 + 1$, $i^9 = i$, so $1/i = -i$.
2. $i^{-22} = 1/i^{22}$. Since $22 = 4 \times 5 + 2$, $i^{22} = i^2 = -1$, so $1/(-1) = -1$.

Answer: $i^{-9} = -i$, $i^{-22} = -1$

Q4. Find the value of $i + i^2 + i^3 + \dots + i^{100}$. [2 Marks]

Solution:

1. $i + i^2 + i^3 + i^4 = i - 1 - i + 1 = 0$, so every group of four consecutive terms adds to zero.
2. 100 terms form 25 such groups, so the total is $25 \times 0 = 0$.

Answer: 0

Q5. State the condition for equality of two complex numbers. Hence find x and y if $(2x - 1) + i(y + 3) = 5 + 2i$. [2 Marks]

Solution:

1. $a + ib = c + id$ if and only if $a = c$ and $b = d$ (real parts are equal and imaginary parts are equal).
2. Real parts: $2x - 1 = 5$, so $x = 3$. Imaginary parts: $y + 3 = 2$, so $y = -1$.

Answer: $x = 3$, $y = -1$

Q6. Write the conjugate of (i) $3 + 4i$ (ii) $-2 - 5i$ (iii) $7i$ (iv) 6 . [2 Marks]

Solution:

1. The conjugate of $a + ib$ is $a - ib$ (change the sign of the imaginary part).
2. (i) $3 - 4i$ (ii) $-2 + 5i$ (iii) $7i = 0 + 7i$, so conjugate $= -7i$ (iv) $6 = 6 + 0i$, so conjugate $= 6$.

Answer: (i) $3 - 4i$ (ii) $-2 + 5i$ (iii) $-7i$ (iv) 6

Type 2: Algebra of Complex Numbers

Q7. Find the sum and the product of $2 + 3i$ and $4 - 5i$. [2 Marks]

Solution:

1. Sum $= (2 + 4) + i(3 - 5) = 6 - 2i$.
2. Product $= (2 + 3i)(4 - 5i) = 8 - 10i + 12i - 15i^2 = 8 + 2i + 15 = 23 + 2i$.

Answer: Sum = $6 - 2i$; Product = $23 + 2i$

Q8. Simplify $(1 + i)^2 + (1 - i)^2$ and find $(1 + i)^4$. [2 Marks]

Solution:

1. $(1 + i)^2 = 1 + 2i + i^2 = 2i$ and $(1 - i)^2 = 1 - 2i + i^2 = -2i$.
2. Sum $= 2i + (-2i) = 0$. $(1 + i)^4 = [(1 + i)^2]^2 = (2i)^2 = 4i^2 = -4$.

Answer: 0 and -4

Q9. Express $1/(2 + 3i)$ in the form $a + ib$. [2 Marks]

Solution:

1. Multiply numerator and denominator by the conjugate $2 - 3i$.
2. $1/(2 + 3i) = (2 - 3i)/[(2 + 3i)(2 - 3i)] = (2 - 3i)/(4 + 9) = (2 - 3i)/13$.

Answer: $2/13 - (3/13)i$

Q10. Express $(3 + i)/(3 - i)$ in the form $a + ib$. [2 Marks]

Solution:

1. Multiply numerator and denominator by $(3 + i)$: $(3 + i)^2/[(3 - i)(3 + i)]$.
2. Numerator = $9 + 6i + i^2 = 8 + 6i$. Denominator = $9 + 1 = 10$.

Answer: $4/5 + (3/5)i$

Q11. If $z = 3 + 4i$, find $z + \bar{z}$, $z - \bar{z}$ and $z\bar{z}$. [2 Marks]

Solution:

1. Here $\bar{z} = 3 - 4i$.
2. $z + \bar{z} = 6$, $z - \bar{z} = 8i$, $z\bar{z} = 9 + 16 = 25$.

Answer: 6, 8i, 25

Q12. Show that $(1 + i)/(1 - i)$ is purely imaginary. [2 Marks]

Solution:

1. $(1 + i)/(1 - i) = (1 + i)^2/[(1 - i)(1 + i)] = 2i/2 = i$.
2. Its real part is 0 and its imaginary part is 1, so it is purely imaginary.

Answer: The number equals i , which is purely imaginary.

Q13. Find the real numbers x and y if $(x + iy)(1 + i) = 2 - i$. [2 Marks]

Solution:

1. $x + iy = (2 - i)/(1 + i) = (2 - i)(1 - i)/[(1 + i)(1 - i)] = (2 - 2i - i + i^2)/2 = (1 - 3i)/2$.
2. Compare real and imaginary parts.

Answer: $x = 1/2$, $y = -3/2$

Q14. Find the real and imaginary parts of $(2 + 3i)^2$. [2 Marks]

Solution:

1. $(2 + 3i)^2 = 4 + 12i + 9i^2 = 4 + 12i - 9$.
2. $= -5 + 12i$.

Answer: $\text{Re} = -5$, $\text{Im} = 12$

Q15. Solve the equation $x^2 - 2x + 5 = 0$. [2 Marks]

Solution:

1. $x = [2 \pm \sqrt{(4 - 20)}]/2 = [2 \pm \sqrt{-16}]/2$.
2. $\sqrt{-16} = 4i$, so $x = (2 \pm 4i)/2 = 1 \pm 2i$.

Answer: $x = 1 + 2i$ or $x = 1 - 2i$

Q16. Find the additive inverse and the multiplicative inverse of $z = 1 - i$. [2 Marks]

Solution:

1. Additive inverse = $-z = -1 + i$ (since $z + (-z) = 0$).

2. Multiplicative inverse $= 1/z = 1/(1 - i) = (1 + i)/[(1 - i)(1 + i)] = (1 + i)/2$.

Answer: Additive: $-1 + i$; Multiplicative: $1/2 + (1/2)i$

Q17. If $z_1 = 4 + 3i$ and $z_2 = 1 + 2i$, find z_1/z_2 . [2 Marks]

Solution:

1. Multiply numerator and denominator by the conjugate of z_2 , i.e. $1 - 2i$.
2. Numerator: $(4 + 3i)(1 - 2i) = 4 - 8i + 3i - 6i^2 = 10 - 5i$. Denominator: $1 + 4 = 5$.

Answer: $z_1/z_2 = 2 - i$

Type 3: Modulus and Argument

Q18. Find the modulus of (i) $5 + 12i$ (ii) $-\sqrt{3} + i$ (iii) -7 (iv) $4i$. [2 Marks]

Solution:

1. Use $|a + ib| = \sqrt{a^2 + b^2}$.
2. (i) $\sqrt{25 + 144} = 13$ (ii) $\sqrt{3 + 1} = 2$ (iii) $|-7| = 7$ (iv) $|0 + 4i| = 4$.

Answer: (i) 13 (ii) 2 (iii) 7 (iv) 4

Q19. Find the principal argument of (i) $-1 - i$ (ii) $\sqrt{3} - i$ (iii) -2 . [2 Marks]

Solution:

1. (i) $\alpha = \tan^{-1}(1) = \pi/4$; the point $(-1, -1)$ is in the third quadrant, so $\theta = -(\pi - \pi/4) = -3\pi/4$.
2. (ii) $\alpha = \tan^{-1}(1/\sqrt{3}) = \pi/6$; the point $(\sqrt{3}, -1)$ is in the fourth quadrant, so $\theta = -\pi/6$.
3. (iii) -2 lies on the negative real axis, so $\theta = \pi$.

Answer: (i) $-3\pi/4$ (ii) $-\pi/6$ (iii) π

Q20. Find $|(3 + 4i)/(5 - 12i)|$. [2 Marks]

Solution:

1. Use $|z_1/z_2| = |z_1|/|z_2|$.
2. $|3 + 4i| = 5$ and $|5 - 12i| = 13$.

Answer: $5/13$

Q21. Find the modulus of $(1 + i)(2 - i)$. [2 Marks]

Solution:

1. Use $|z_1 z_2| = |z_1| |z_2|$.
2. $|1 + i| = \sqrt{2}$ and $|2 - i| = \sqrt{5}$, so the modulus $= \sqrt{2} \times \sqrt{5}$.

Answer: $\sqrt{10}$

Q22. If $z = 3 + 4i$, find $|\bar{z}|$ and $|-z|$. [2 Marks]

Solution:

1. $\bar{z} = 3 - 4i$, so $|\bar{z}| = \sqrt{9 + 16} = 5$.

2. $-z = -3 - 4i$, so $|-z| = \sqrt{9 + 16} = 5$.

Answer: $|\bar{z}| = 5$, $|-z| = 5$ (in general $|\bar{z}| = |-z| = |z|$)

Q23. If $|z| = 1$, prove that $\bar{z} = 1/z$. [2 Marks]

Solution:

1. We know $z\bar{z} = |z|^2$.
2. Since $|z| = 1$, $z\bar{z} = 1$, and dividing by z gives $\bar{z} = 1/z$.

Answer: Hence proved.

Q24. Write the principal argument of 5, -5, 5i and -5i. [2 Marks]

Solution:

1. 5 lies on the positive real axis and -5 on the negative real axis.
2. 5i lies on the positive imaginary axis and -5i on the negative imaginary axis.

Answer: 0, π , $\pi/2$, $-\pi/2$ respectively

Q25. Find the modulus and argument of i^7 . [2 Marks]

Solution:

1. $i^7 = i^4 \cdot i^3 = i^3 = -i = 0 - i$.
2. Modulus = 1. The point (0, -1) is on the negative imaginary axis, so the argument is $-\pi/2$.

Answer: Modulus = 1, argument = $-\pi/2$

Type 4: Polar and Exponential Form

Q26. Express $\sqrt{3} + i$ in polar form. [2 Marks]

Solution:

1. $r = \sqrt{3 + 1} = 2$. $\theta = \tan^{-1}(1/\sqrt{3}) = \pi/6$ (first quadrant).
2. Use $z = r(\cos\theta + i \sin\theta)$.

Answer: $2(\cos \pi/6 + i \sin \pi/6)$

Q27. Express -2 and 3i in polar form. [2 Marks]

Solution:

1. -2: modulus 2 and it lies on the negative real axis, so $\theta = \pi$.
2. 3i: modulus 3 and it lies on the positive imaginary axis, so $\theta = \pi/2$.

Answer: $-2 = 2(\cos\pi + i \sin\pi)$; $3i = 3(\cos \pi/2 + i \sin \pi/2)$

Q28. Express $-\sqrt{3} + i$ in polar form. [2 Marks]

Solution:

1. $r = \sqrt{3 + 1} = 2$. $\alpha = \tan^{-1}(1/\sqrt{3}) = \pi/6$.

2. The point $(-\sqrt{3}, 1)$ is in the second quadrant, so $\theta = \pi - \pi/6 = 5\pi/6$.

Answer: $2(\cos 5\pi/6 + i \sin 5\pi/6)$

Q29. Express $4(\cos \pi/3 + i \sin \pi/3)$ in the form $a + ib$. [2 Marks]

Solution:

1. $\cos \pi/3 = 1/2$ and $\sin \pi/3 = \sqrt{3}/2$.
2. $4(1/2 + i\sqrt{3}/2) = 2 + 2\sqrt{3}i$.

Answer: $2 + 2\sqrt{3}i$

Q30. Express $-1 + i$ in exponential form. [2 Marks]

Solution:

1. $r = \sqrt{2}$. $\alpha = \tan^{-1}(1) = \pi/4$; the point $(-1, 1)$ is in the second quadrant, so $\theta = 3\pi/4$.
2. Use $z = re^{i\theta}$.

Answer: $\sqrt{2} e^{i3\pi/4}$

Q31. Find the values of $e^{i\pi/2}$, $e^{-i\pi}$ and $e^{2\pi i}$. [2 Marks]

Solution:

1. Use $e^{i\theta} = \cos\theta + i \sin\theta$.
2. $e^{i\pi/2} = \cos\pi/2 + i \sin\pi/2 = i$. $e^{-i\pi} = \cos\pi - i \sin\pi = -1$. $e^{2\pi i} = \cos 2\pi + i \sin 2\pi = 1$.

Answer: $i, -1, 1$

Q32. State Euler's formula and deduce $\cos\theta = (e^{i\theta} + e^{-i\theta})/2$. [2 Marks]

Solution:

1. Euler's formula: $e^{i\theta} = \cos\theta + i \sin\theta$. Replacing θ by $-\theta$: $e^{-i\theta} = \cos\theta - i \sin\theta$.
2. Add the two: $e^{i\theta} + e^{-i\theta} = 2\cos\theta$. (Subtracting gives $\sin\theta = (e^{i\theta} - e^{-i\theta})/2i$.)

Answer: $\cos\theta = (e^{i\theta} + e^{-i\theta})/2$

Q33. Find the product of $2(\cos \pi/6 + i \sin \pi/6)$ and $3(\cos \pi/3 + i \sin \pi/3)$. [2 Marks]

Solution:

1. Multiply the moduli and add the arguments: modulus $= 2 \times 3 = 6$, argument $= \pi/6 + \pi/3 = \pi/2$.
2. Product $= 6(\cos \pi/2 + i \sin \pi/2) = 6(0 + i)$.

Answer: $6i$

Q34. Find the quotient $6(\cos 5\pi/6 + i \sin 5\pi/6) \div 2(\cos \pi/3 + i \sin \pi/3)$. [2 Marks]

Solution:

1. Divide the moduli and subtract the arguments: modulus $= 6/2 = 3$, argument $= 5\pi/6 - \pi/3 = \pi/2$.
2. Quotient $= 3(\cos \pi/2 + i \sin \pi/2) = 3(0 + i)$.

Answer: $3i$

Type 5: De Moivre's Theorem

Q35. State De Moivre's theorem. [2 Marks]

Solution:

1. For any integer n and any real θ : $(\cos\theta + i \sin\theta)^n = \cos n\theta + i \sin n\theta$.
2. For a fraction $n = p/q$, the right side gives one of the values of the left side.

Answer: $(\cos\theta + i \sin\theta)^n = \cos n\theta + i \sin n\theta$

Q36. Simplify $(\cos 2\theta + i \sin 2\theta)^4 / (\cos 3\theta + i \sin 3\theta)^2$. [2 Marks]

Solution:

1. $(\cos 2\theta + i \sin 2\theta)^4 = \cos 8\theta + i \sin 8\theta$ and $(\cos 3\theta + i \sin 3\theta)^2 = \cos 6\theta + i \sin 6\theta$.
2. Dividing: $\cos(8\theta - 6\theta) + i \sin(8\theta - 6\theta)$.

Answer: $\cos 2\theta + i \sin 2\theta$

Q37. Simplify $(\cos\theta - i \sin\theta)^{-4}$. [2 Marks]

Solution:

1. $\cos\theta - i \sin\theta = (\cos\theta + i \sin\theta)^{-1}$.
2. So $(\cos\theta - i \sin\theta)^{-4} = (\cos\theta + i \sin\theta)^4 = \cos 4\theta + i \sin 4\theta$.

Answer: $\cos 4\theta + i \sin 4\theta$

Q38. Evaluate $(\cos \pi/6 + i \sin \pi/6)^{12}$. [2 Marks]

Solution:

1. By De Moivre: $= \cos(12\pi/6) + i \sin(12\pi/6)$.
2. $= \cos 2\pi + i \sin 2\pi = 1 + 0$.

Answer: 1

Q39. Evaluate $(1 + i)^8$. [2 Marks]

Solution:

1. $(1 + i)^2 = 2i$.
2. $(1 + i)^8 = (2i)^4 = 16i^4 = 16$.

Answer: 16

Note: By polar form: $(\sqrt{2})^8(\cos 2\pi + i \sin 2\pi) = 16$, the same answer.

Q40. If $z = \cos\theta + i \sin\theta$, find $z + 1/z$ and $z - 1/z$. [2 Marks]

Solution:

1. $1/z = \cos\theta - i \sin\theta$.
2. $z + 1/z = 2\cos\theta$ and $z - 1/z = 2i \sin\theta$.

Answer: $2\cos\theta$ and $2i \sin\theta$

Q41. Using De Moivre's theorem for $n = 2$, deduce the formulae for $\cos 2\theta$ and $\sin 2\theta$. [2 Marks]

Solution:

1. $\cos 2\theta + i \sin 2\theta = (\cos \theta + i \sin \theta)^2 = \cos^2 \theta - \sin^2 \theta + 2i \sin \theta \cos \theta$.
2. Equate real and imaginary parts.

Answer: $\cos 2\theta = \cos^2 \theta - \sin^2 \theta$; $\sin 2\theta = 2 \sin \theta \cos \theta$

Q42. Evaluate $(\cos 2\pi/5 + i \sin 2\pi/5)^5$. [2 Marks]

Solution:

1. By De Moivre: $\cos(5 \times 2\pi/5) + i \sin(5 \times 2\pi/5)$.
2. $= \cos 2\pi + i \sin 2\pi = 1$.

Answer: 1

Type 6: Roots of Complex Numbers

Q43. Find the square roots of $5 + 12i$. [2 Marks]

Solution:

1. Let $\sqrt{5 + 12i} = p + iq$. Then $p^2 - q^2 = 5$, $2pq = 12$ (so $pq = 6$), and $p^2 + q^2 = |5 + 12i| = 13$.
2. Add and subtract: $p^2 = 9$, $q^2 = 4$, so $p = \pm 3$ and $q = \pm 2$. Since $pq = 6$ is positive, p and q have the same sign.

Answer: $\pm(3 + 2i)$, i.e. $3 + 2i$ and $-3 - 2i$

Q44. Find the fourth roots of unity. [2 Marks]

Solution:

1. $x^4 = 1 = \cos 2k\pi + i \sin 2k\pi$, so $x = \cos(k\pi/2) + i \sin(k\pi/2)$, $k = 0, 1, 2, 3$.
2. $k = 0: 1$; $k = 1: i$; $k = 2: -1$; $k = 3: -i$.

Answer: 1, i, -1, -i

Q45. Find the cube roots of -1 . [2 Marks]

Solution:

1. $x^3 = -1 = \cos(\pi + 2k\pi) + i \sin(\pi + 2k\pi)$, so $x = \cos((2k + 1)\pi/3) + i \sin((2k + 1)\pi/3)$, $k = 0, 1, 2$.
2. $k = 0: \cos \pi/3 + i \sin \pi/3 = (1 + i\sqrt{3})/2$; $k = 1: \cos \pi + i \sin \pi = -1$; $k = 2: \cos 5\pi/3 + i \sin 5\pi/3 = (1 - i\sqrt{3})/2$.

Answer: $(1 + i\sqrt{3})/2$, -1 , $(1 - i\sqrt{3})/2$

Q46. Write the formula for the n th roots of $z = r(\cos \theta + i \sin \theta)$. How many distinct roots are there? [2 Marks]

Solution:

1. $z^{1/n} = r^{1/n} [\cos((\theta + 2k\pi)/n) + i \sin((\theta + 2k\pi)/n)]$, $k = 0, 1, 2, \dots, n - 1$.

2. Values of k beyond $n - 1$ only repeat the same roots.

Answer: There are exactly n distinct n th roots, equally spaced on a circle of radius $r^{1/n}$.

Q47. How many distinct values does $(1 + i)^{1/5}$ have? Write their modulus and arguments. [2 Marks]

Solution:

1. $1 + i = \sqrt{2} [\cos(\pi/4 + 2k\pi) + i \sin(\pi/4 + 2k\pi)]$.
2. The 5th roots have modulus $(\sqrt{2})^{1/5} = 2^{1/10}$ and arguments $(\pi/4 + 2k\pi)/5 = \pi/20 + 2k\pi/5$, $k = 0, 1, 2, 3, 4$.

Answer: 5 values, each of modulus $2^{1/10}$, with arguments $\pi/20 + 2k\pi/5$ ($k = 0$ to 4)

Type 7: Cube Roots of Unity

Q48. Find the cube roots of unity and show that $\omega^3 = 1$. [2 Marks]

Solution:

1. $x^3 = 1$ gives $(x - 1)(x^2 + x + 1) = 0$, so $x = 1$ or $x = (-1 \pm i\sqrt{3})/2$. Let $\omega = (-1 + i\sqrt{3})/2$, so $\omega^2 = (-1 - i\sqrt{3})/2$.
2. $\omega^3 = \omega \cdot \omega^2 = [(-1 + i\sqrt{3})(-1 - i\sqrt{3})]/4 = (1 + 3)/4 = 1$.

Answer: Roots: 1, $(-1 + i\sqrt{3})/2$, $(-1 - i\sqrt{3})/2$; $\omega^3 = 1$

Q49. If ω is a cube root of unity, find $\omega^{28} + \omega^{29} + 1$. [2 Marks]

Solution:

1. $28 = 3 \times 9 + 1$, so $\omega^{28} = \omega$. $29 = 3 \times 9 + 2$, so $\omega^{29} = \omega^2$.
2. Sum $= \omega + \omega^2 + 1 = 0$.

Answer: 0

Q50. If ω is a cube root of unity, find $(1 + \omega)^3$. [2 Marks]

Solution:

1. $1 + \omega + \omega^2 = 0$, so $1 + \omega = -\omega^2$.
2. $(1 + \omega)^3 = (-\omega^2)^3 = -\omega^6 = -(\omega^3)^2 = -1$.

Answer: -1

Q51. If ω is a cube root of unity, find $1/\omega + 1/\omega^2$. [2 Marks]

Solution:

1. Since $\omega^3 = 1$: $1/\omega = \omega^2$ and $1/\omega^2 = \omega$.
2. Sum $= \omega^2 + \omega = -1$.

Answer: -1

Q52. If ω is a cube root of unity, find $(1 + \omega - \omega^2)^2$. [2 Marks]

Solution:

1. $1 + \omega = -\omega^2$, so $1 + \omega - \omega^2 = -\omega^2 - \omega^2 = -2\omega^2$.
2. Square: $(-2\omega^2)^2 = 4\omega^4 = 4\omega$.

Answer: 4ω

Q53. If $\omega = (-1 + i\sqrt{3})/2$, show that the conjugate of ω is ω^2 . [2 Marks]

Solution:

1. Conjugate of $\omega = (-1 - i\sqrt{3})/2$ (change the sign of the imaginary part).
2. This is exactly the value of ω^2 .

Answer: Conjugate of $\omega = \omega^2 = 1/\omega$

Q54. If ω is a cube root of unity, find $1 + \omega^2 + \omega^4$. [2 Marks]

Solution:

1. $\omega^4 = \omega^3 \cdot \omega = \omega$.
2. So $1 + \omega^2 + \omega^4 = 1 + \omega^2 + \omega = 0$.

Answer: 0

Q55. If ω is a cube root of unity, find $(1 + \omega)(1 + \omega^2)$. [2 Marks]

Solution:

1. $1 + \omega = -\omega^2$ and $1 + \omega^2 = -\omega$.
2. Product = $(-\omega^2)(-\omega) = \omega^3 = 1$.

Answer: 1

Type 8: Locus in the Argand Plane

Q56. Describe the locus of z if (i) $|z| = 3$ (ii) $|z - 2| = 1$. [2 Marks]

Solution:

1. Put $z = x + iy$. (i) $|z| = 3$ gives $x^2 + y^2 = 9$.
2. (ii) $|z - 2| = |(x - 2) + iy| = 1$ gives $(x - 2)^2 + y^2 = 1$.

Answer: (i) Circle with centre (0, 0) and radius 3. (ii) Circle with centre (2, 0) and radius 1.

Q57. Describe the locus of z if (i) $\text{Re}(z) = 3$ (ii) $\text{Im}(z) = -2$. [2 Marks]

Solution:

1. Put $z = x + iy$, so $\text{Re}(z) = x$ and $\text{Im}(z) = y$.
2. (i) $x = 3$. (ii) $y = -2$.

Answer: (i) The vertical line $x = 3$. (ii) The horizontal line $y = -2$.

Q58. Find the locus of z if $|z - 1| = |z + 1|$. [2 Marks]

Solution:

1. Squaring: $(x - 1)^2 + y^2 = (x + 1)^2 + y^2$.
2. Expand: $-2x = 2x$, so $4x = 0$, i.e. $x = 0$.

Answer: The imaginary axis (the line $x = 0$), the perpendicular bisector of the points 1 and -1 .

Q59. Describe the locus of z if $\arg z = \pi/4$. [2 Marks]

Solution:

1. $z = x + iy$ has argument $\pi/4$ when $y/x = \tan(\pi/4) = 1$, with $x > 0$ and $y > 0$.
2. So $y = x$ with $x > 0$.

Answer: A ray (half-line) $y = x$, $x > 0$, starting at the origin (origin excluded).

Exam Tips for Short Questions

- Always write the formula first, then substitute. Even a one-line formula earns a mark.
- In division, multiply by the conjugate of the denominator. Do not leave i in the denominator.
- Check the quadrant before writing the argument. Give the principal value between $-\pi$ and π .
- Reduce powers of i by the remainder on division by 4, and powers of ω by the remainder on division by 3.
- Write the final answer clearly in the form $a + ib$, or in the polar form asked for.